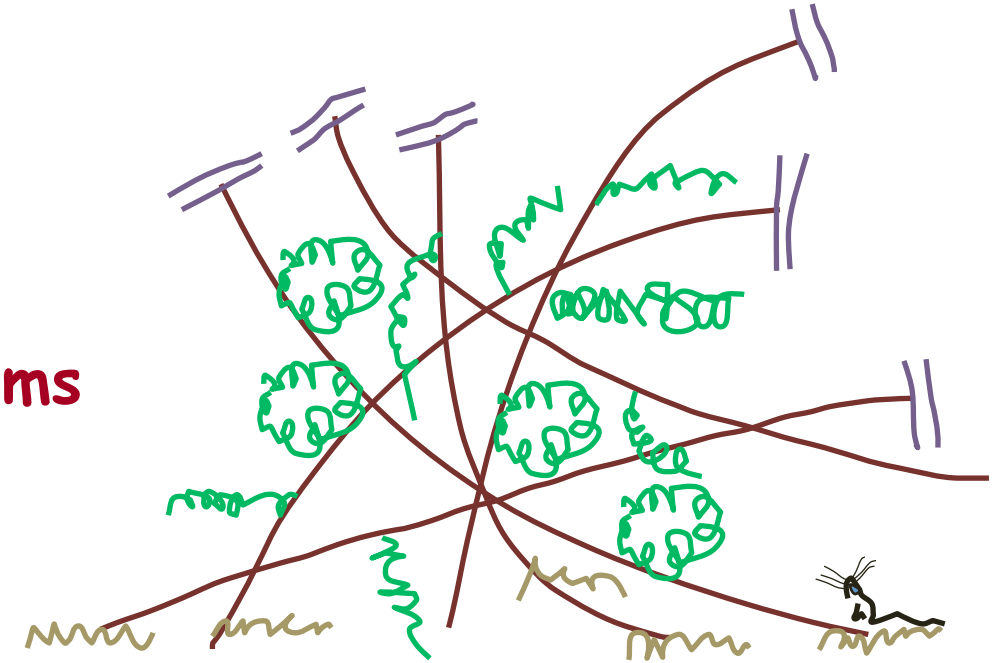
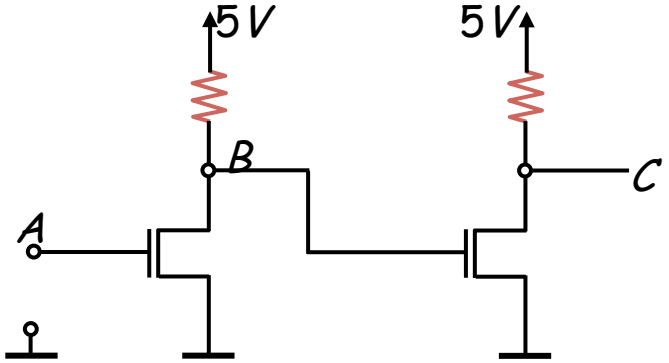
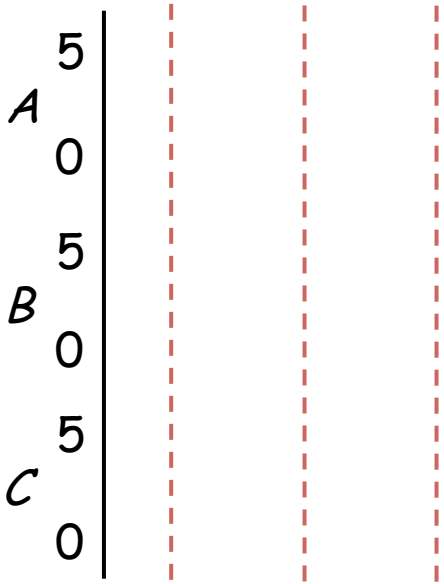
6.002x CIRCUITS AND ELECTRONICS

Capacitors and First-Order Systems



Motivation

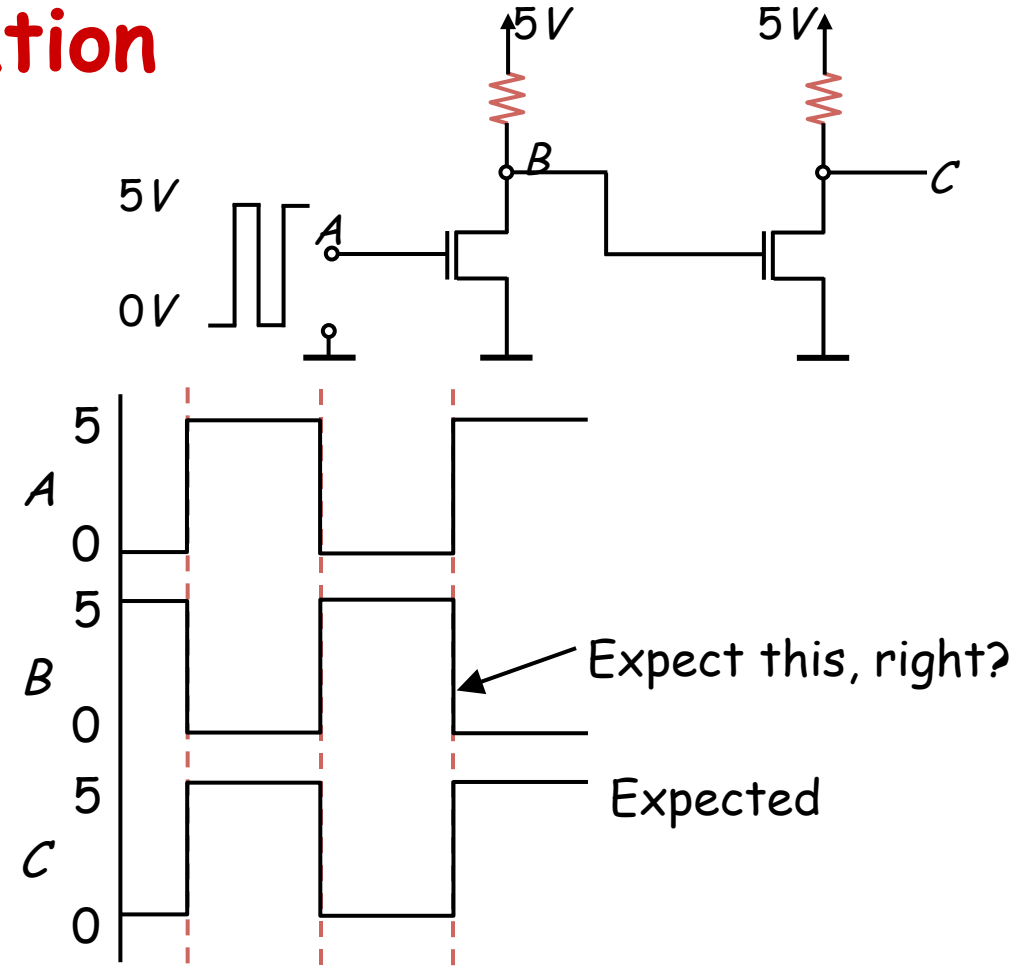




Demo

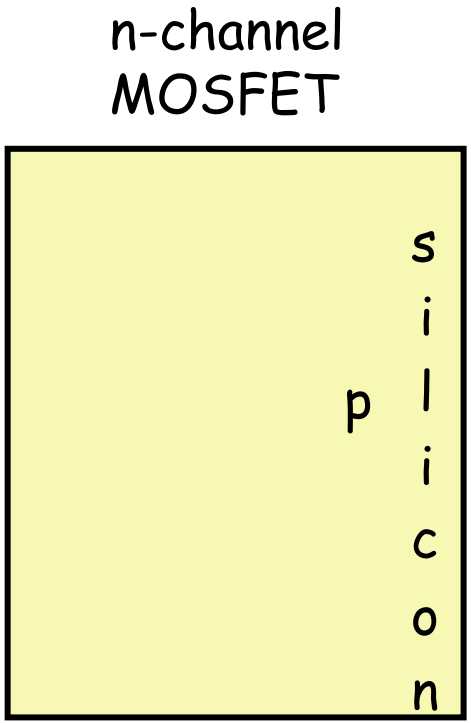
Chapters 9 & 10

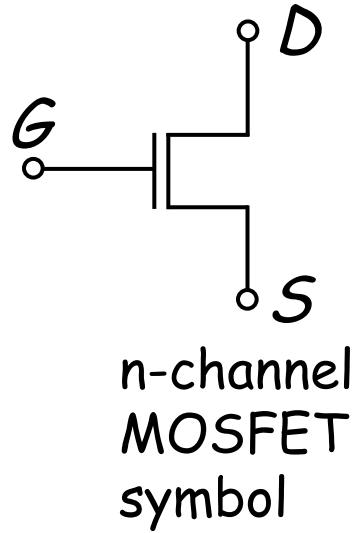
Motivation



Chapters 9 & 10

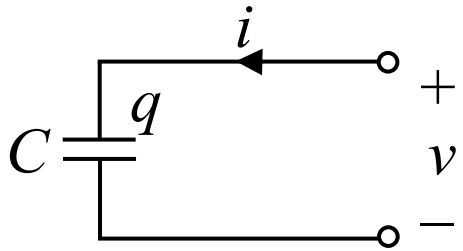
The Capacitor





Ideal Linear Capacitor

Ideal Linear Capacitor

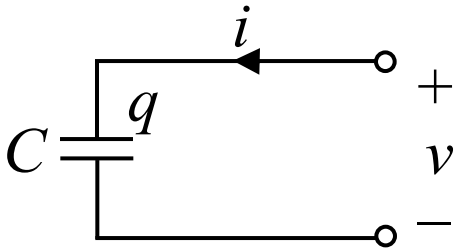


$$q = C\ v$$

Capacitors are linear devices*

*Verify for yourselves that capacitors satisfy both the properties of homogeneity and superposition

Ideal Linear Capacitor



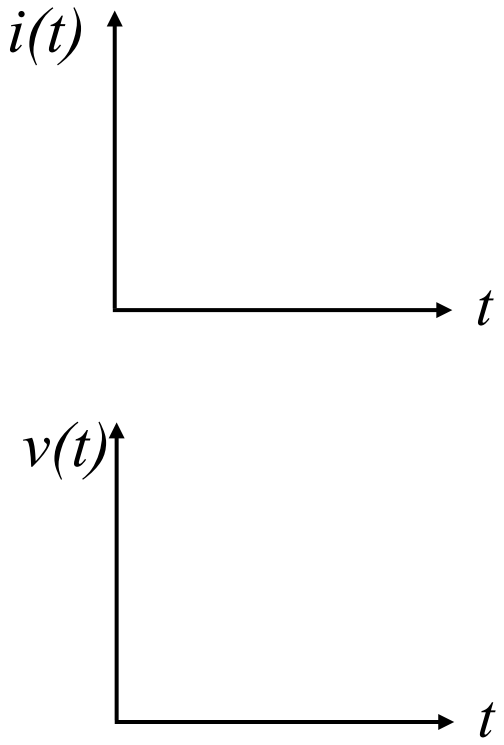
$$q = C\,v$$

$$i = C\frac{dv}{dt}$$

Demo

A capacitor is an energy storage device → memory device → history matters!

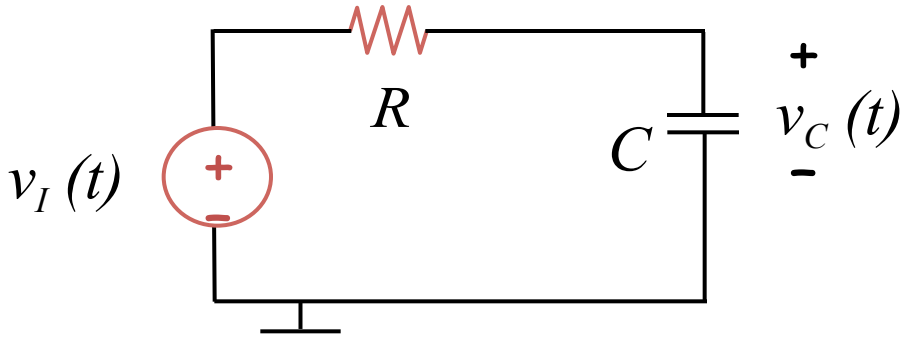
Capacitor and a Current Source



Analyzing an RC circuit

Circuits with one energy storage element are called first order circuits

Let's do an example RC circuit



How to solve the differential equation

Method of homogeneous and particular solutions:

1. Find the particular solution.
2. Find the homogeneous solution.
3. The total solution is the sum of the particular and homogeneous solutions. Use the initial conditions to solve for the remaining constants.

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \text{ given}$$

$$RC\frac{dv_C}{dt} + v_C = V_I$$



Solving differential equations is actually quite easy – commonly guesswork and applying patterns

① Particular solution

v_{CP} : any solution that satisfies the original equation ⊗

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \text{ given}$$

$$RC\frac{dv_C}{dt} + v_C = V_I$$

⊗

In general, use trial and error (guesswork!)

2 Homogeneous solution

v_{CH}: solution to the homogeneous equation (set drive to zero) (y)

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \text{ given}$$

$$RC\frac{dv_C}{dt} + v_C = V_I$$

(x)

③ Total solution

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \text{ given}$$

$$RC\frac{dv_C}{dt} + v_C = V_I$$



$$v_{CP} = V_I$$

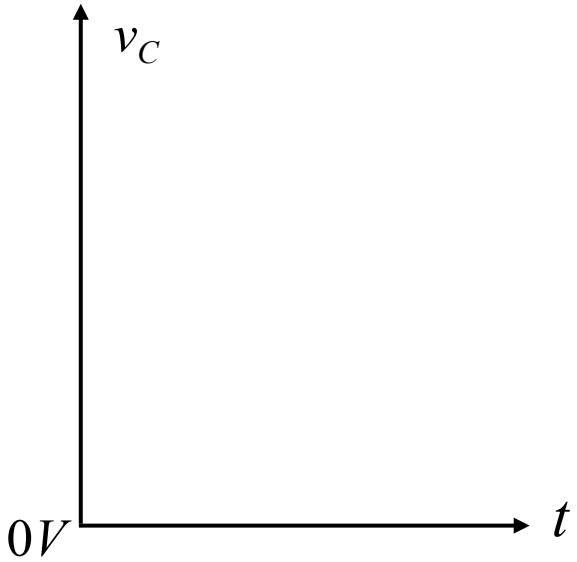
$$v_{CH} = Ae^{\frac{-t}{RC}}$$

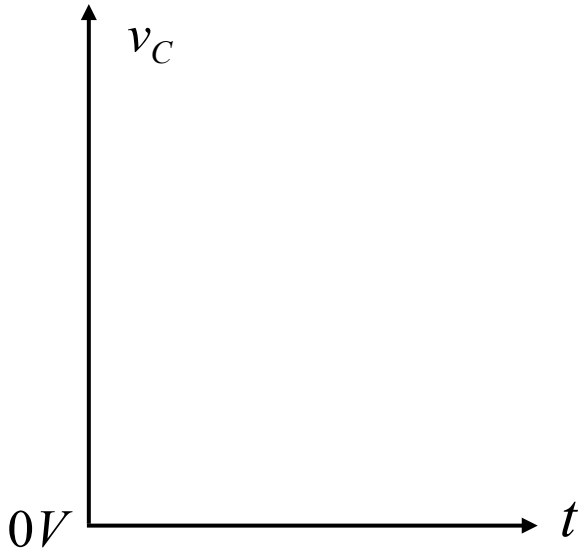
Plot Solution

$$v_C = V_I + (V_0 - V_I)\, e^{\frac{-t}{RC}}$$

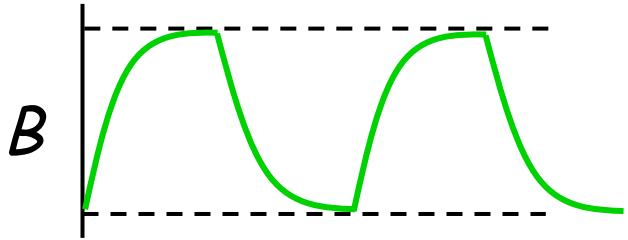


Examples





Remember demo



Demo