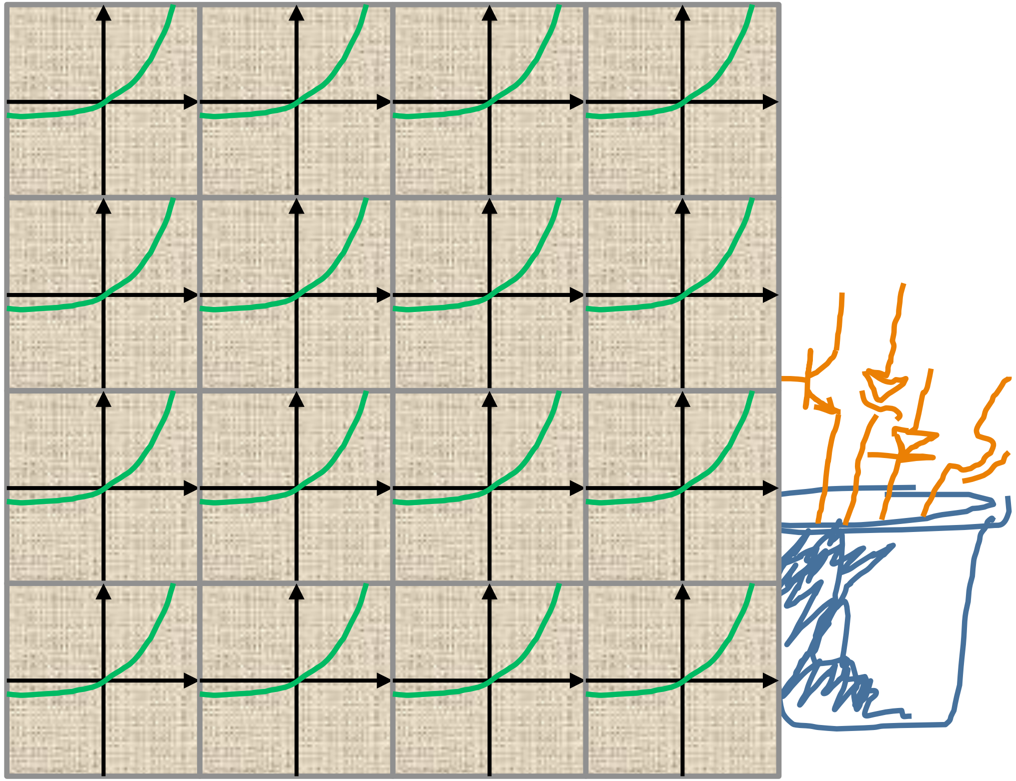
6.002x CIRCUITS AND ELECTRONICS

Nonlinear Circuits

$i = a v^2$

Reading Chap 4.1 – 4.3



Review

- Discretize matter → Lumped circuit abstraction

m1 ▶ KVL, KCL, *i-v*

m2 ▶ Composition rules

m3 ▶ Node method

m4 ▶ Superposition

m5 ▶ Thévenin, Norton

any circuit

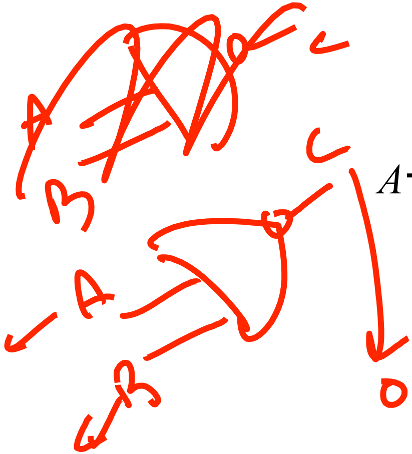
linear circuits

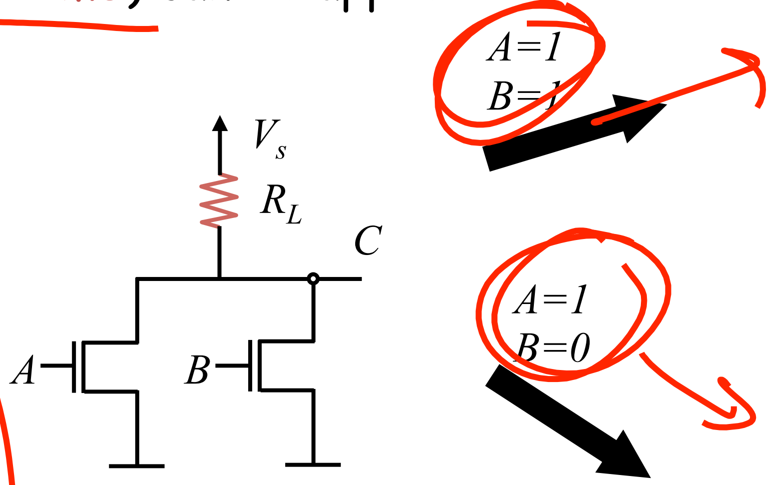
linear subckts as well

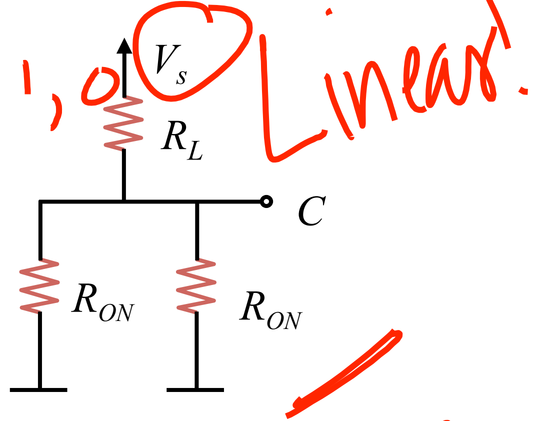
linear

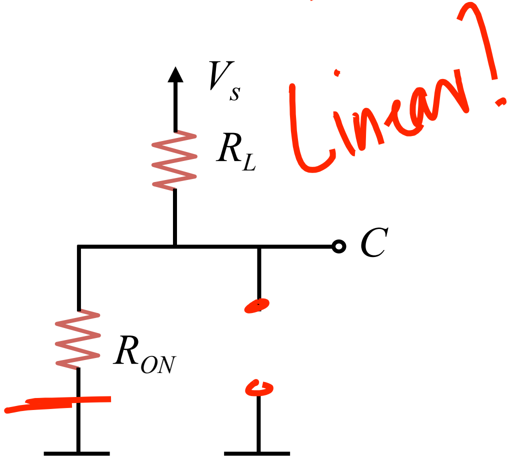
Review

- Discretize value → Digital abstraction
  - Subcircuits for given "switch" setting are linear! So, all 5 methods (m1 - m5) can be applied









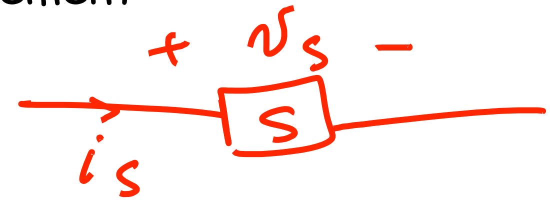
Use SR MOSFET Model

Today

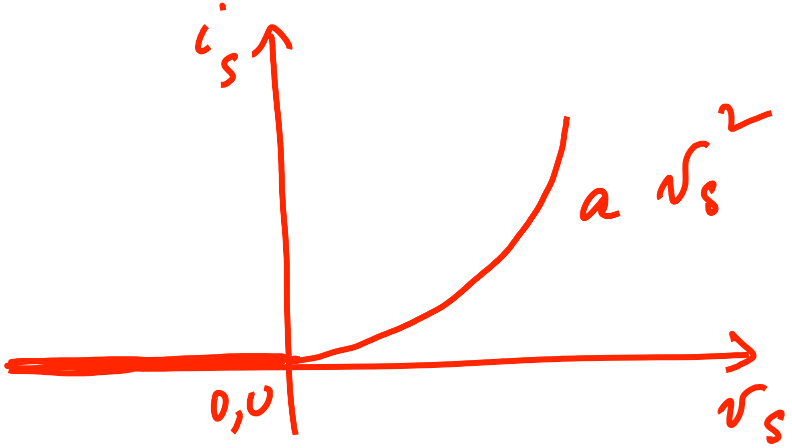
- Nonlinear circuits and their analysis
  - Analytical method based on m1, m2, m3 of circuit analysis ~~m4~~, ~~m5~~
  - Graphical method
  - Piecewise linear method — not a focus area for 6.002x
  - Introduction to incremental analysis, or small signal method

Non-Linear Elements

A square law two-terminal element

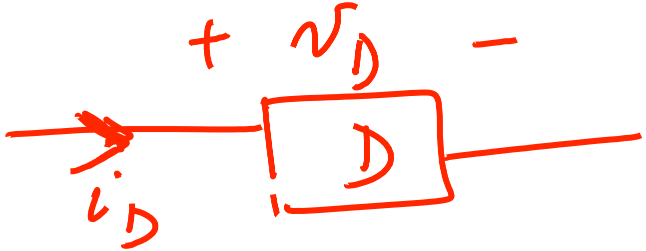


$$i_s = a v_s^2 \quad \text{for } v_s \geq 0$$
$$= 0 \quad \text{for } v_s < 0$$



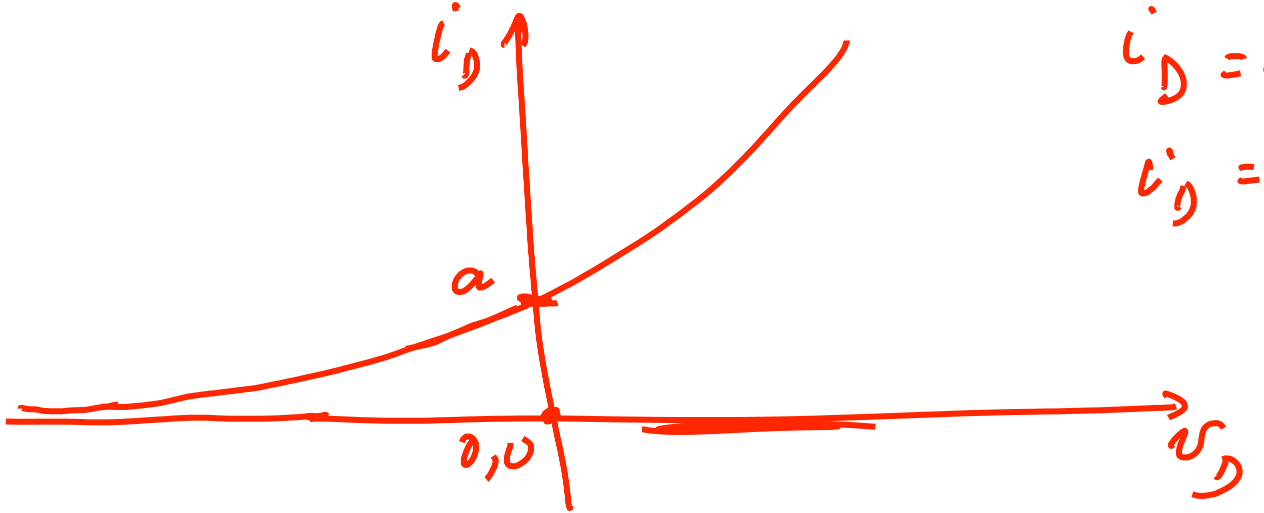
Non-Linear Elements

Hypothetical nonlinear device
(ExpoDweeb ☺)



$i_D = a \cdot e^{b v_D}$

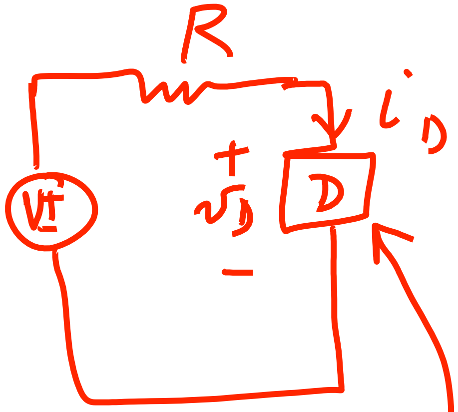
$i_D = a$ for $v_D = 0$

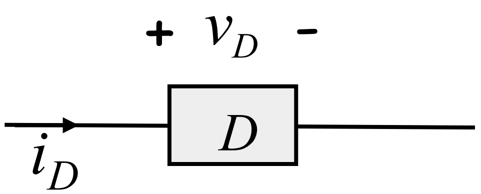


(Curiously, this funky device supplies power when v_D is negative!)

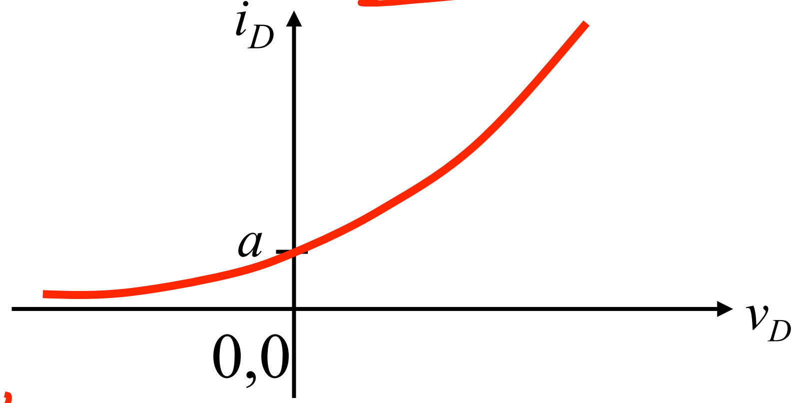
How do we analyze nonlinear circuits

For example:



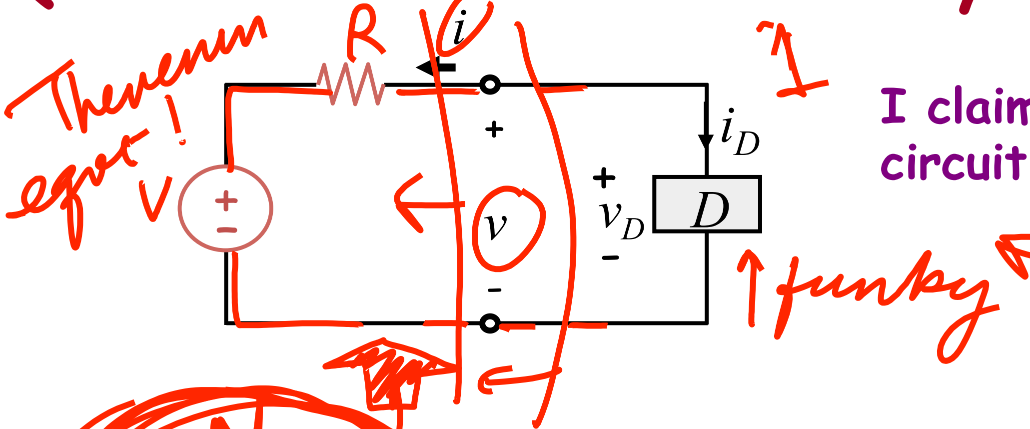


$$i_D = ae^{bv_D}$$



hypothetical nonlinear device, the ExpoDweeb!

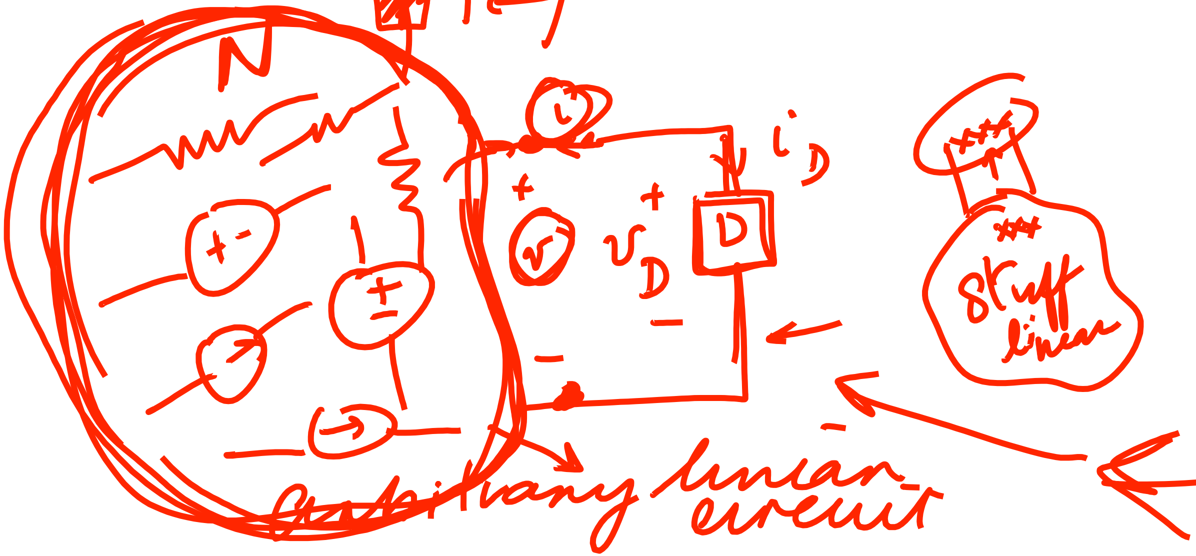
Quick Aside: Note this Key Circuits Hack



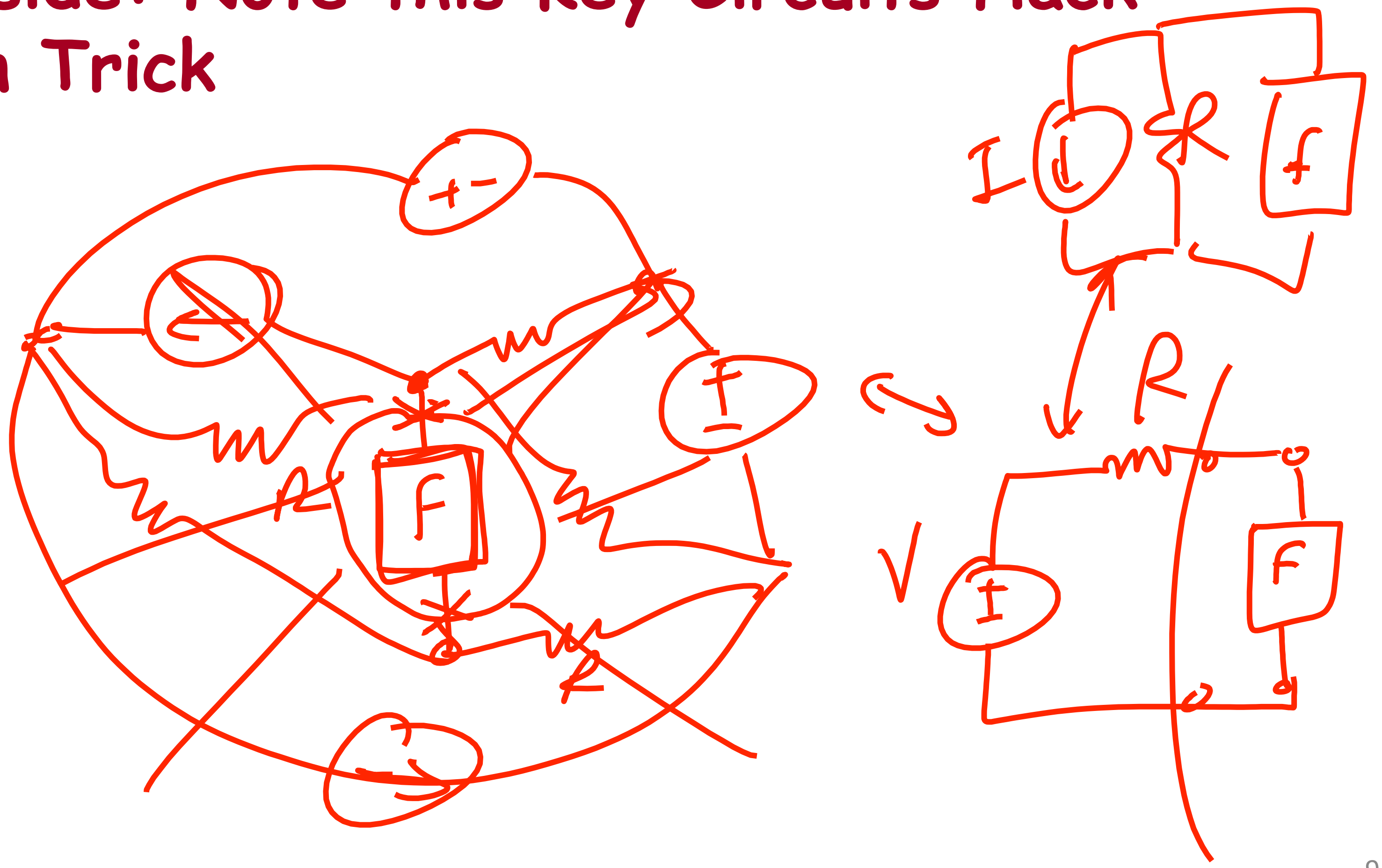
I claim this is a very important circuit pattern! Why?

Powerful!!

If you know how to analyze the above circuit pattern, you can analyze any of these more complex circuits below



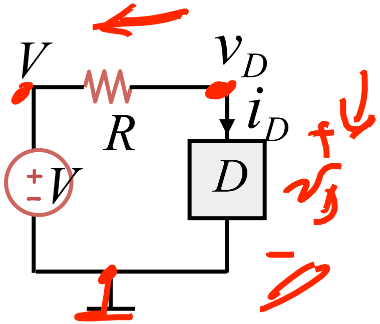
Quick Aside: Note this Key Circuits Hack
Thevenin Trick



Method 1: Analytical Method

Using the node method,

(remember the node method applies for linear or nonlinear circuits)



$$i_D = a e^{b v_D}$$

$$\frac{v_D - V}{R} + i_D = 0 \quad \text{— ①}$$

2 unknowns

2 equations

$$i_D = a e^{b v_D} \quad \text{— ②}$$

Solve by
- trial and error
- numerical techniques

Solve by trial and error

$$\frac{v_D - V}{R} + i_D = 0 \quad ①$$

$$i_D = ae^{bv_D} \quad ②$$

fixed point

$$\frac{v_D - V}{R} + ae^{bv_D} = 0$$

E.g., for $V=1V$, $R=1$ Ohm, $a=1/4$ A, $b=1V^{-1}$

$$v_D = V - Rae^{bv_D}$$

$$i_D = ae^{bv_D}$$

analytical method

$$v_D = 1 - \frac{1}{4}e^{v_D}$$

1V → 0.32V → 0.65V → 0.52V → 0.58V → 0.55V → 0.56V → 0.56V

Corresponding i_D is 0.44A

$$\boxed{\begin{aligned} v_D &= 0.56V \\ i_D &= 0.44A \end{aligned}}$$

Method 2: Graphical Method

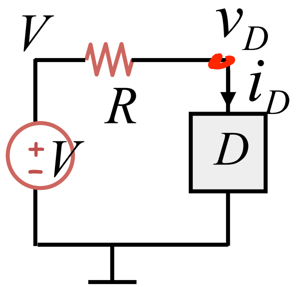
Using the node method

$$\frac{v_D - V}{R} + i_D = 0 \qquad (1)$$

$$i_D = \frac{V}{R} - \frac{v_D}{R} \qquad (1')$$

rearrange

$$i_D = ae^{bv_D} \qquad (2)$$



$$i_D = ae^{bv_D}$$

2 unknowns, 2 equations

Can also solve by the graphical method

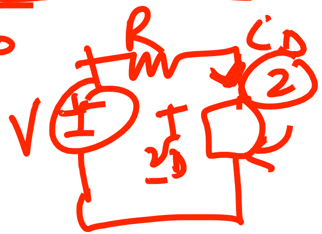
Method 2: Graphical Method

Notice: the solution satisfies equations (1') and (2)

$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$

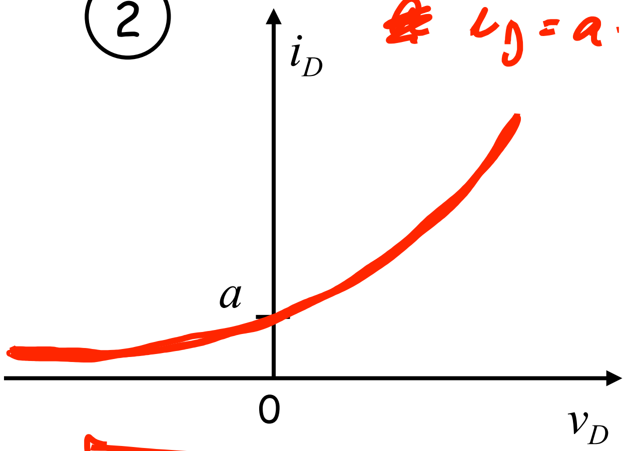
$$i_D = a e^{b v_D} \quad (2)$$

$0 \leftarrow i_D = \frac{V}{R} - \frac{v_D}{R} \rightarrow 0$



(2)

$i_D = a \cdot e^{b v_D}$



Constraint on i_D and v_D
imposed by device

(1')

Constraint on on i_D and v_D
imposed by the rest of the circuit
(Thevenin equivalent circuit)

Combine the two constraints

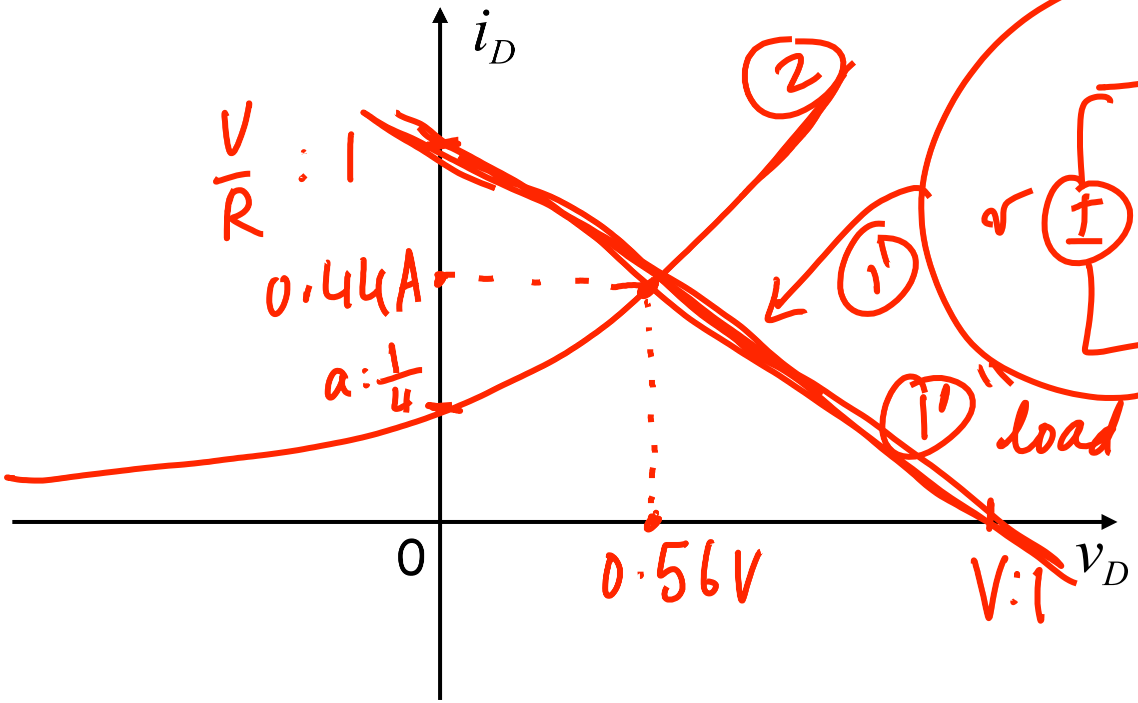
$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$

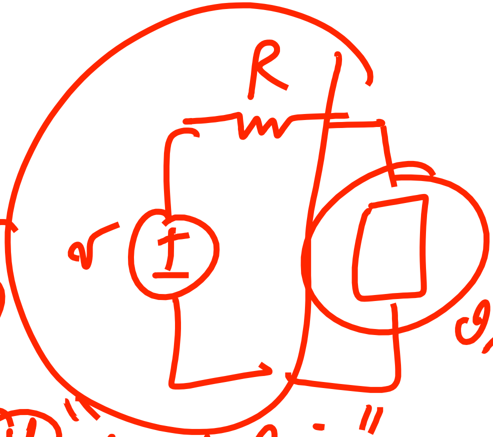
$$i_D = ae^{bv_D} \quad (2)$$

E.g., for $V=1V$, $R=1$ Ohm, $a=1/4$ A, $b=1V^{-1}$

$$i_D = 1 - v_D \quad (1')$$

$$i_D = \frac{1}{4}e^{v_D} \quad (2)$$





Interesting

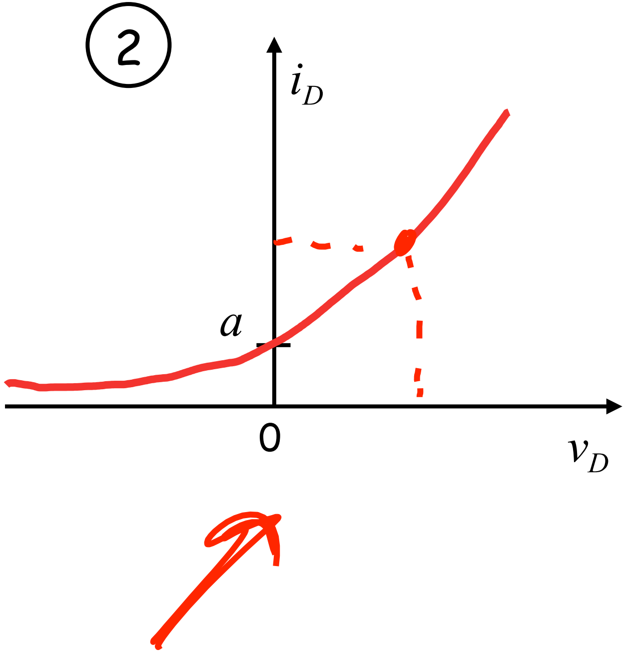
$i_D = 0.44A$

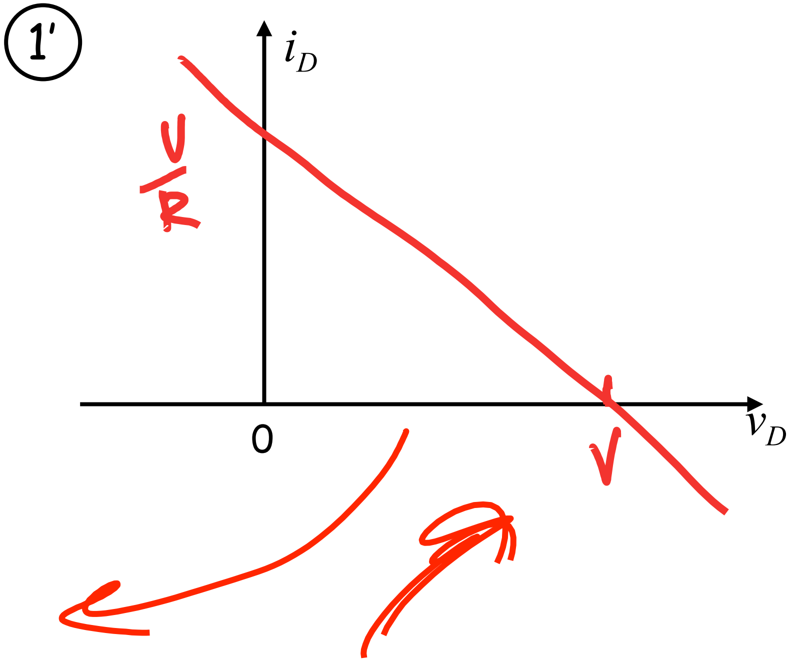
$v_D = 0.56V$

Combining the two constraints

$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$

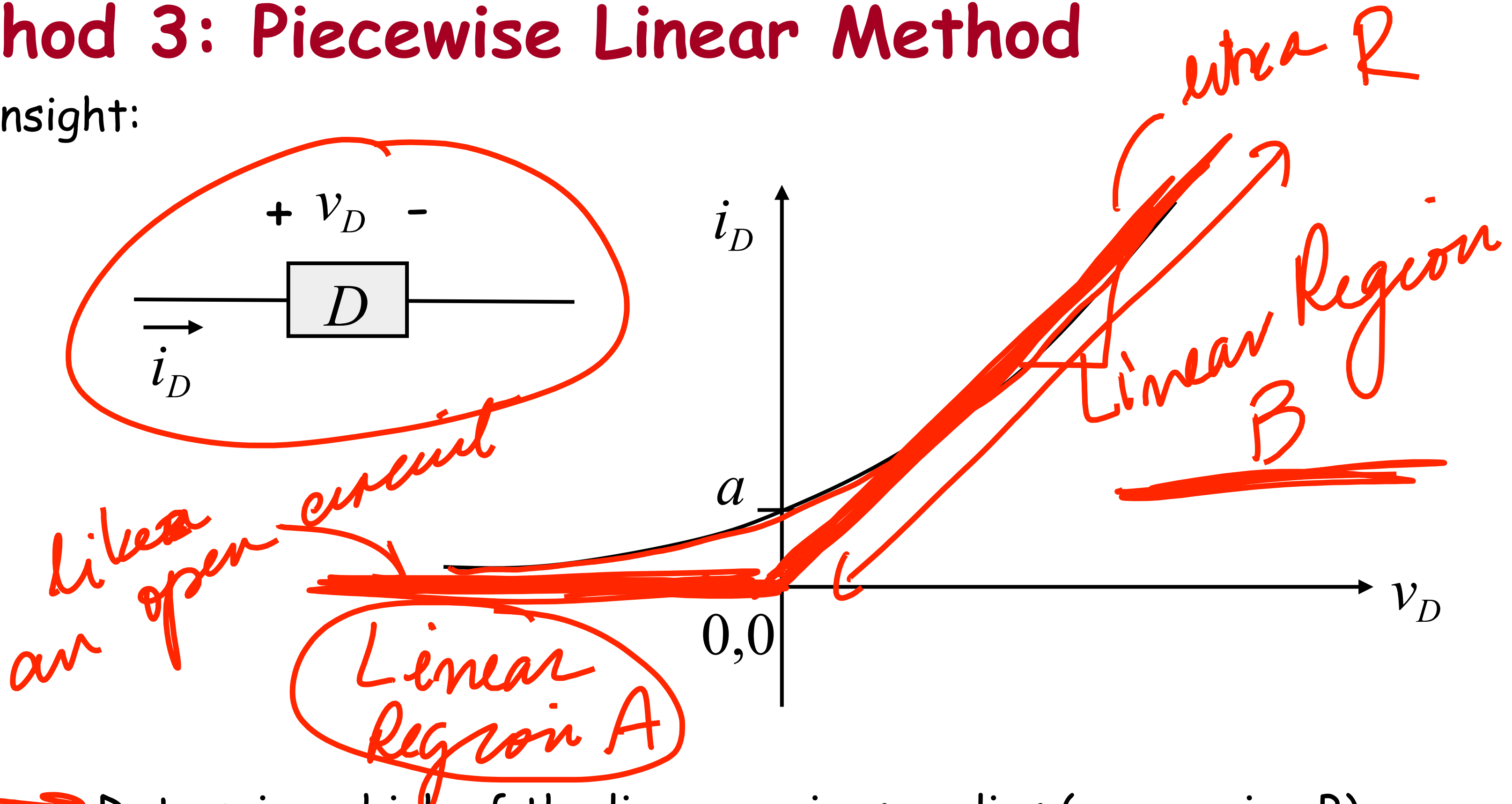
$$i_D = a e^{b v_D} \quad (2)$$





Method 3: Piecewise Linear Method

Insight:

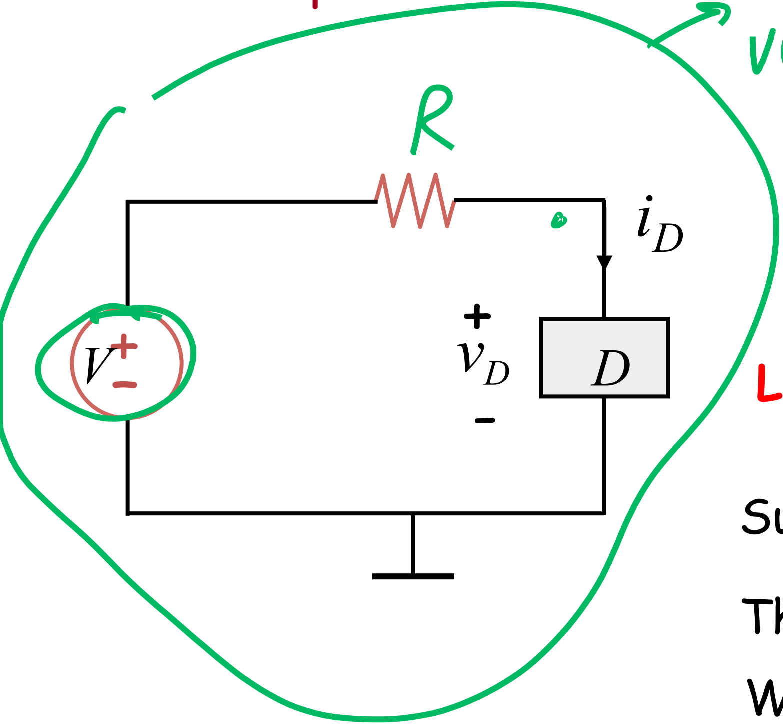


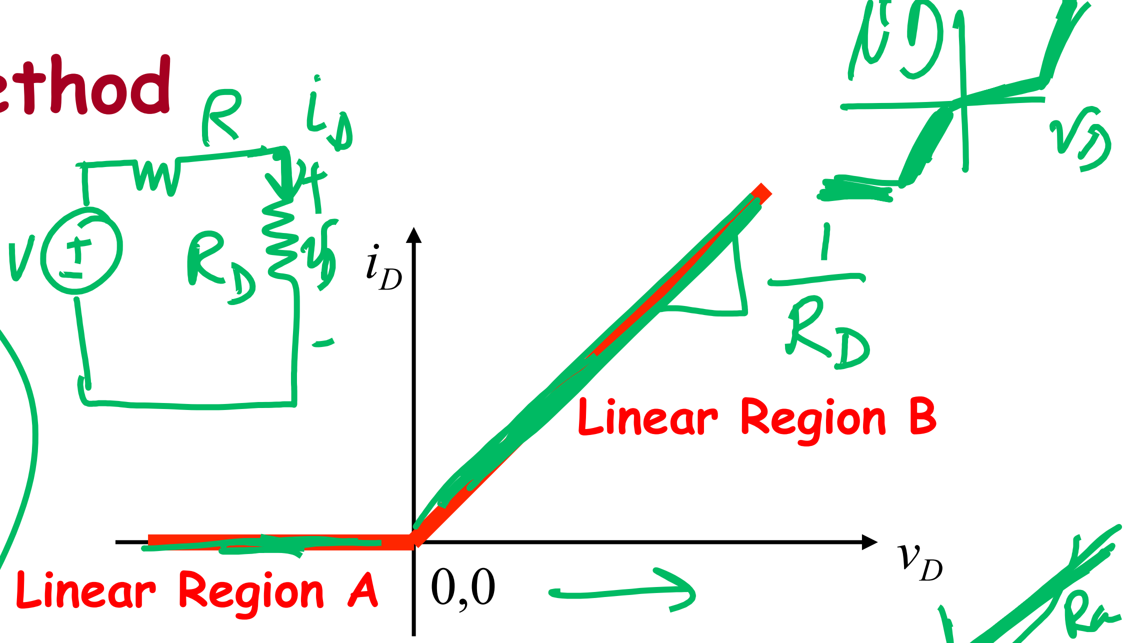
- Determine which of the linear regions applies (e.g., region B)
- Solve circuit assuming that linear relation (from region B)

See Sec 4.4 of the text for details and examples

Piecewise Linear Method

For example:





Suppose we are given that V is positive

Then D will operate in Linear Region B

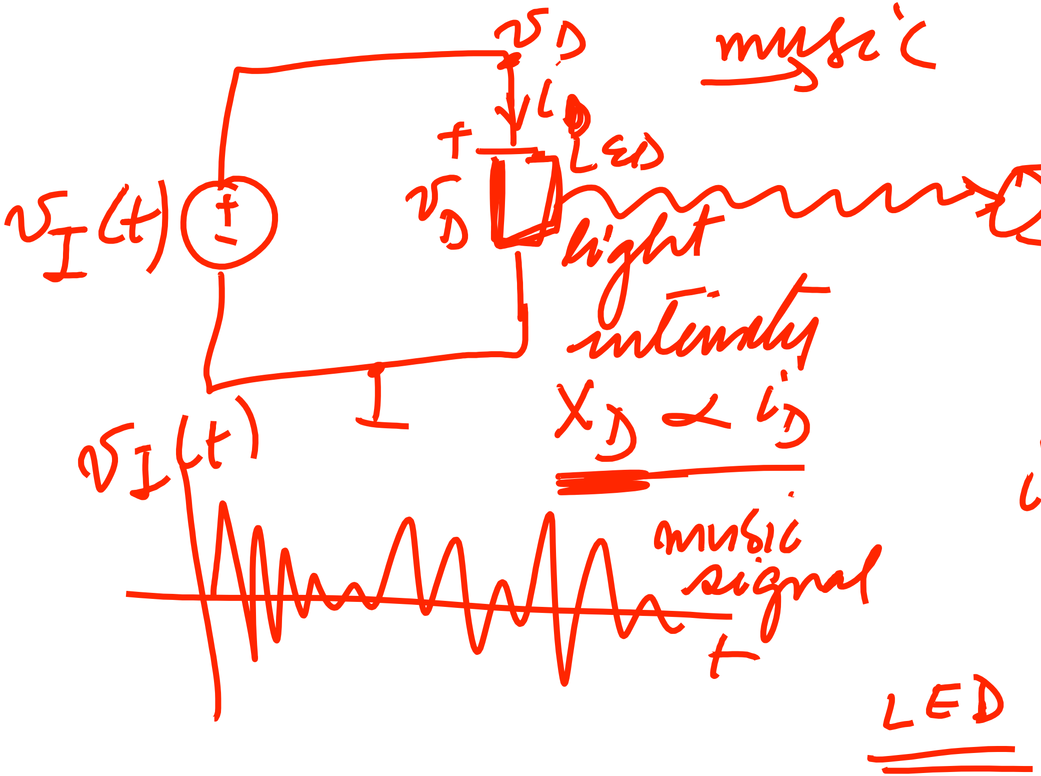
We can replace D in the circuit with a resistance of value $1/R_D$ when D operates in region B

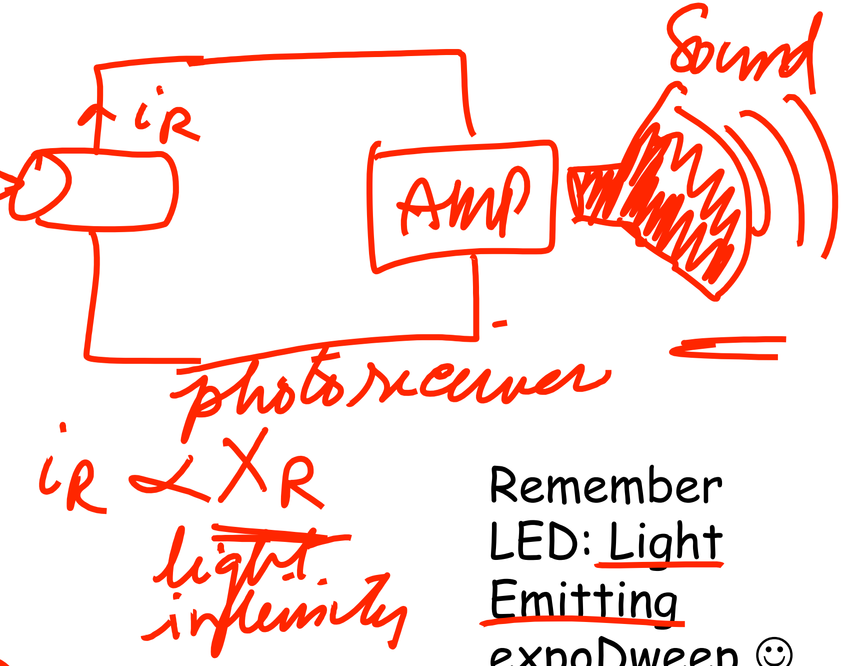
Method 3: Incremental Analysis

Small Signal method

(Actually, a disciplined way of using a circuit called small signal method)

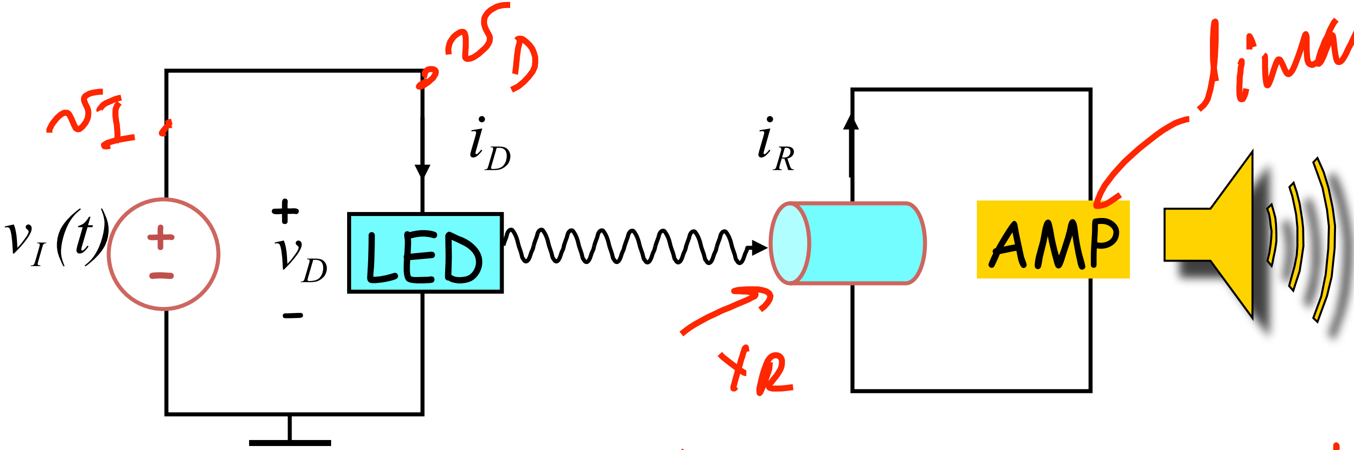
Motivation: music over a light beam. Can we pull this off?





Remember
LED: Light
Emitting
expoDweep ☺

Method 3: Incremental Analysis



$i_D = a e^{v_D}$

nonlinear

$v_I(t) \rightarrow i_D(t) \rightsquigarrow$ light $\rightsquigarrow i_R(t) \rightarrow$ sound

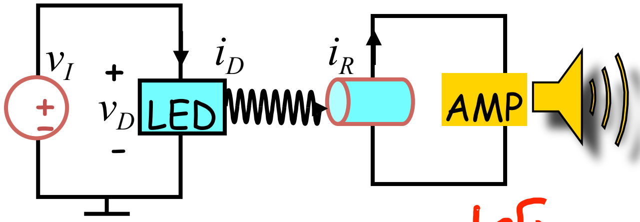
$v_I = v_D$

non linear!

linear

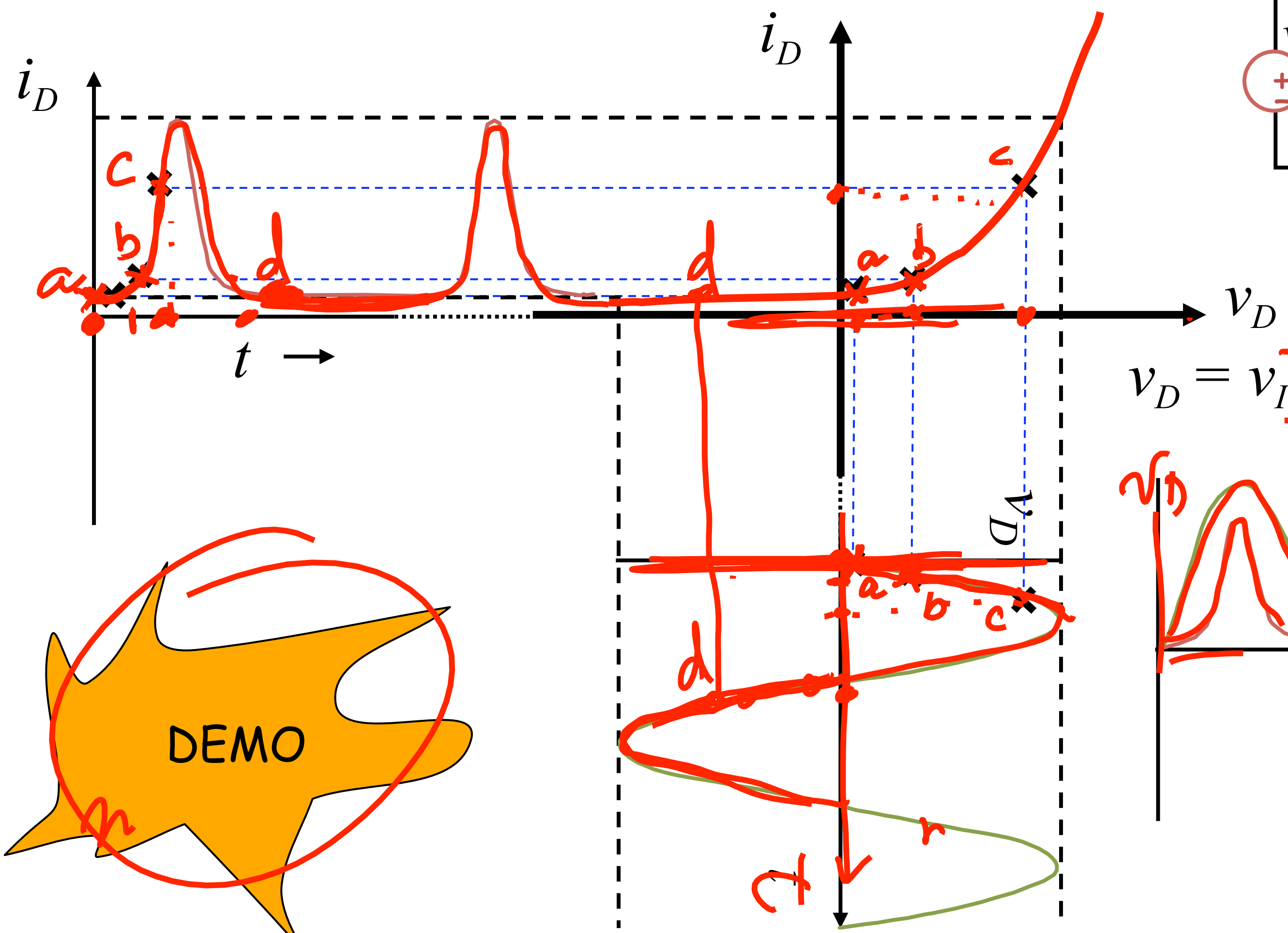
problem → will result in distortion

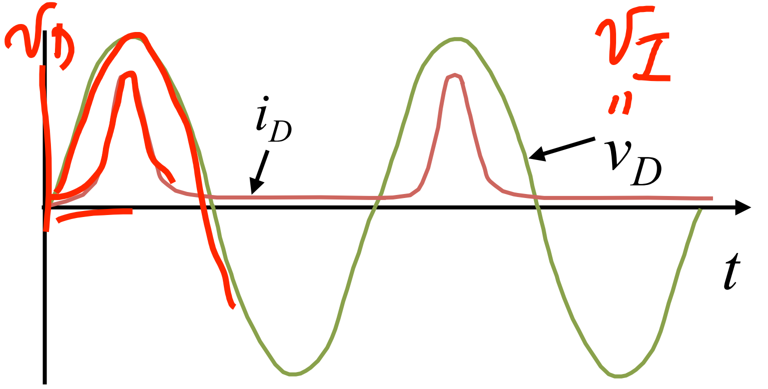
Problem: The LED is nonlinear → distortion



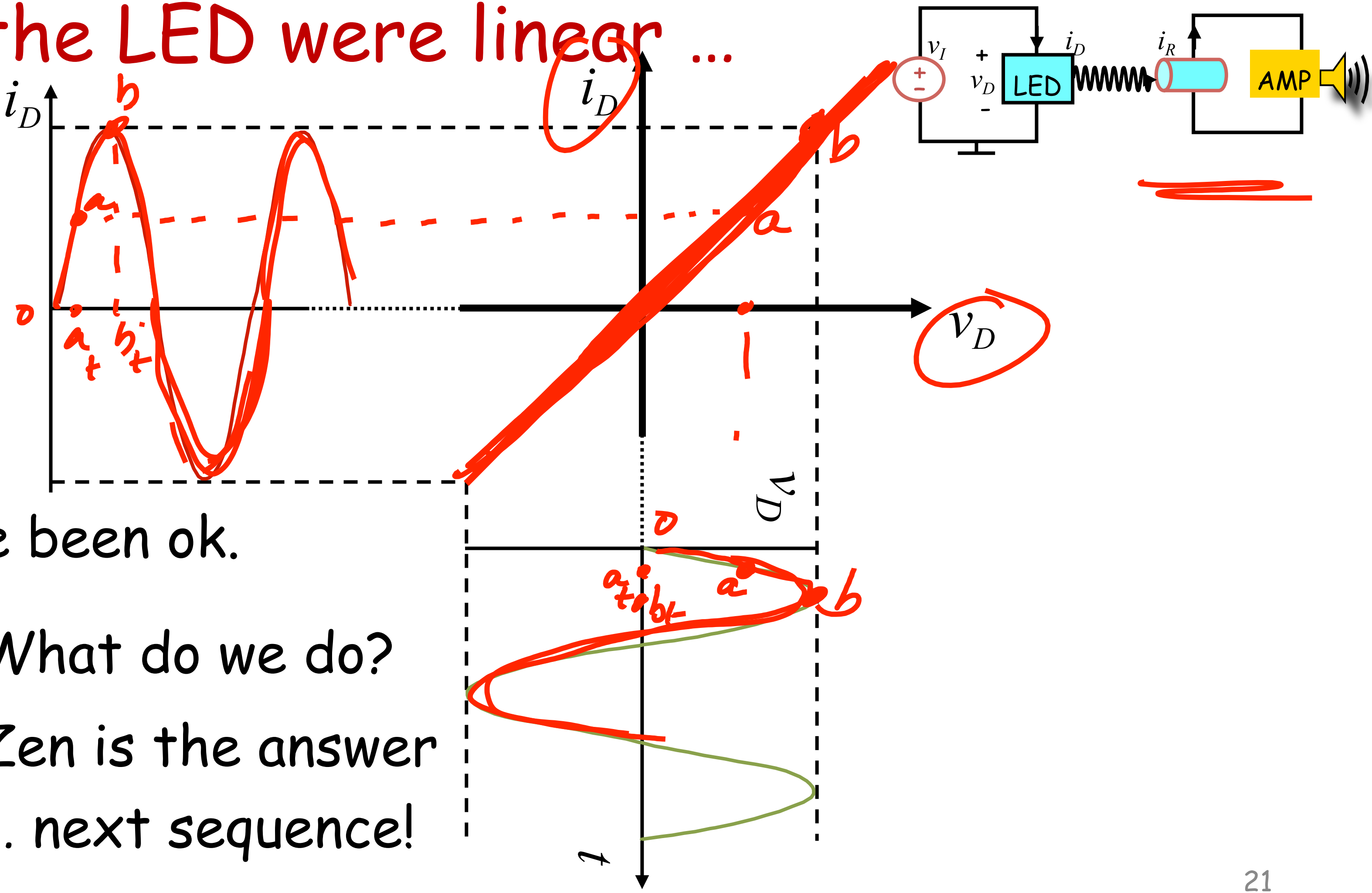
$i_D = a e^{b v_D}$

$v_D = v_I$





If only the LED were linear ...



it would' ve been ok.

What do we do?

Zen is the answer

... next sequence!