Physics laws or "abstractions"
- Maxwell's abstraction for tables of data
- Ohm's abstraction

Lumped circuit abstraction

Simple amplifier abstraction
- Operational amplifier abstraction
- Digital abstraction
- Combinational logic

Filters
- Clock digital abstraction

Analog subsystems
- Modulators, oscillators, RF amps, power supplies

Mice, toasters, sonar, stereos, angry birds, space shuttle, iPAD

Instruction set abstraction
- Pentium, MIPS 6.004, 6.846

Programming languages
- Java, C++, Matlab, Python

Software systems
- 6.033
- Operating systems, Browsers

6.002.1x
Circuits and Electronics 1
Consider

\[ V \quad \text{?} \quad I \]

Suppose we wish to answer this question:
What is the current through the bulb?

Reading: Skim through Chapter 1 of A&L
We could do it the Hard Way...

Apply Maxwell's

Differential form                           Integral form

Faraday's  \[ \nabla \times E = -\frac{\partial B}{\partial t} \]  \[ \oint E \cdot dl = -\frac{\partial \phi_B}{\partial t} \]

Continuity  \[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} \]  \[ \oint J \cdot dS = -\frac{\partial q}{\partial t} \]

Others  \[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]  \[ \oint E \cdot dS = \frac{q}{\varepsilon_0} \]
Instead, there is an Easy Way…

First, let us build some insight:

Analogy

I ask you: What is the acceleration?
You quickly ask me: What is the mass?
I tell you: \[ m \]
You respond: \[ a = \frac{f}{m} \]

Done!!!
Instead, there is an Easy Way...

In doing so, you ignored

- the object’s shape
- its temperature
- its color
- point of force application
- ...

→ Point-mass discretization

\[ F \rightarrow a? \]
The Easy Way...

Consider the filament of the light bulb.

We do not care about
- how current flows inside the filament
- its temperature, shape, orientation, etc.

We can replace the bulb with a discrete resistor for the purpose of calculating the current.
The Easy Way...

Replace the bulb with a **discrete resistor** for the purpose of calculating the current.

$$I = \frac{V}{R}$$
The Easy Way...

\[ I = \frac{V}{R} \]

In EECS, we do things the easy way...

\( R \) represents the only property of interest!
Like with point-mass:
replace objects with their mass \( m \) to find
\( a = \frac{F}{m} \)
V-I Relationship

\[ I = \frac{V}{R} \]

\( R \) represents the only property of interest!

\( R \) relates element \( V \) and \( I \)

called element v-i relationship

\[ I = \frac{V}{R} \]
$R$ is a lumped element abstraction for the bulb.
Lumped Elements

Lumped circuit element described by its $vi$ relation

Power consumed by element = $vi$

Resistor

Voltage source
Lumped element examples whose behavior is completely captured by their $V-I$ relationship.

Demo only for the sorts of questions we as EEs would like to ask!

Demo

Exploding resistor demo can’t predict that!

Pickle demo can’t predict light, smell
Not so fast, though …

Although we will take the easy way using lumped abstractions for the rest of this course, we must make sure (at least for the first time) that our abstraction is reasonable.

In this case, ensuring that $V$ and $I$ are defined for the element...
must be defined.

\[ I_{\text{in}} = S_A = I_{\text{out}} + S_B \]
I must be defined. True when

\[ I_{\text{into}} S_A = I_{\text{out of}} S_B \]

True only when \( \frac{\partial q}{\partial t} = 0 \) in the filament!

So, we stuck?

\[
\oint_{S_A} \mathbf{J} \cdot d\mathbf{s} - \oint_{S_B} \mathbf{J} \cdot d\mathbf{s} = \oint_{S_B} \frac{\partial \mathbf{W}}{\partial t} \]

\[ I_A = I_B \quad \text{only if} \quad \frac{\partial q}{\partial t} = 0 \]

We're engineers! So, let's make it true!
Must also be defined.

\[ V_{AB} \text{ defined when } \frac{\partial \Phi_B}{\partial t} = 0 \]

\[ V_{AB} = \oint_{AB} E \cdot dl \]

So let's assume this too!

Also, signal speeds of interest should be way lower than speed of light.
Welcome to the EECS Playground

The world

The EECS playground

Our self imposed constraints in this playground

Outside

\[ \frac{\partial \phi_B}{\partial t} = 0 \]

Inside elements

Bulb, wire, battery

Where good things happen

\[ \frac{\partial q}{\partial t} = 0 \]
Connecting using ideal wires lumped elements that obey LMD to form an assembly results in the lumped circuit abstraction.

**Lumped Matter Discipline (LMD)**

Or self imposed constraints:

\[ \frac{\partial \phi_B}{\partial t} = 0 \quad \text{outside} \]

\[ \frac{\partial q}{\partial t} = 0 \quad \text{inside elements} \]

bulb, wire, battery

More in Chapter 1 of A & L
So, what does LMD buy us?

Replace the differential equations with simple algebra using lumped circuit abstraction (LCA).

For example:

$$2a + 3b = 0$$

What can we say about voltages in a loop under the lumped matter discipline?

Reading: Chapter 2.1 - 2.2.2 of A&L
What can we say about voltages in a loop under LMD?

Kirchhoff's Voltage Law (KVL):

The sum of the voltages in a loop is 0.

\[ \oint \mathbf{E} \cdot d\mathbf{l} = \frac{\Phi B}{\partial t} \]

\[ \sum_{\text{ca}} \mathbf{E} \cdot d\mathbf{l} + \sum_{\text{ab}} \mathbf{E} \cdot d\mathbf{l} + \sum_{\text{bc}} \mathbf{E} \cdot d\mathbf{l} = 0 \]

\[ V_{ca} + V_{ab} + V_{bc} = 0 \]

Remember, this is not true everywhere, only in our EECS playground.
What can we say about currents?

\[ I_{ca} \quad I_{da} \]

\[ R_1 \]

\[ I_{ba} \]
What can we say about currents?

Kirchhoff's Current Law (KCL):
The sum of the currents into a node is 0.

Simply conservation of charge
KVL and KCL Summary

KVL:
\[ \sum v_j = 0 \]
loop

KCL:
\[ \sum i_i = 0 \]
node
Remember, our EECS playground

Lumped Matter Discipline LMD: Constraints we impose on ourselves to simplify our analysis

\[ \frac{\partial \phi_B}{\partial t} = 0 \]

Outside elements

\[ \frac{\partial q}{\partial t} = 0 \]

Inside elements

Also, signals speeds of interest should be way lower than speed of light

Allows us to create the lumped circuit abstraction

Summary
Summary

Lumped circuit element

\[ i = f(v) \]
\[ i = \frac{v}{R} \]
\[ i = 6.7v^2 + \frac{v^3}{2} \]

power consumed by element = \( vi \)
Maxwell's equations simplify to algebraic KVL and KCL under LMD.

KVL: \[ \sum_j v_j = 0 \]

KCL: \[ \sum_j i_j = 0 \]

This is amazing!