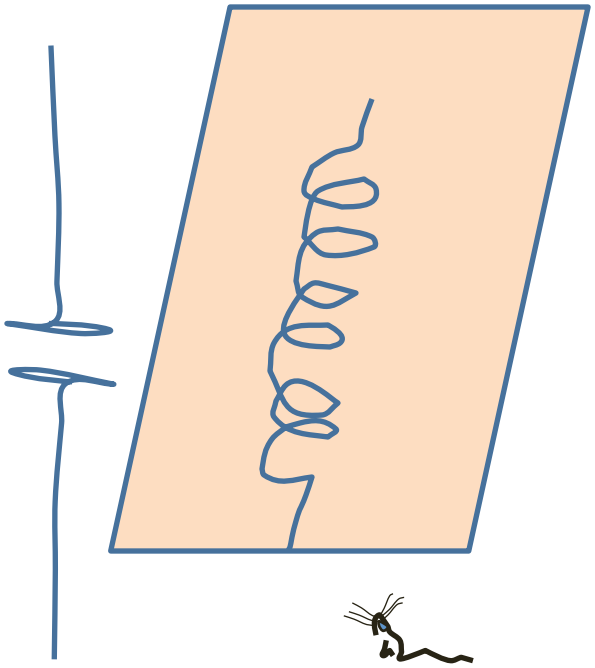
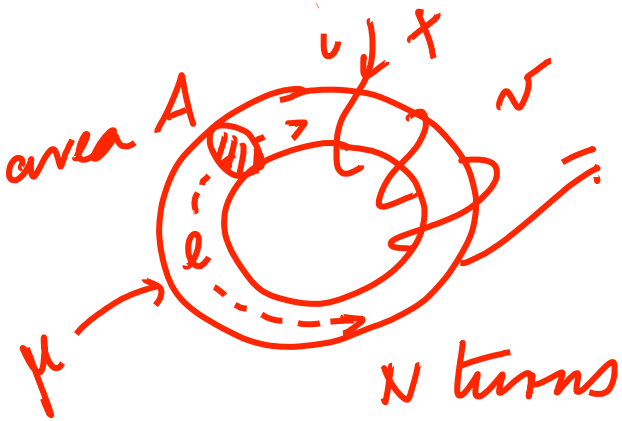
# 6.002x CIRCUITS AND ELECTRONICS

## Inductors and First-Order Systems

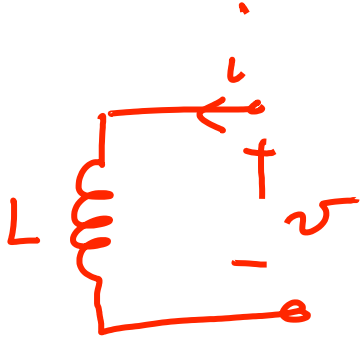


# Ideal Linear Inductor




$$L = \frac{\mu N^2 A}{l}$$

"$C = \frac{\varepsilon A}{d}$" capacitor




$\lambda$ = total flux linked

$\lambda = L i$ "$q = Cv$"

Webers — Inductance Henries — Amps — capacitor

Sec 9.1.2 of the textbook

# Ideal Linear Inductor




$\lambda = Li$

Inductors are linear

$v = \frac{d\lambda}{dt}$ → from Maxwell $\frac{d\Phi_B}{dt}$

$= \frac{d\,Li}{dt}$

$= L\frac{di}{dt}$

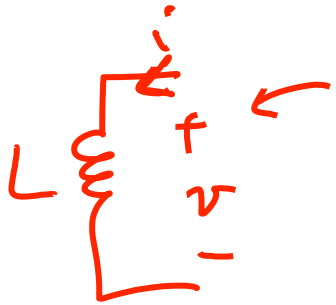
$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$

600Ω assumptions & L is time inv.

$i = C\frac{dv}{dt}$

note dual relations

# Energy Storage in Inductor




Inductors & capacitors do not dissipate energy — they store it, they can also supply it

$\lambda = Li$

$v = L\frac{di}{dt}$

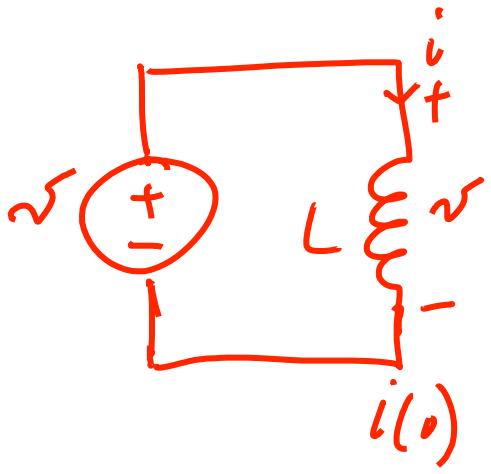
$$p = vi = i \cdot L\frac{di}{dt} = \frac{dLi^2}{dt}\frac{1}{2}$$

$$E = \frac{1}{2}Li^2$$

for cap $E = \frac{1}{2}CV^2$

An inductor stores energy → memory device — history matters.

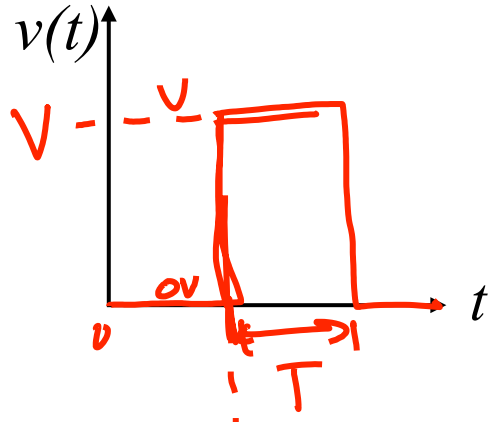
# Inductor and a Voltage Source




$i_y$

$i_x$

instantaneous open to an external change!



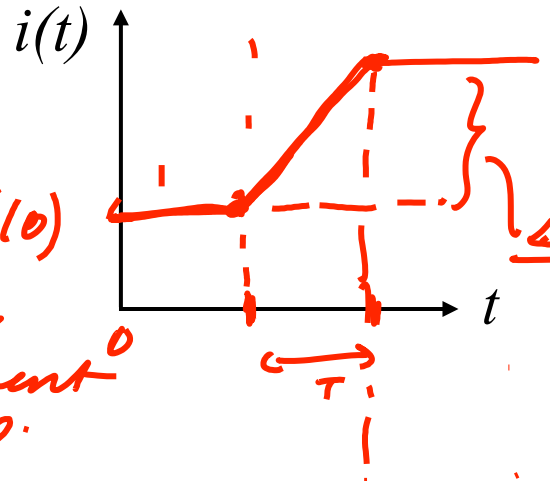

$v = L\frac{di}{dt}$

$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$

$= \frac{1}{L}\int_{-\infty}^{0} v\,dt + \frac{1}{L}\int_{0}^{t} v\,dt$

$= i(0) + \frac{Vt}{L}$

Inductors like to hold current the same — for continuous inputs, & for non infinite jumps in input, inductor current is continuous.




$\Delta q = IT$

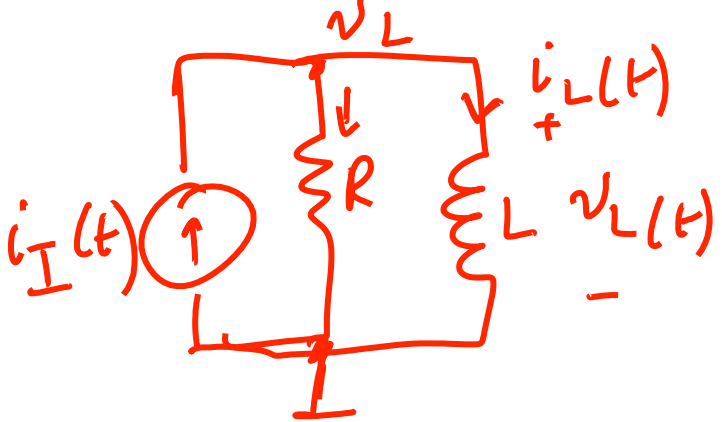
$\Delta\lambda = VT$

$\lambda = Li$

$v = \frac{d\lambda}{dt}$

$VT = \Delta\lambda$

# Analyzing an RL circuit




$$v_L = L\frac{di_L}{dt}$$

Node:

$$-i_I + \frac{v_L}{R} + i_L = 0$$

$$-i_I + \frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

$$\frac{L}{R}\frac{di_L}{dt} + i_L = i_I$$

$\frac{L}{R}$ → units of time

Time constant $\tau$ for RL circuit

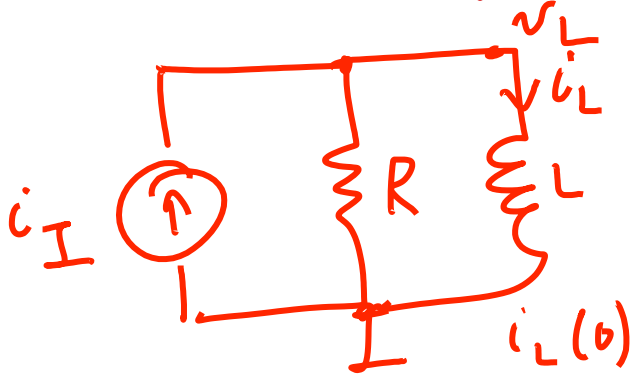
$$\tau = \frac{L}{R}$$

$$R\frac{dv_c}{dt} + v_c = v_I$$

$\tau$ → time const

$\tau \rightarrow RC$

# Let's do an example RL circuit



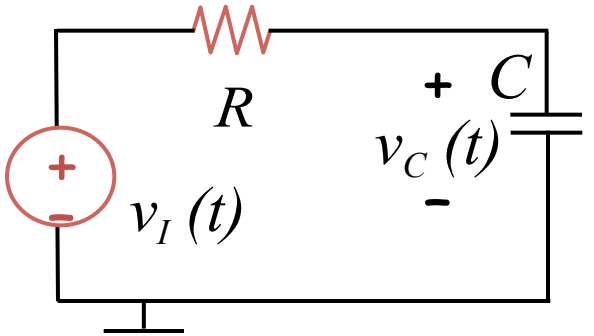

$i_I(t) = I_I$ ←

$i_L(0) = I_0$ given ←

$$\frac{L}{R}\frac{di_L}{dt} + i_L = I_I$$

→ time const. $\tau$

Capacitor



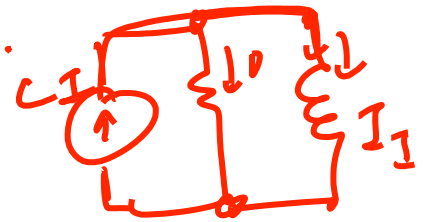

→ $v_I(t) = V_I$

→ $v_C(0) = V_0$

$$RC\frac{dv_C}{dt} + v_C = V_I$$

$\tau$

# Plot Solution

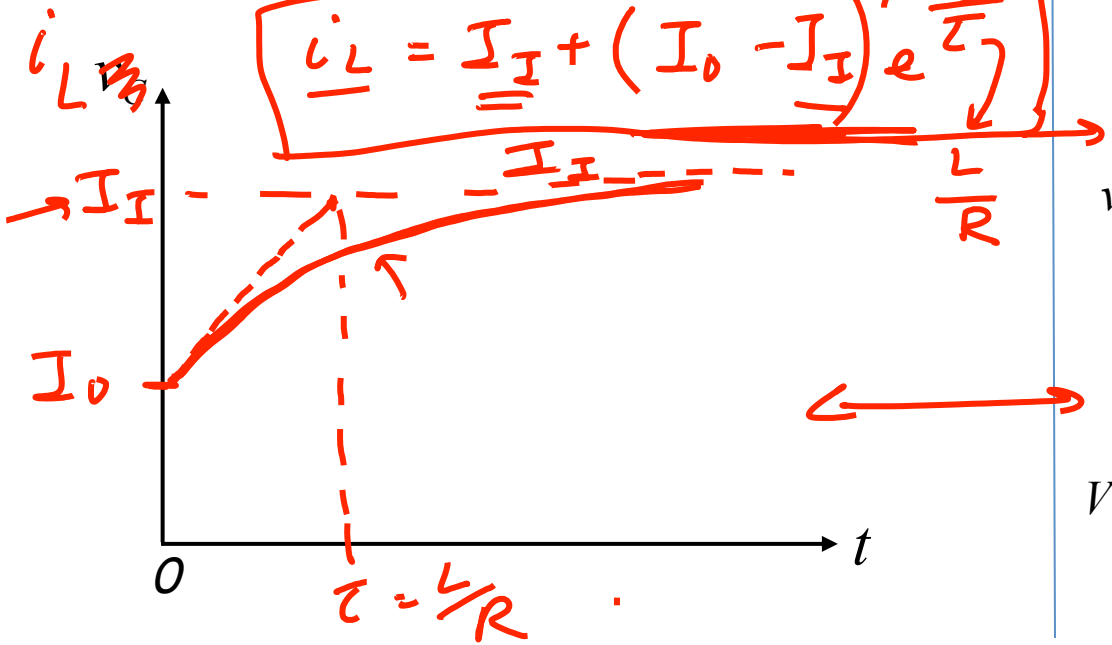


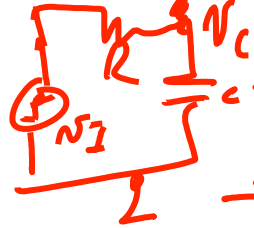

$i_I(t) = I_I$

$i_L(0) = I_0$

$\frac{L}{R}\frac{di_L}{dt} + i_L = I_I$

$i_L = I_I + (I_0 - I_I)\,e^{-\frac{t}{\tau}}$




$v_I(t) = V_I$

$v_C(0) = V_0$

$RC\frac{dv_C}{dt} + v_C = V_I$

$v_C = V_I + (V_0 - V_I)\,e^{\frac{-t}{RC}}$

