6.002x

CIRCUITS AND ELECTRONICS

Small Signal Circuits



Review:

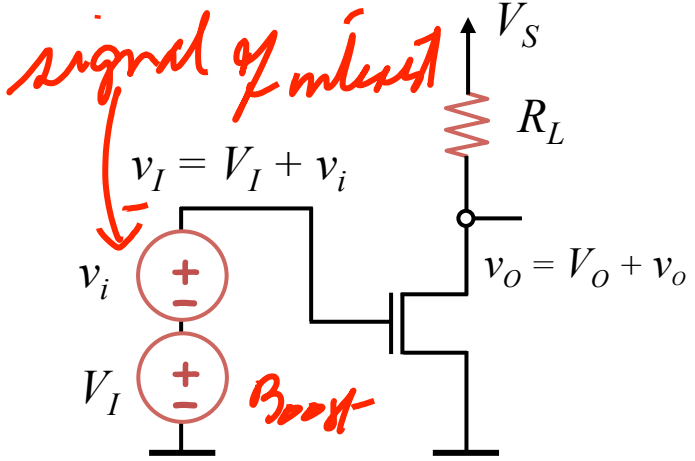
- Small signal notation

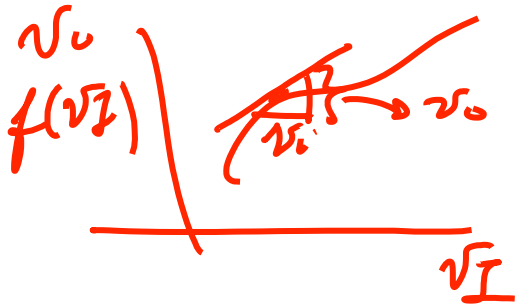
$$v_A = V_A + v_a$$

total — operating point — small signal

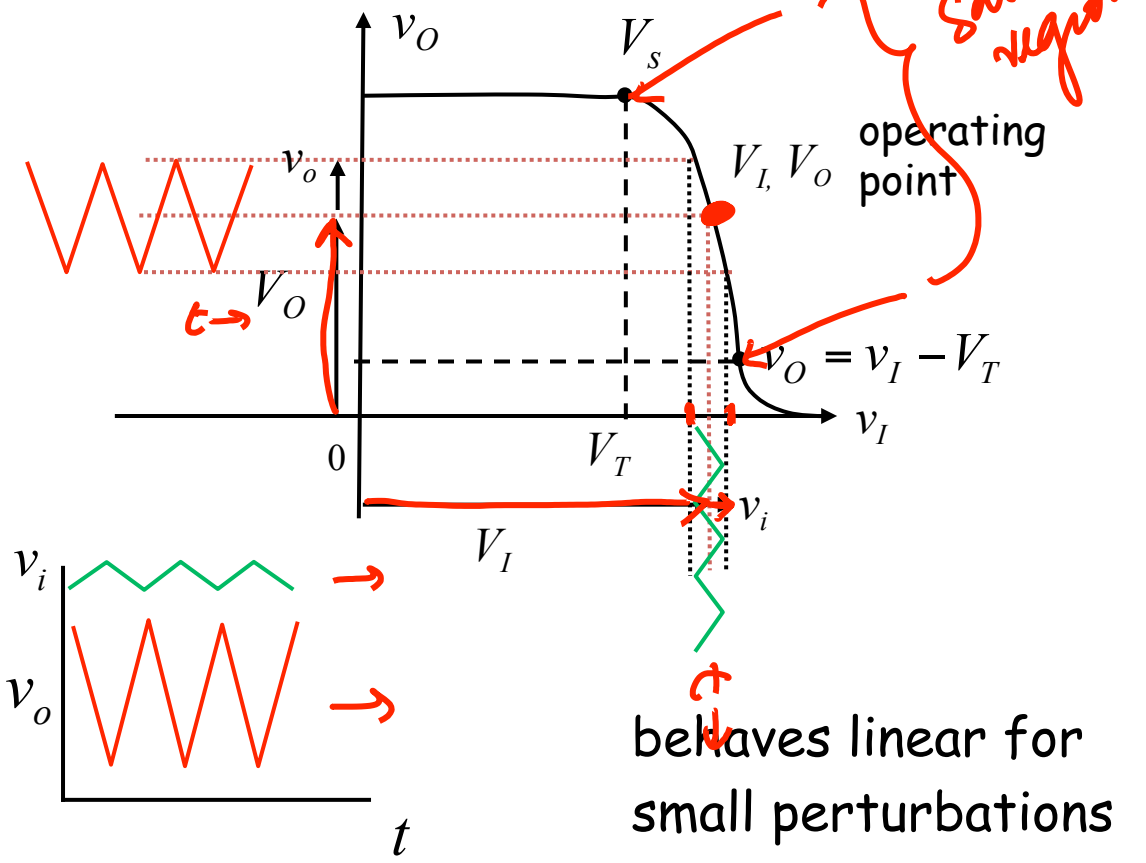
- $$v_{OUT} = f(v_I)$$

$$v_{out} = \frac{d}{dv_I} f(v_I)\Big|_{v_I = V_I} \cdot v_i$$

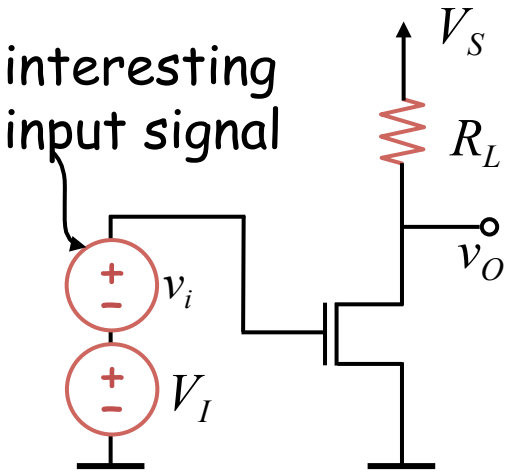




Review: I Graphical View



sat region



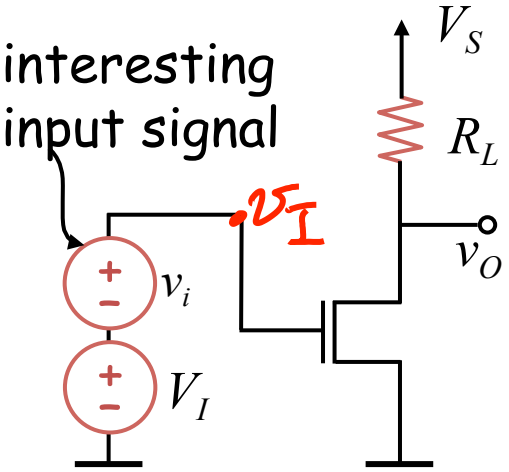
How to choose op pt?

behaves linear for small perturbations

Review: II Mathematical view

$$v_O = V_S - \frac{K(v_I - V_T)^2}{2} R_L$$

$v_O = f(v_I)$



$$v_o = \frac{d}{dv_I}\left[V_S - \frac{K}{2}(v_I - V_T)^2 R_L\right]\Bigg|_{v_I = V_I} \cdot v_i$$

at op. pt.

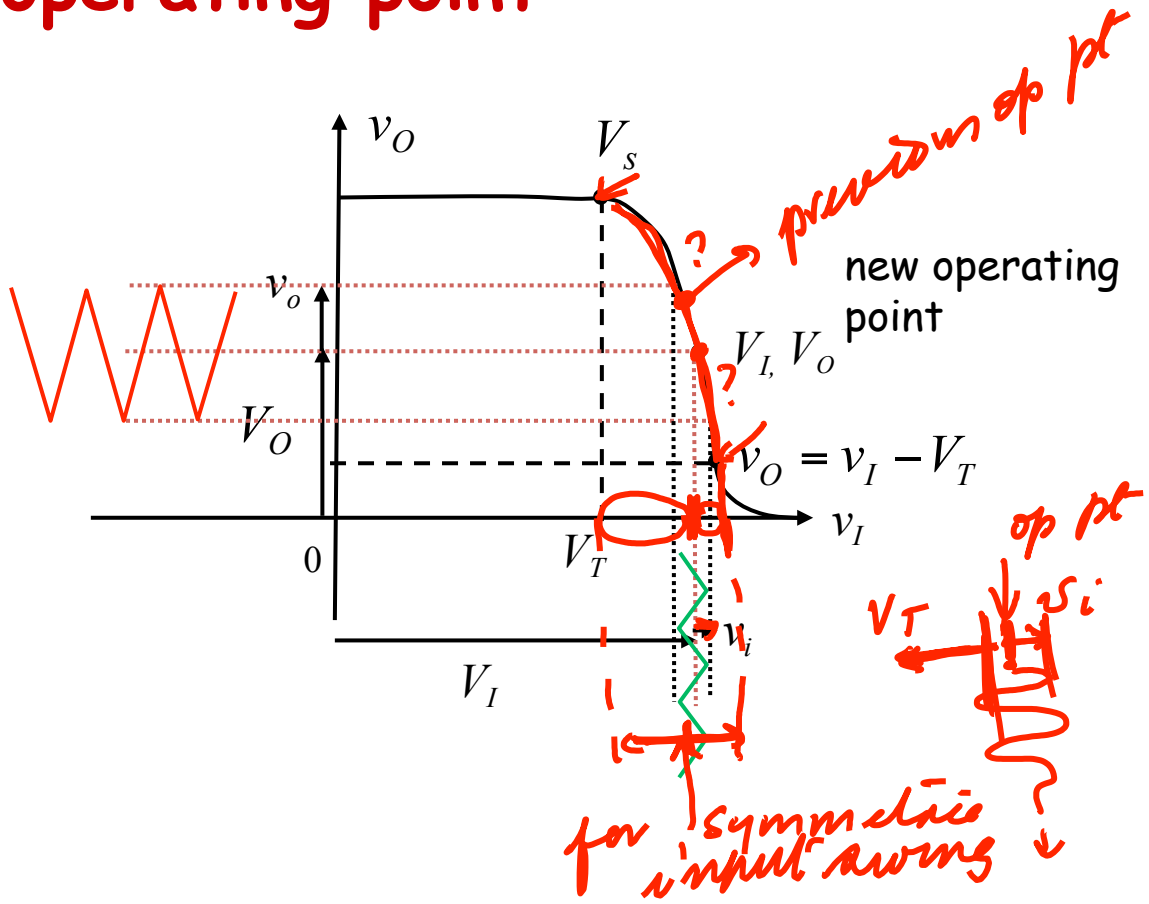
$$v_o = -K(V_I - V_T) R_L \cdot v_i$$

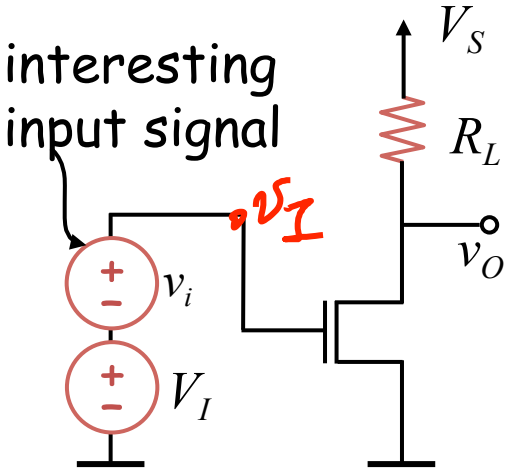
g_m

related to V_I

~~constant~~ for fixed DC bias

How to choose the bias point aka operating point





How to choose the bias point

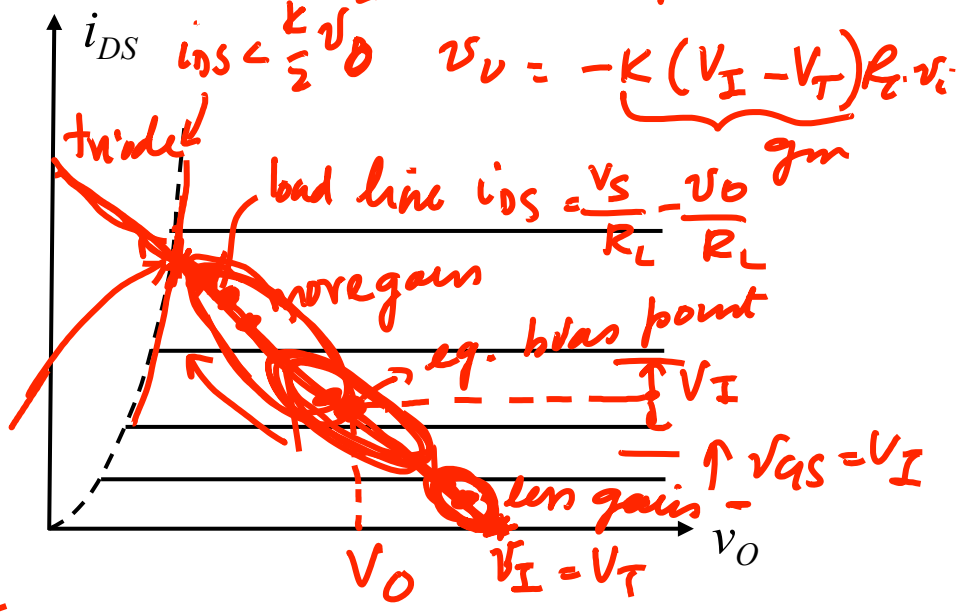
$$v_o = -K(V_I - V_T)R_L \cdot v_i$$

$$= g_m$$

1. Gain $g_m R_L \propto V_I$

2. Input swing and symmetry

Bias — select gain
— input swing



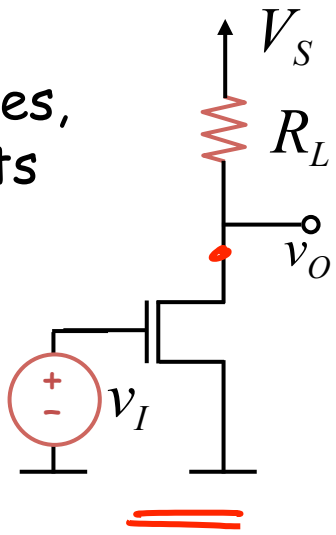
$v_O \geq v_I - V_T$

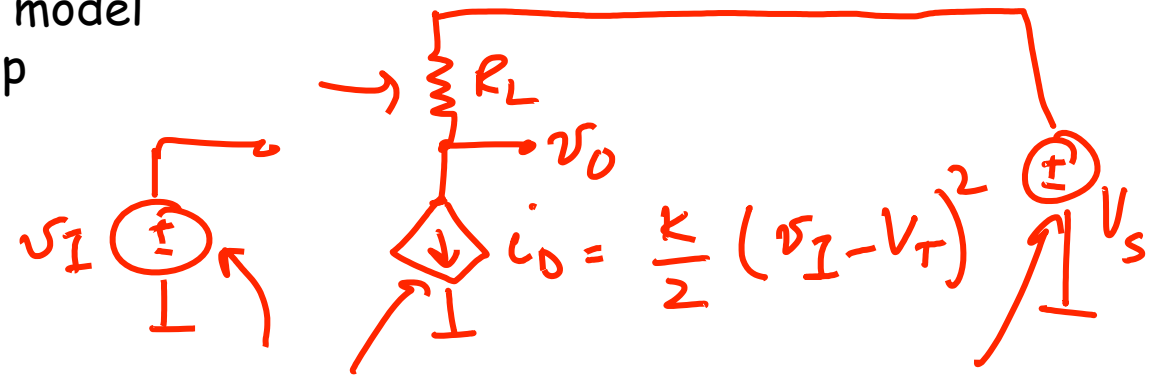
Demo

III The Small Signal Circuit View

We can derive small circuit equivalent models for our devices, and thereby conduct small signal analysis directly on circuits

e.g. large signal circuit model for amp





We can replace large signal element models with small signal element models in the same circuit topology

Textbook

Foundations: Section 8.2.1, and also at the end of this lecture sequence

Small Signal Circuit Analysis

1. Find operating point using DC bias inputs using large signal model.

$V_I \rightarrow V_O$
$\rightarrow I_{DS}$

2. Develop small signal (linearized) models for elements.

3. Replace original elements with small signal models to get the small signal circuit

Analyze resulting linearized circuit…

Key: Can use superposition and other linear circuit tools with linearized circuit!

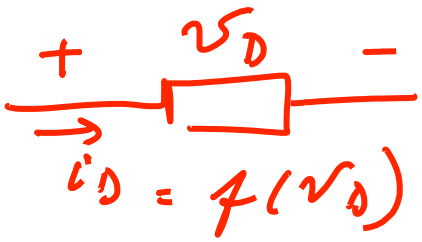
Small Signal Circuit Analysis

1. Find operating point using DC bias inputs using large signal model.

2. How to develop small signal (linearized) models for each of the elements around the operating point.

Large signal element model

Small signal model

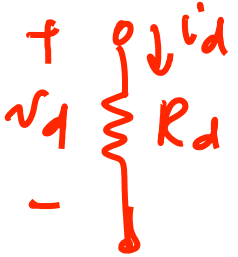




$$i_d = \left.\frac{\partial f(v_D)}{\partial v_D}\right|_{v_D = V_D} \cdot v_d$$

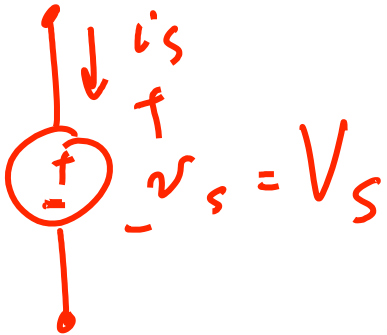
const. w.r.t. v_d

$$R_d = \frac{1}{\left.\frac{\partial f(v_D)}{\partial v_D}\right|_{v_D = V_D}}$$

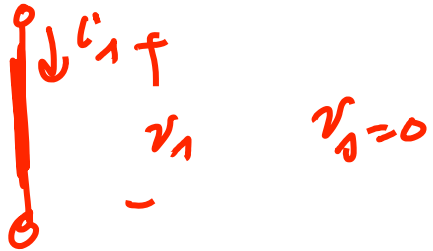


② Voltage Sources and DC Supply V_S

Large signal



Small signal

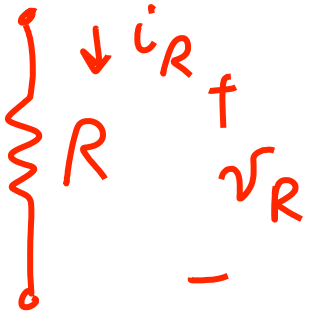


DC voltage source behaves as short to small signals.

DC current source behaves as open to small signals.

② Similarly, R

large signal

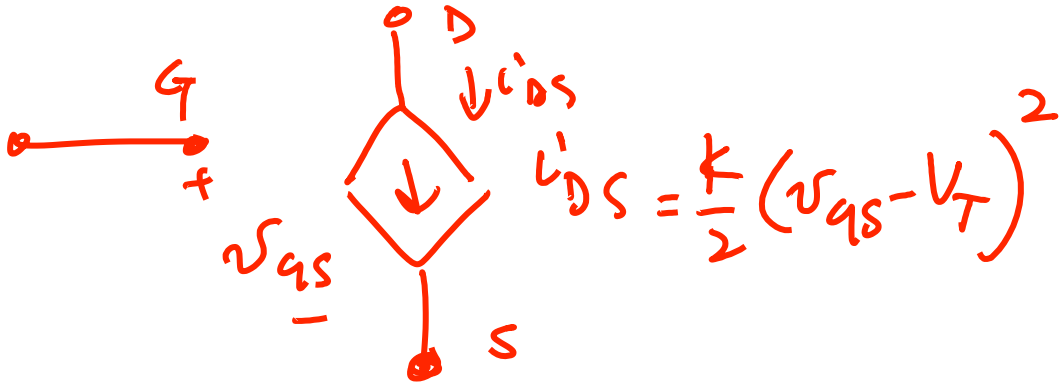


small signal



② MOSFET

Large signal



Small signal model ?

② MOSFET

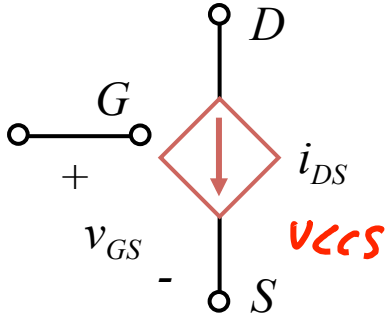
Small signal

LS: $i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$

i_{ds} $\qquad$ v_{gs}

$i_{ds} = \frac{\partial}{\partial v_{GS}}(\quad)\Big|_{v_{GS}=V_{GS}} \cdot v_{gs}$

large signal



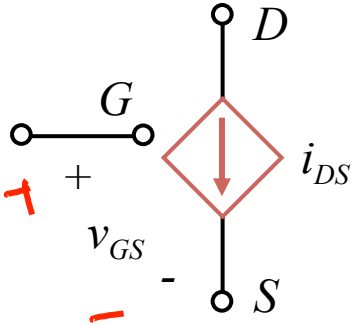
$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

② MOSFET

Small signal

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

large signal



$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

$$i_{ds} = \frac{\partial}{\partial v_{GS}}\left[\frac{K}{2}(v_{GS}-V_T)^2\right]\Big|_{v_{GS}=V_{GS}} \cdot v_{gs}$$

$$= \frac{K}{2} \cdot 2(V_{GS}-V_T)\Big|_{v_{GS}=V_{GS}} \cdot v_{gs}$$

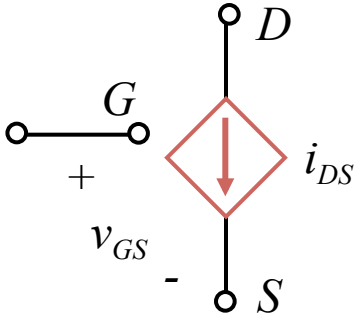
$$i_{ds} = K(V_{GS}-V_T)\cdot v_{gs} \Longrightarrow i_{ds} \text{ is linear in } v_{gs}$$

What is this device? 3T

Hint: It is not a resistor!

② MOSFET

Large signal

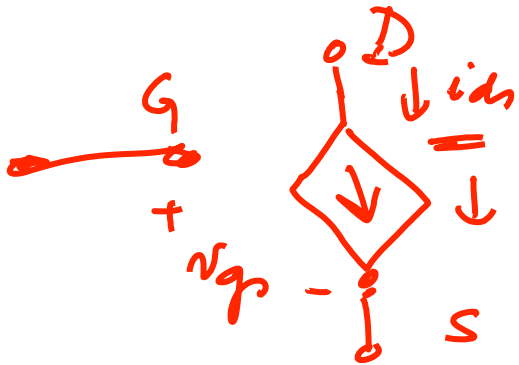


$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

small signal

$$i_{ds} = K(V_{GS} - V_T)v_{gs}$$

3T



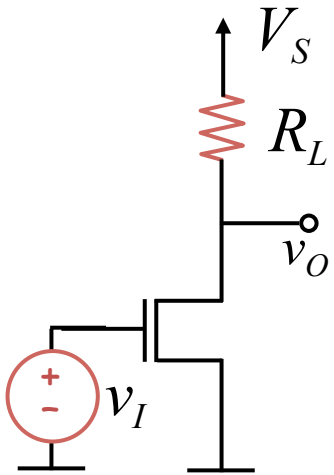
VCCS
Linear!

$i_{ds} = g_m \cdot v_{gs}$

transconductance

Amplifier example

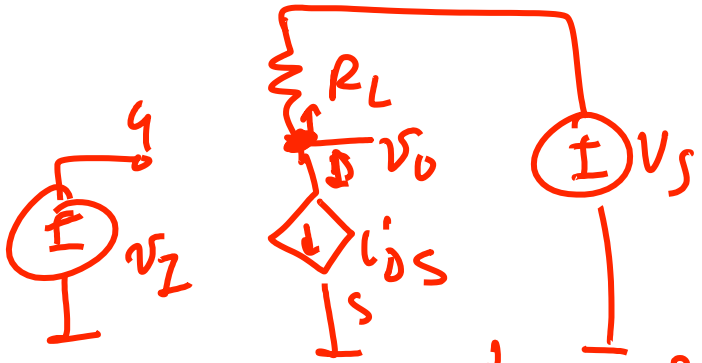
Let us perform a small signal analysis of our MOSFET amplifier using the small signal circuit method



Amplifier example: Step 1

Find operating point from large signal model

Large signal model

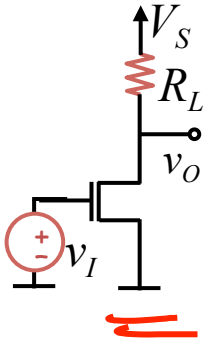


$$I_{DS} \rightarrow i_{DS} = \frac{k}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{k}{2}(v_I - V_T)R_L$$

$v_O \uparrow V_O$ $\quad$ $v_I \uparrow V_I$

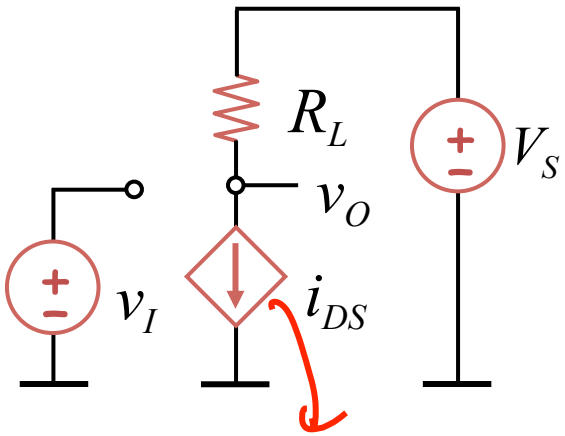
Small signal model



Amplifier example: Step 2

Find small signal models for elements

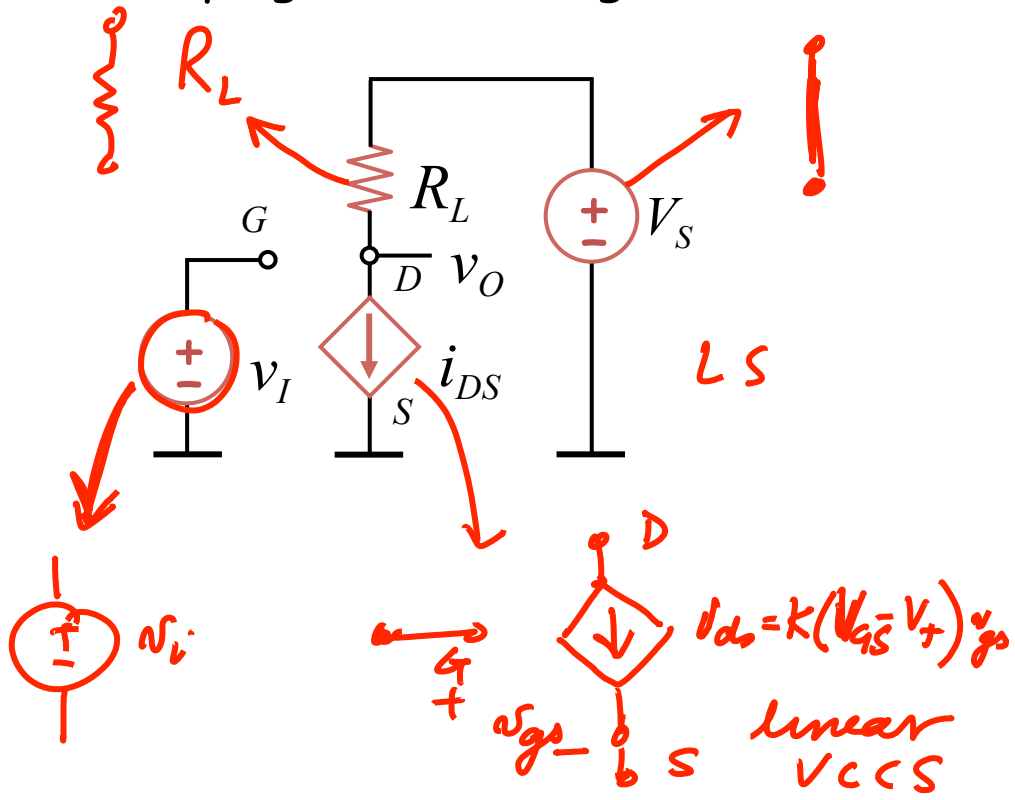
Large signal model



$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

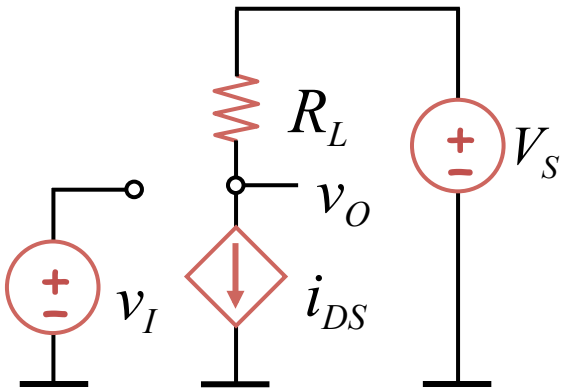
Developing the small signal model



Amplifier example: Step 3

Replace elements in original circuit with small signal models

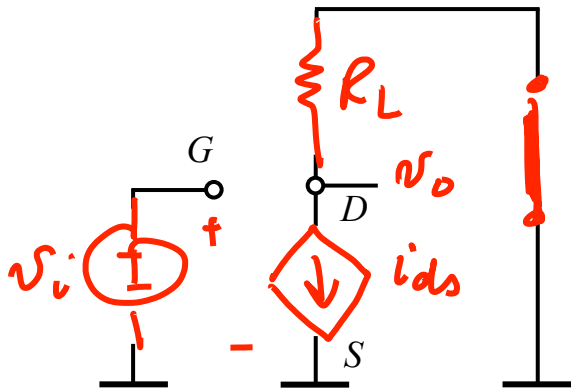
Large signal model



$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

Developing the small signal model



$$i_{ds} = K(V_{GS} - V_T)\, v_{gs}$$

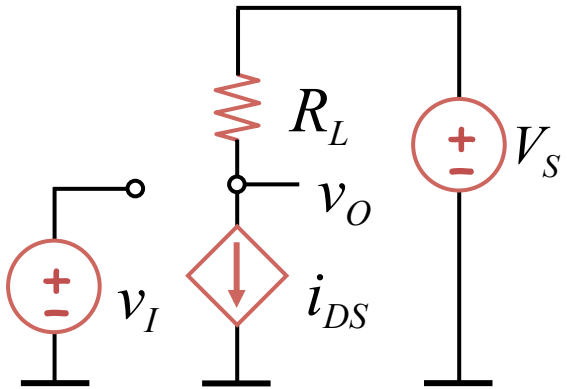
$v_{gs} = v_i$

$$i_{ds} = g_m v_i$$

Here is the SS circuit!

Amplifier example: Analyze resulting SS circuit

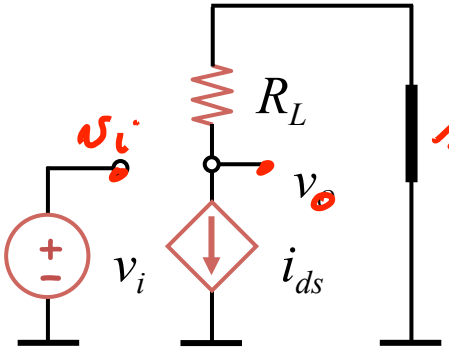
Large signal model



$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

Small signal model



short

V_{GS}

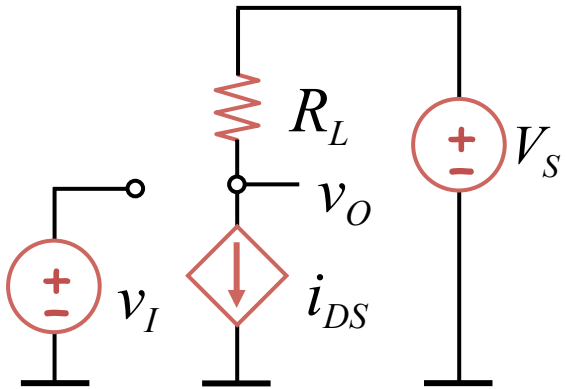
$$i_{ds} = K(V_I - V_T)\cdot v_i$$

node method:

$v_o = f(v_i)$?

Amplifier example: Analyze resulting SS circuit

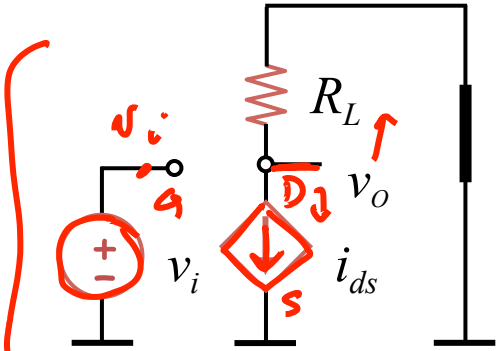
Large signal model



$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

Small signal model



Since small signal models are linear, our linear tools will now apply…

$$i_{ds} = K(V_I - V_T)\cdot v_i$$

Node method:

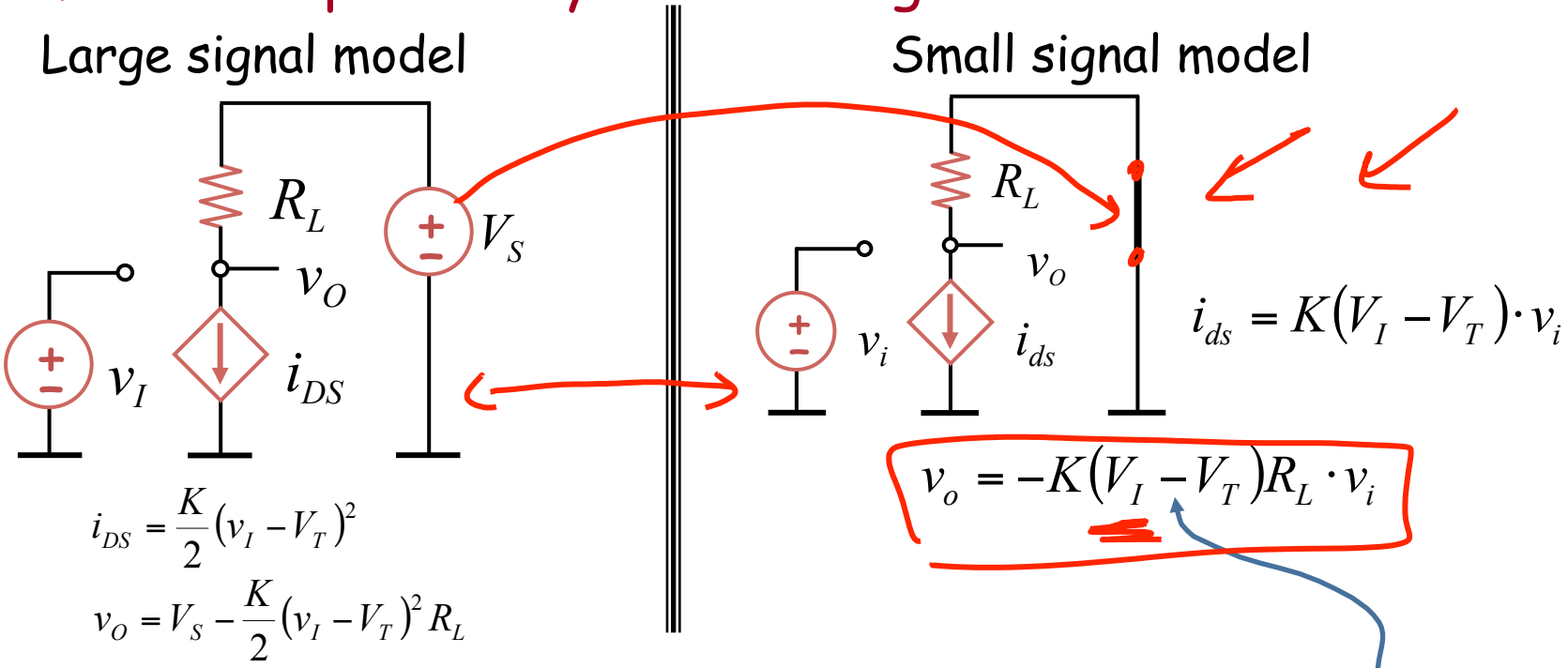
$$\frac{v_o}{R_L} + i_{ds} = 0$$

$$v_o = -i_{ds} R_L$$

$$v_o = -K(V_I - V_T)\cdot R_L \cdot v_i$$

$$v_o = -\underbrace{g_m R_L}_{\text{const}} \cdot v_i$$

Amplifier example: Analyze resulting SS circuit



Notice, first we need to find operating point voltages and currents.
Get these from large signal analysis

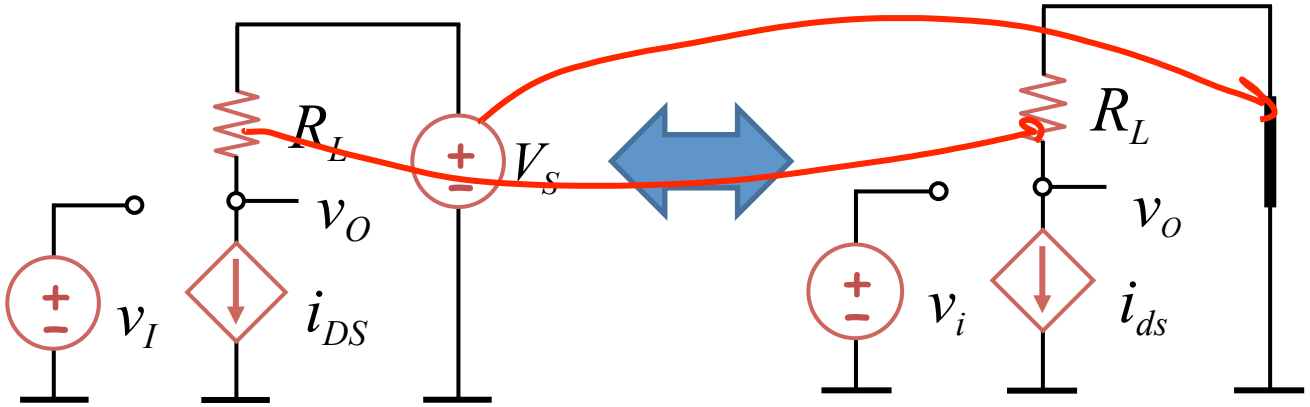
But, note that we can obtain insight into small-signal behavior directly from the small-signal circuit, without having to do a large signal analysis

The Small Signal Circuit View – A Perspective

To find relationship between small signal parameters of circuit:

(1) we replaced large signal device models with corresponding small signal device models in the original circuit

(2) then analyzed the resulting small signal circuit.



Why was this possible?

Also see section 8.2.1 of A&L

The Small Signal Circuit View – Foundations

- KVL, KCL applied to some circuit C yields:

$$\cdots + v_A + \cdots + v_{OUT} + \cdots + v_B + \cdots = 0 \quad (1)$$

- Replace total variables with operating point variables plus small signal variables

$$\cdots + (V_A + v_a) + \cdots + (V_{OUT} + v_{out}) + \cdots + (V_B + v_b) + \cdots = 0$$

- Operating point variables themselves satisfy the same KVL, KCL equations

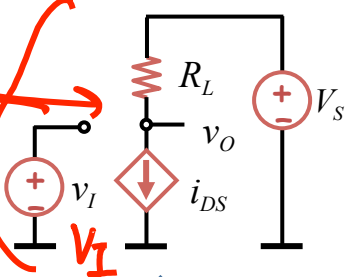
$$\cdots + V_A + \cdots + V_{OUT} + \cdots + V_B + \cdots = 0$$

- so, we can cancel them out, leaving

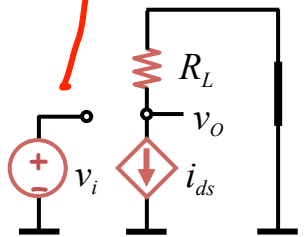
$$\cdots + v_a + \cdots + v_{out} + \cdots + v_b + \cdots = 0 \quad (2)$$

- But (2) is the same equation as (1) with small signal variables replacing total variables, so (2) must reflect same typology as in C, except that small signal models are used.

typology



$v_A = V_A + v_a$

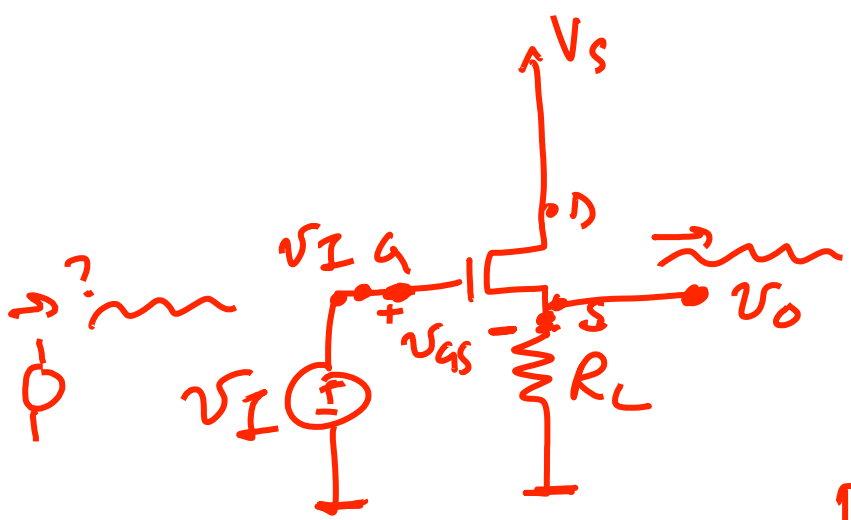


look at (1) & (2)

Caution: v_i vs v_{gs}

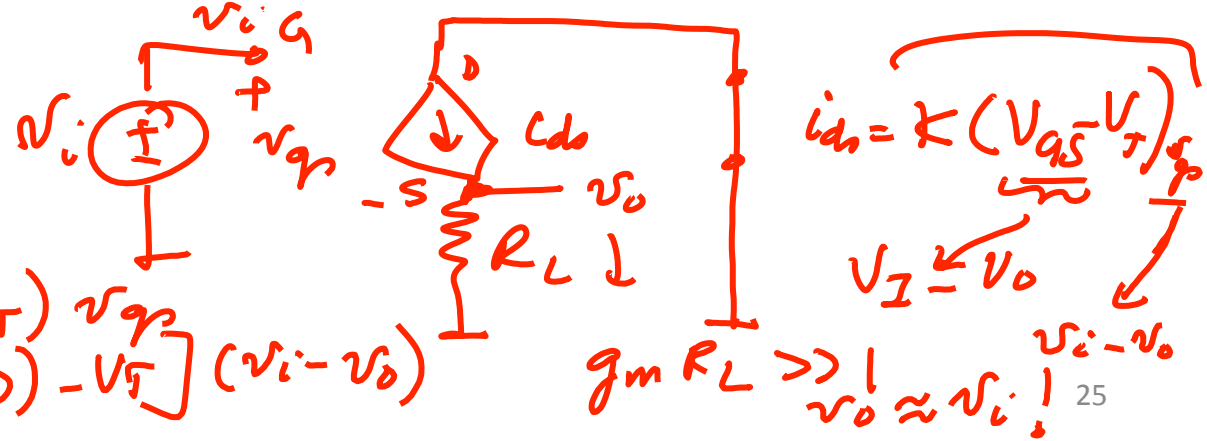
$V_I = V_{GS}$

$v_I \neq v_{gs}$ general



$v_{GS} \neq v_I$

$v_{GS} = v_I - v_o$, $v_{gs} = v_i - v_o$



Node:

$\frac{v_o}{R_L} - i_{ds} = 0$

$i_{ds} = K(V_{GS} - V_T)\, v_{gs}$

$= K\left[(V_I - V_o) - V_T\right](v_i - v_o)$

$g_m R_L \gg 1 \Rightarrow v_o \approx v_i$!