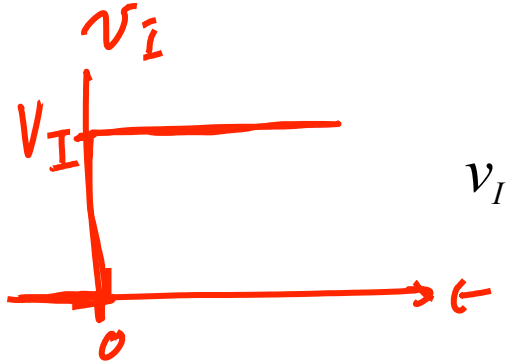
6.002x

CIRCUITS AND ELECTRONICS

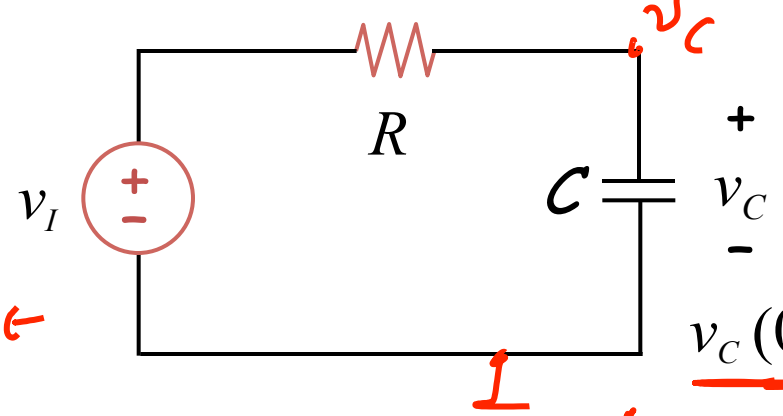
Digital Circuit



Review



$t=0$



v_C

$t=0^-$ $t=0$ $t=0^+$

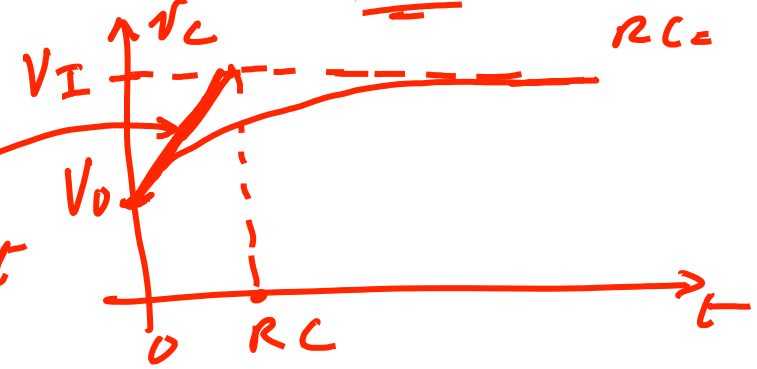
$v_C(0^-) = V_0$

$v_C(0) = V_0$ $t=0^-$ $t=0^+$

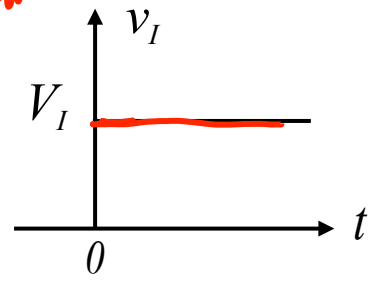
$$v_C = V_I + (V_0 - V_I)\, e^{\frac{-t}{RC}} \quad \text{(1)}$$

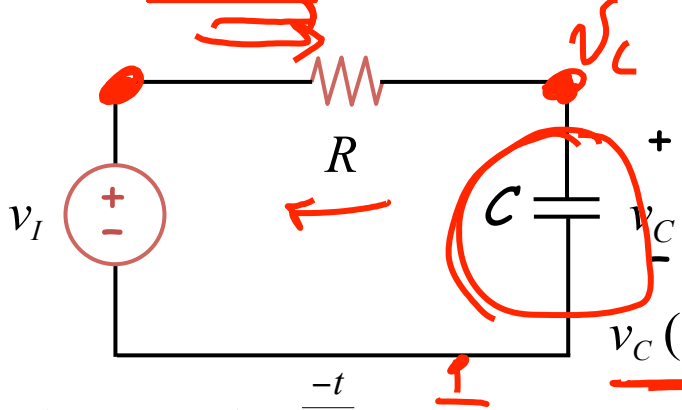
RC = time const.

Time const τ = RC is slope at this point



An Intuitive Approach

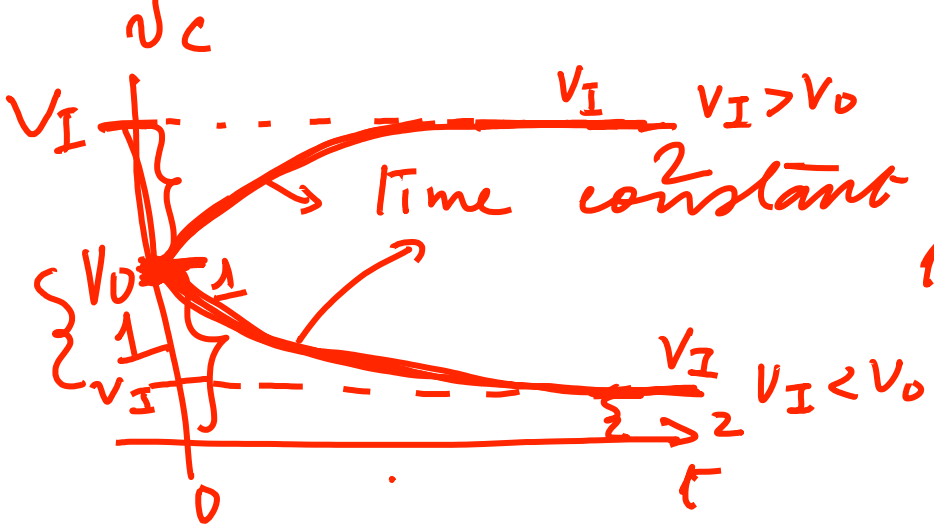




$$a\frac{dv}{dt} + bv = c$$

$$v_C(0) = V_O$$

$$v_C = V_I + (V_O - V_I)\, e^{\frac{-t}{RC}} \qquad (1)$$



the same

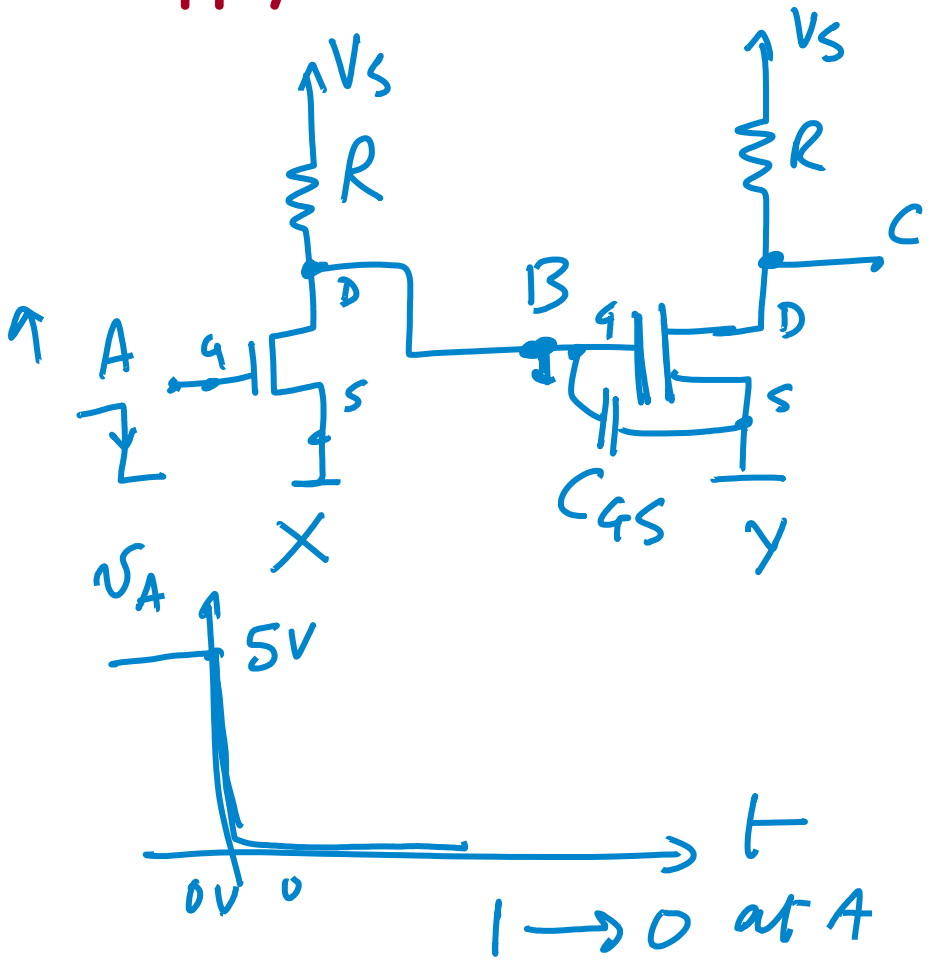
$\tau = RC$

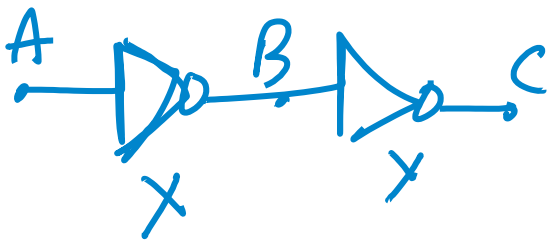
$$v_C = V_O + (V_I - V_O)\left(1 - e^{\frac{-t}{RC}}\right)$$

init value

$$v_C = V_I + (V_O - V_I)\, e^{\frac{-t}{RC}}\Big|_0 \cdot 1\left(1 - e^{\frac{-t}{RC}}\right)$$

Let's apply the result to an inverter





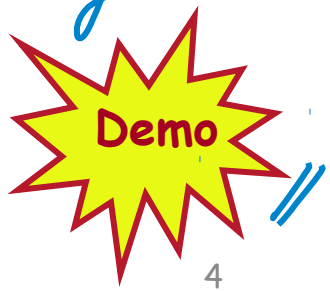
① B ↑ t_r rising delay

② B ↓ t_f falling delay

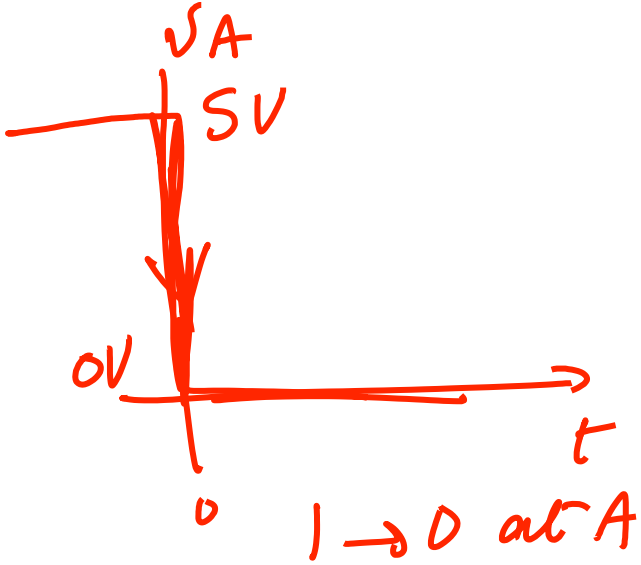
First, rising delay

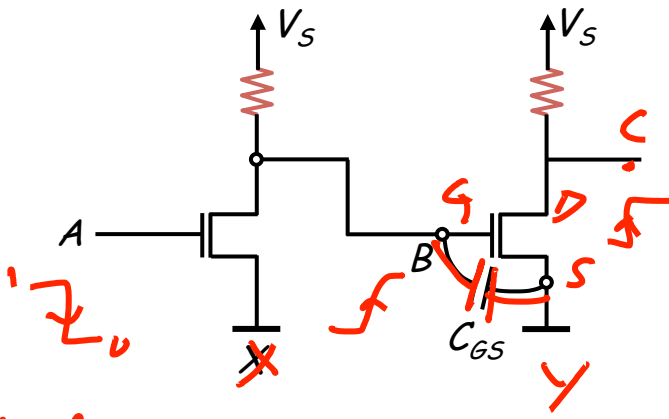
t_r at B

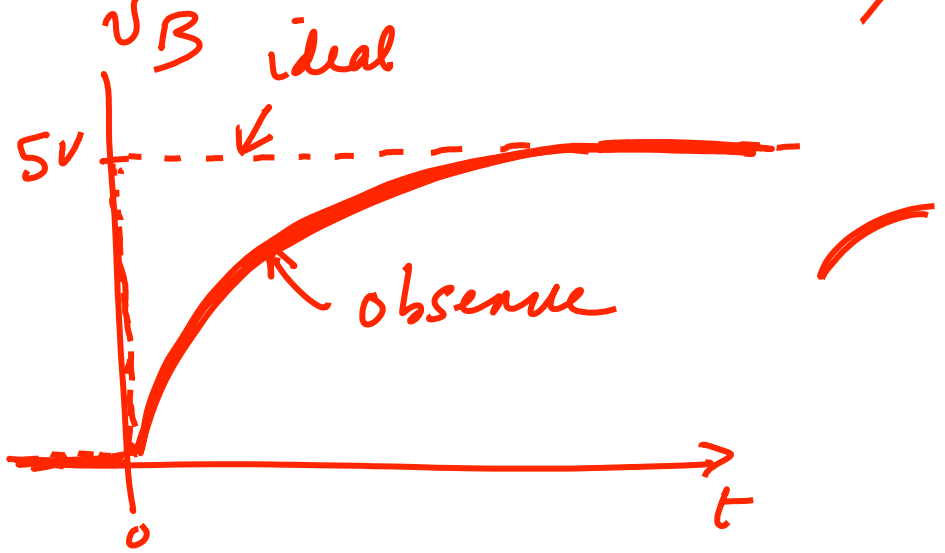




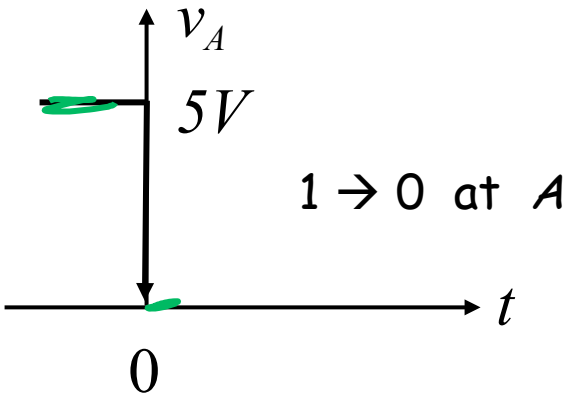
First, rising delay t_r at B

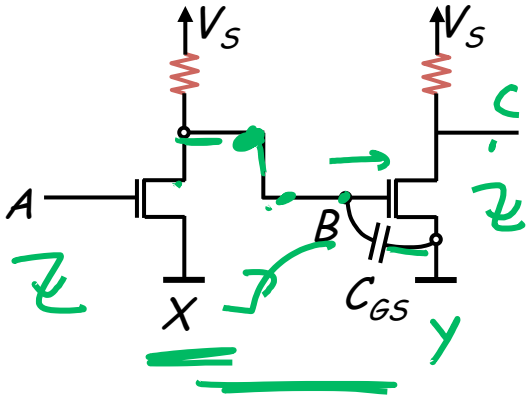


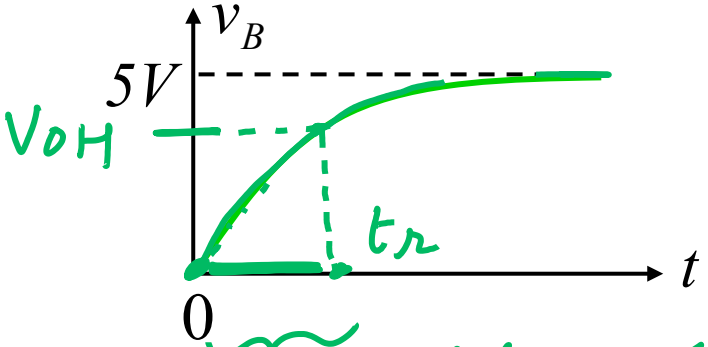




Rising delay t_r at B

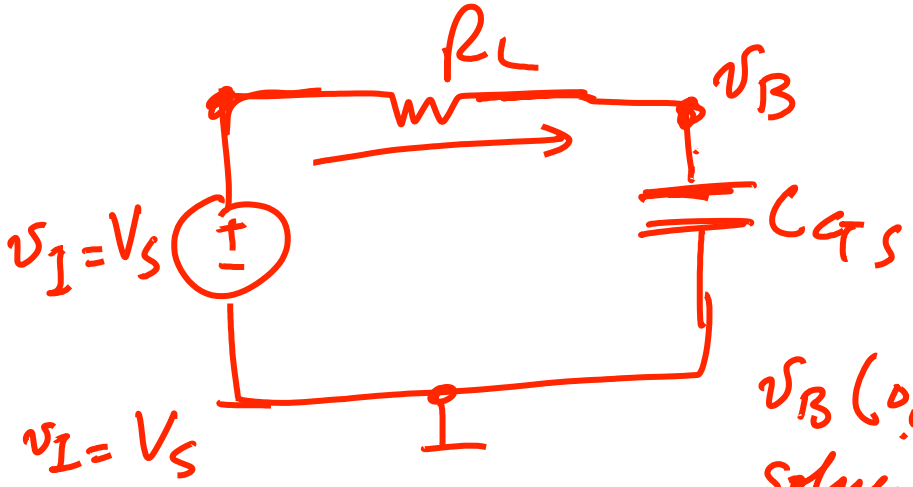


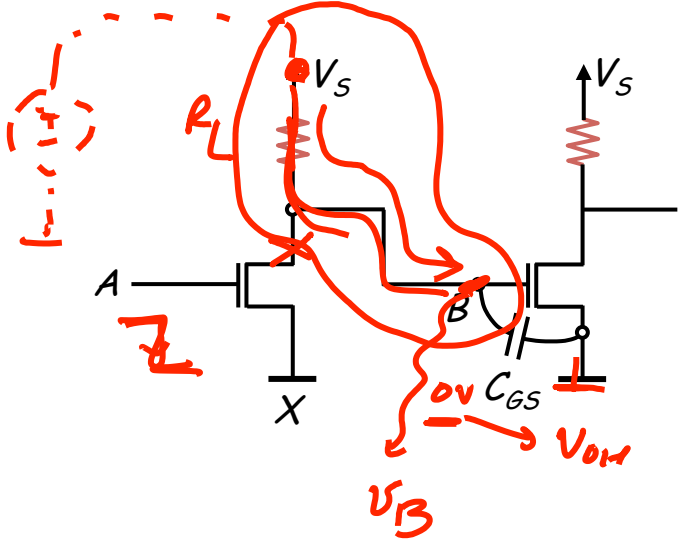




rising delay of 2nd inv. X

Equivalent circuit for 0→1 at *B*





$v_B(0) = \underline{0V}$

solve for $t \geq 0$

V_S

$$v_B = V_S\left(1 - e^{-\frac{t}{R_L C_{GS}}}\right)$$

0V

Now, we need to find t for which $v_B = V_{OH}$ $v_B: 0V \rightarrow V_{OH}$

Rising delay t_r at B

$$v_B = V_S - V_S e^{\frac{-t}{R_L C_{GS}}}$$

$$v_B = V_S\left(1 - e^{\frac{-t}{R_L C_{GS}}}\right)$$

Find t_r:

$$V_{OH} = V_S - V_S e^{\frac{-t_r}{R_L C_{GS}}}$$

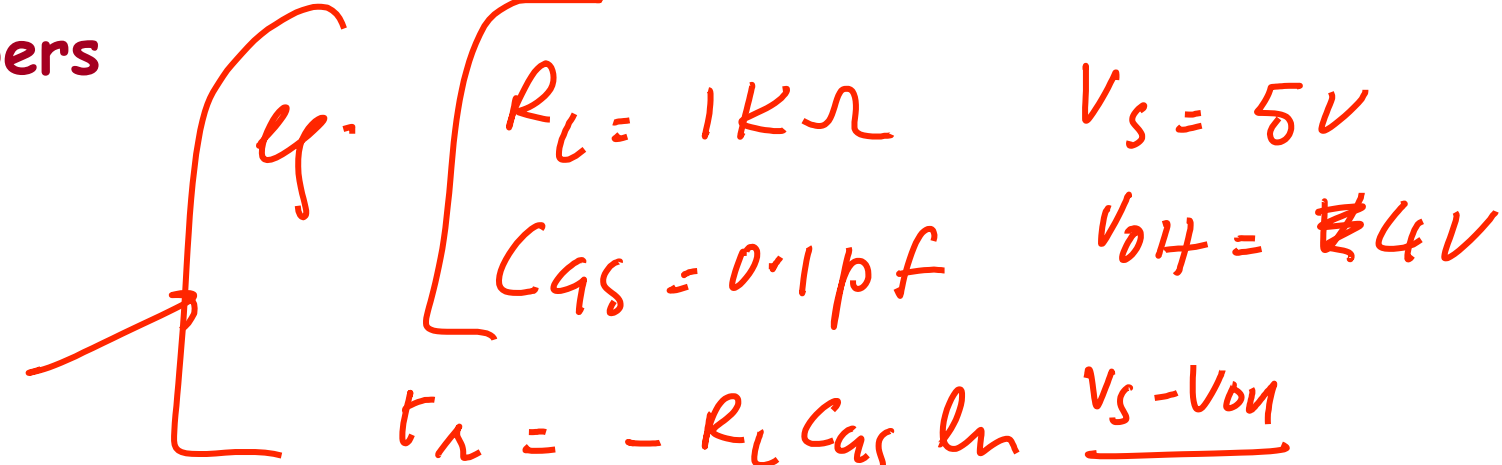
$$e^{-t_r/R_L C_{GS}} = \frac{V_S - V_{OH}}{V_S}$$

ln:

$$\frac{-t_r}{R_L C_{GS}} = \ln \frac{V_S - V_{OH}}{V_S}$$

$$t_r = -R_L C_{GS} \ln \frac{V_S - V_{OH}}{V_S}$$

Numbers



$$t_r = -R_L C_{GS} \ln \frac{V_S - V_{OH}}{V_S}$$

$$= -1\times10^{3} \times 0.1\times10^{-12} \ln \frac{5-4}{5}$$

$\sim 6\,GHz$ — $t_r = 0.16\,nS$

Compare with $R_L C_{GS} = 10^{3} \times 10^{-12} \times 0.1$

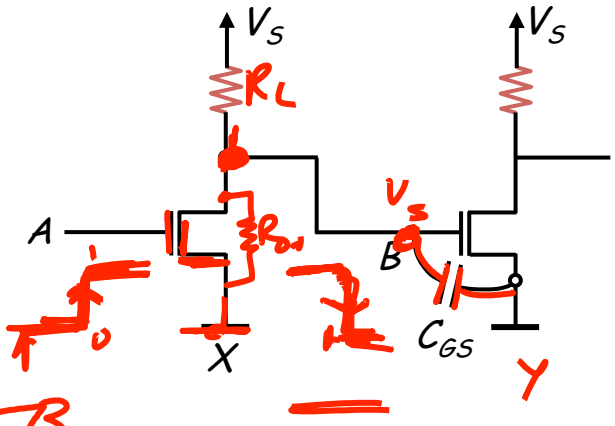
$= 0.1\,nS$

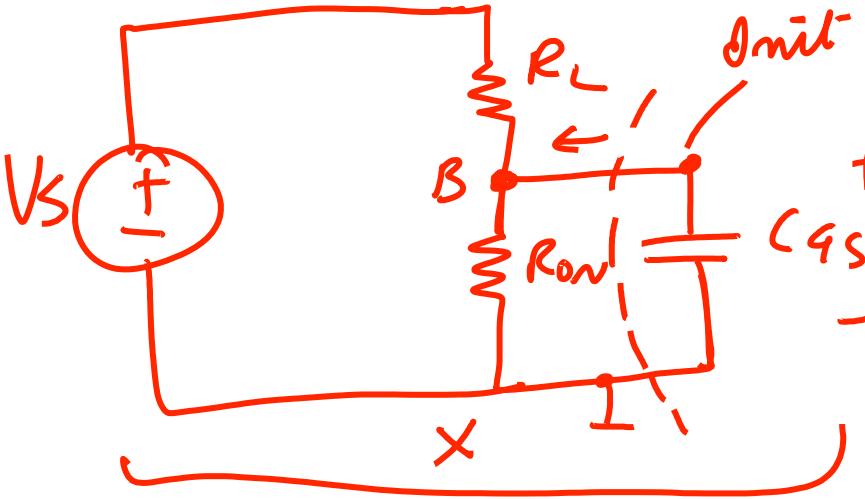
ID = 20 gate delays $\frac{6}{10}$ GHz

Falling Delay at B t_f

t_f : the time t for which v_B falls from V_S to V_{OL}

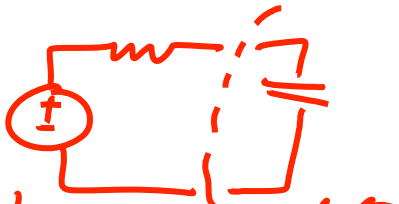
Equivalent ckt for 1→0 transition at B





Initial V_S

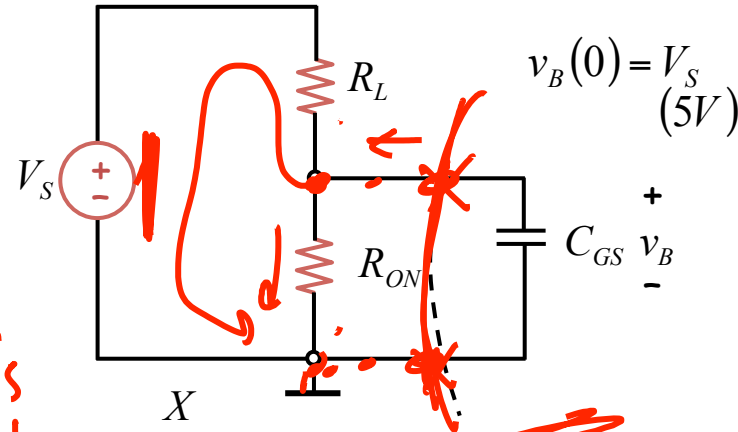
$v_B(0) = V_S$ (5V)



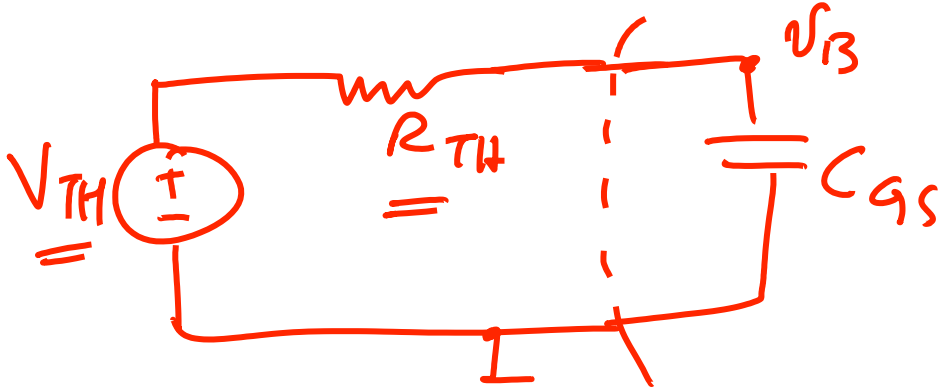
Std Pattern: Series RC ckt

Falling Delay at B t_f

Equivalent circuit for 1 → 0 at *B*



Thevenin Replacement



V_S ↓ V_{OL}

V_{OC}

2

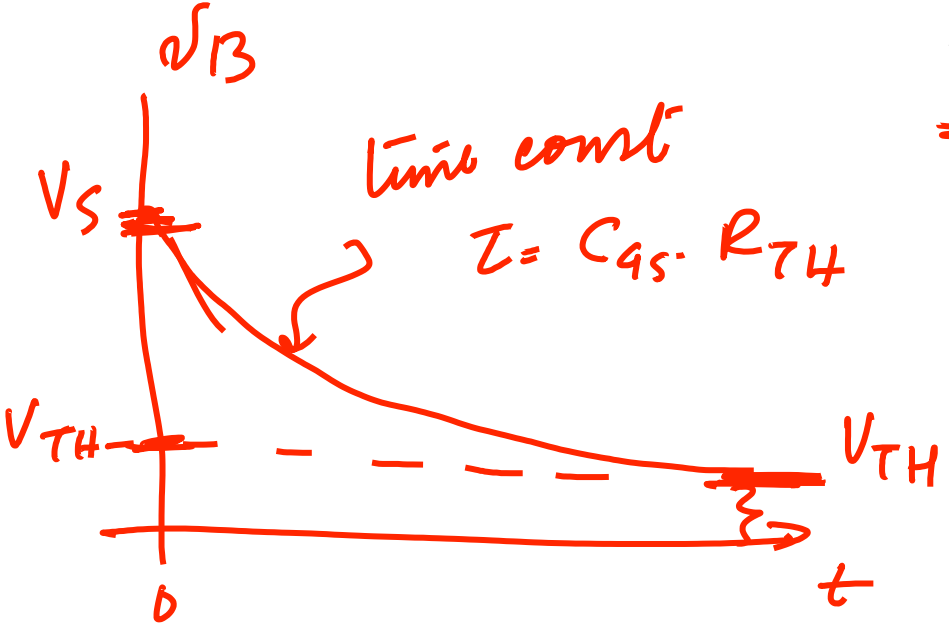
R_{TH}

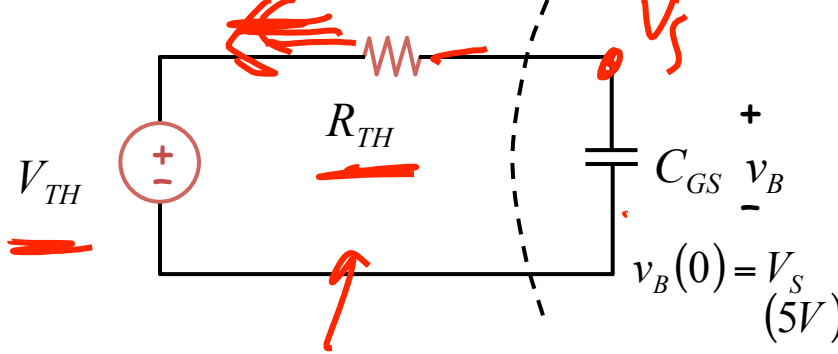
$$V_{TH} = V_S \cdot \frac{R_{ON}}{R_{ON}+R_L}$$

$$R_{TH} = R_L \| R_{ON} = \frac{R_{ON}R_L}{R_{ON}+R_L}$$

V_S ? $1-e^{-t/RC}$ $e^{-t/RC}$

Falling Delay at B t_f

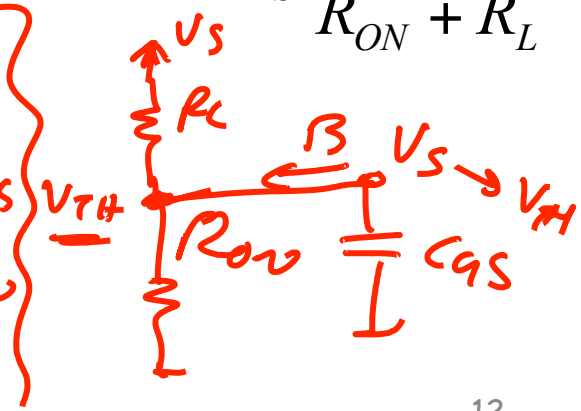




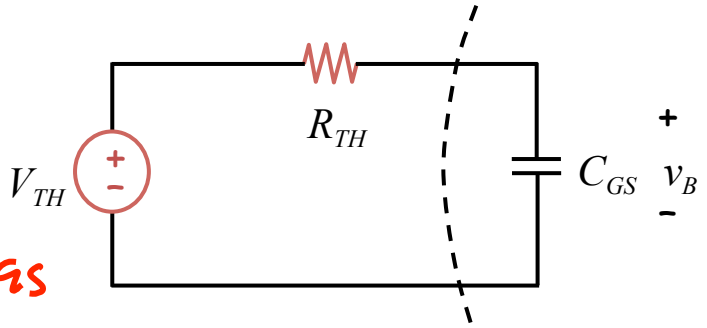
$$\rightarrow R_{TH} = R_L \parallel R_{ON}$$

$$\rightarrow V_{TH} = V_S \frac{R_{ON}}{R_{ON} + R_L}$$

$$v_B = V_{TH} + (V_S - V_{TH})\, e^{\frac{-t}{R_{TH} C_{GS}}}$$



Falling Delay at B t_f



From

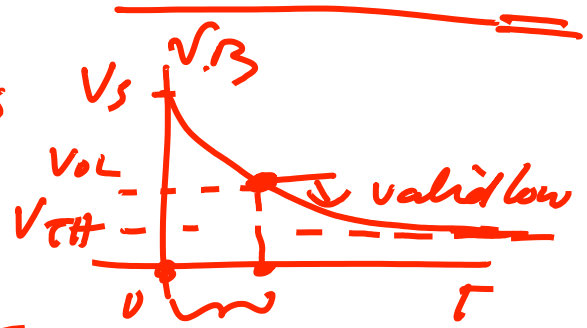
$$v_B = V_{TH} + (V_S - V_{TH})\, e^{\frac{-t}{R_{TH}C_{GS}}}$$

Falling delay t_f is the time t for which v_B falls from V_S to V_{OL}

$$V_{OL} = V_{TH} + (V_S - V_{TH})\, e^{\frac{-t_f}{R_{TH}C_{GS}}}$$

$$e^{-t_f/R_{TH}C_{GS}} = \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$

$$t_f = -R_{TH}C_{GS} \cdot \ln\frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$



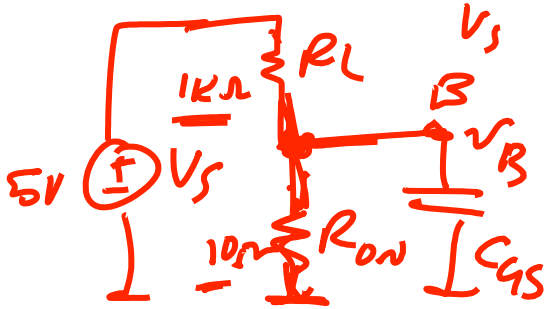
Numbers

eg. $R_L = 1k\Omega$ $V_S = 5V$ $R_{ON} = 10\Omega$

$C_{GS} = 0.1pF$ $V_{OL} = 1V$

$R_{TH} \approx 10\Omega$

$V_{TH} \simeq 0V$

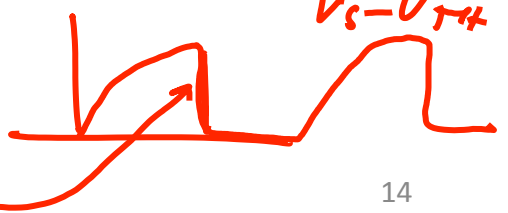


$$t_f = -10 \times 0.1 \times 10^{-12} \ln \frac{1-0}{5-0}$$

$$t_f = 1.6\,pS$$

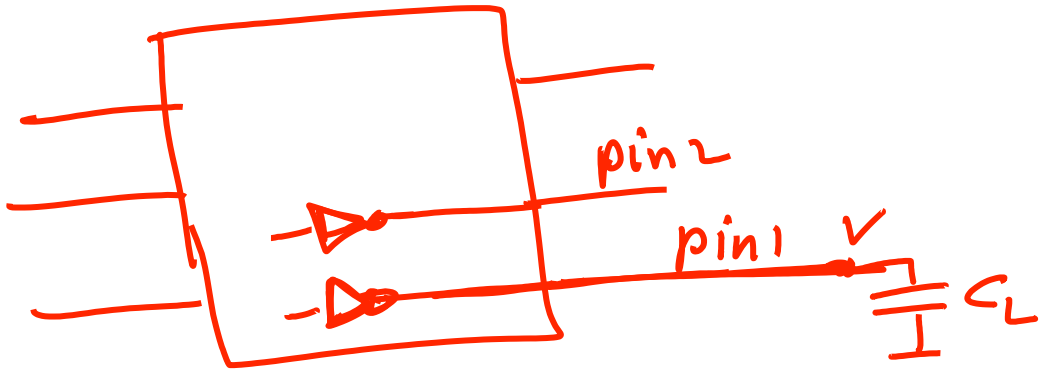
compare with $R_{TH} C_{GS} = 1\,pS$ ($10\Omega \sim 0.1pF$)

$\leq$ "$\frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$"

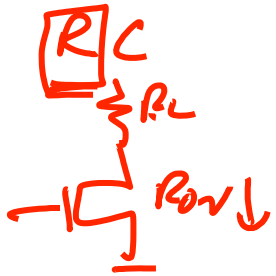


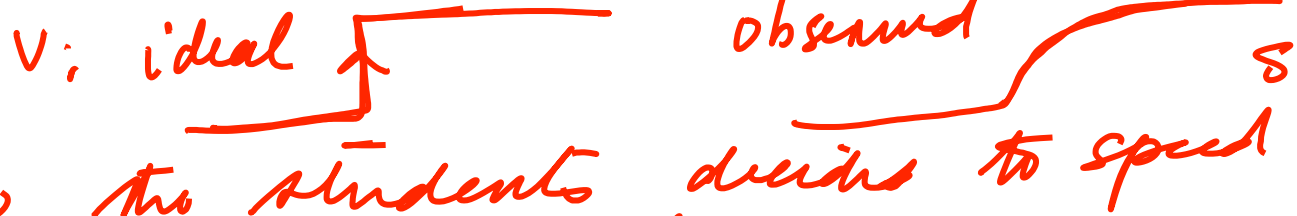
Slower may be better

Problem



t_r, t_f related to τ

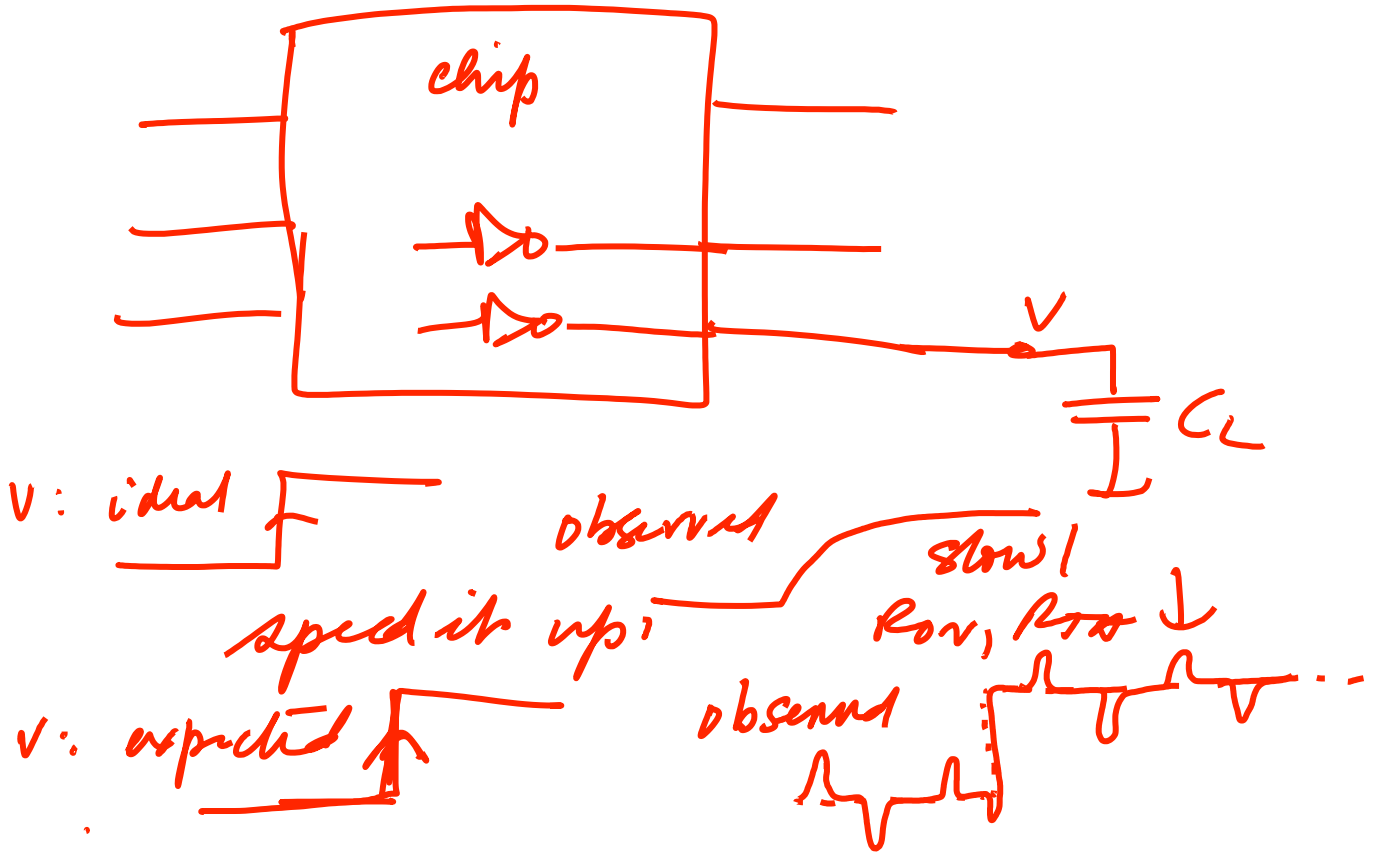




So the students decide to speed it up



Slower may be better

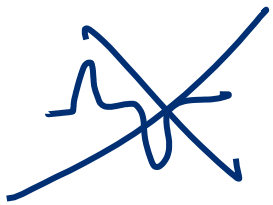


6.002 expert saw the solution!

└ said — make the transition slower

at V: 

Solved — no glitches



Demo