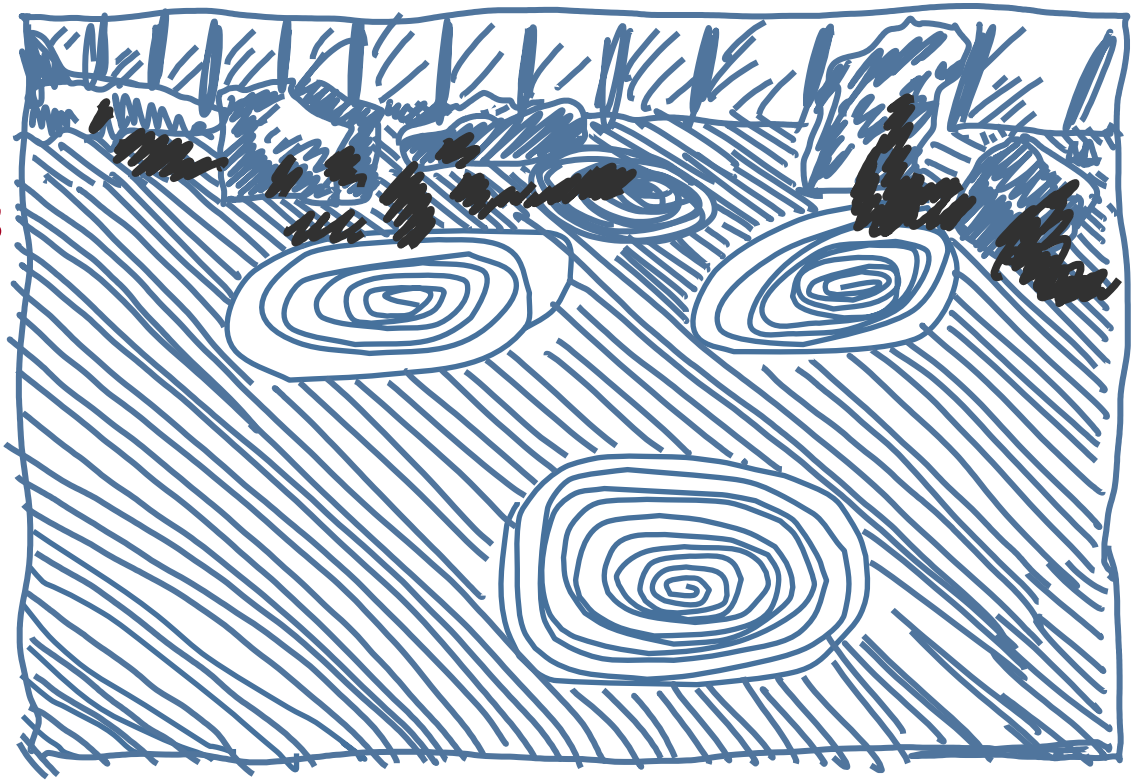
6.002x CIRCUITS AND ELECTRONICS

Incremental Analysis



Review

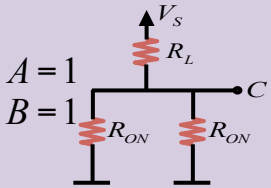
The 6.002x world

Lumped Circuit Abstraction

KVL/KCL
Node
Composition

Linear

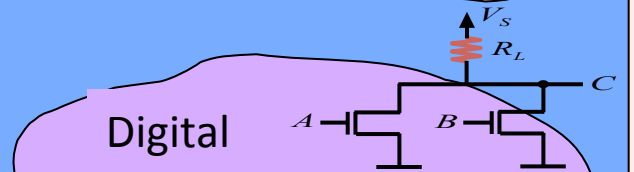
Superposition
Thevenin, Norton



Nonlinear

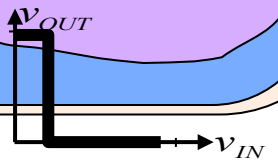
Analytical
Graphical

Small Signal Model



Digital

For fixed input values …



Review

Nonlinear Analysis

- Analytical method
- Graphical method
- Piecewise Linear method

Today

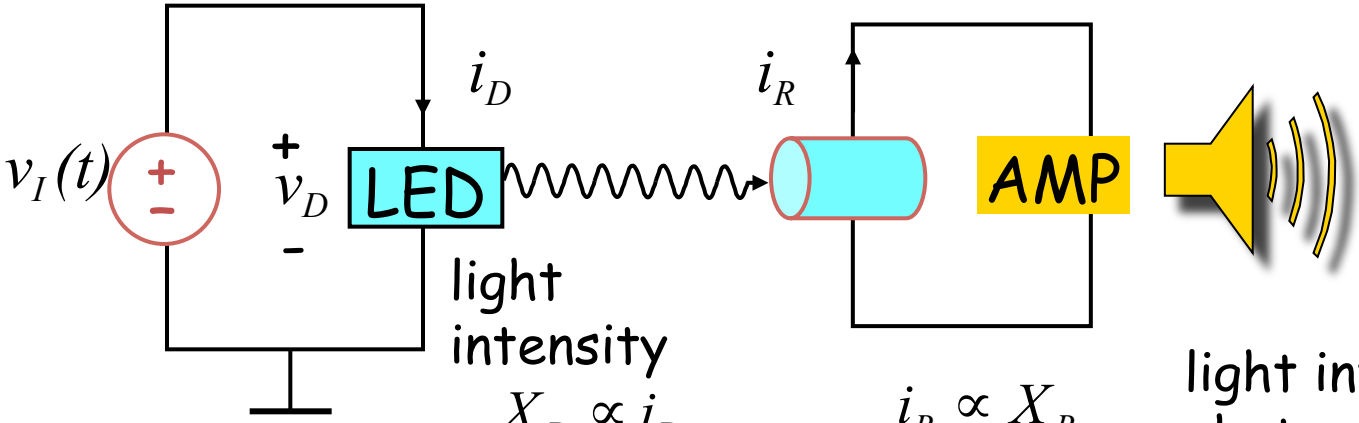
- Incremental analysis or small signal method

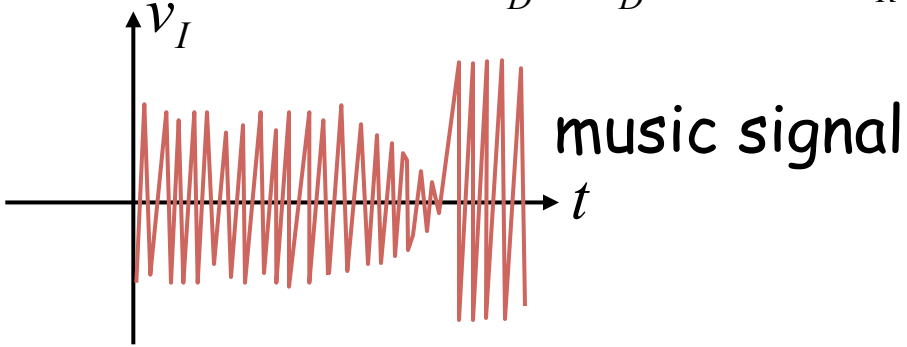
Reading: Section 4.5

Method 3: Incremental Analysis

(Actually a particular disciplined use of a circuit)

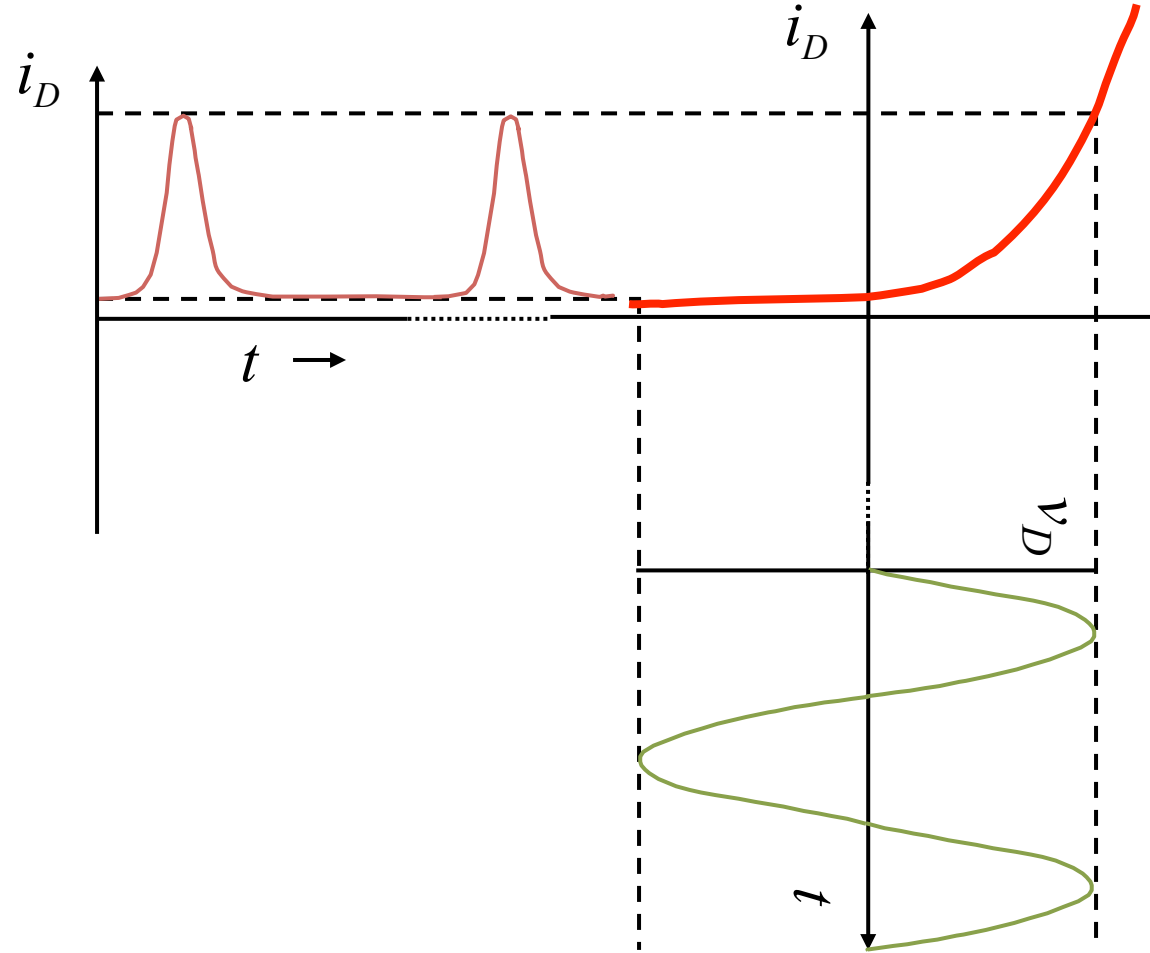
Motivation: music over a light beam. Can we pull this off?

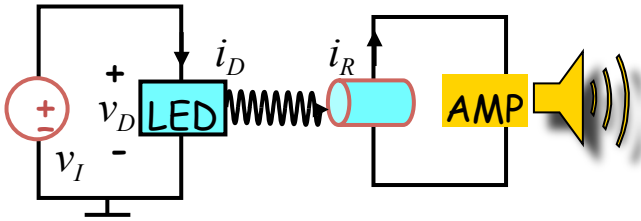


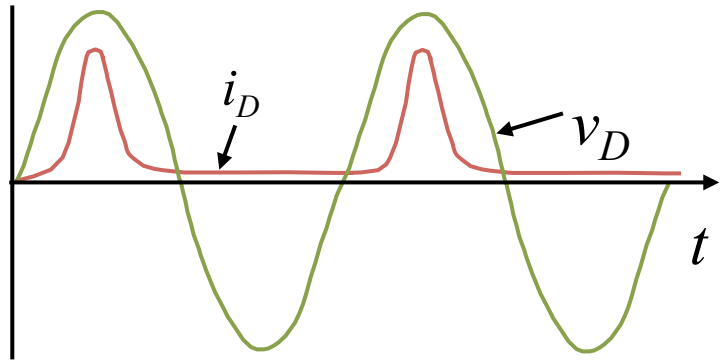


Remember
LED: Light Emitting expoDweep ☺

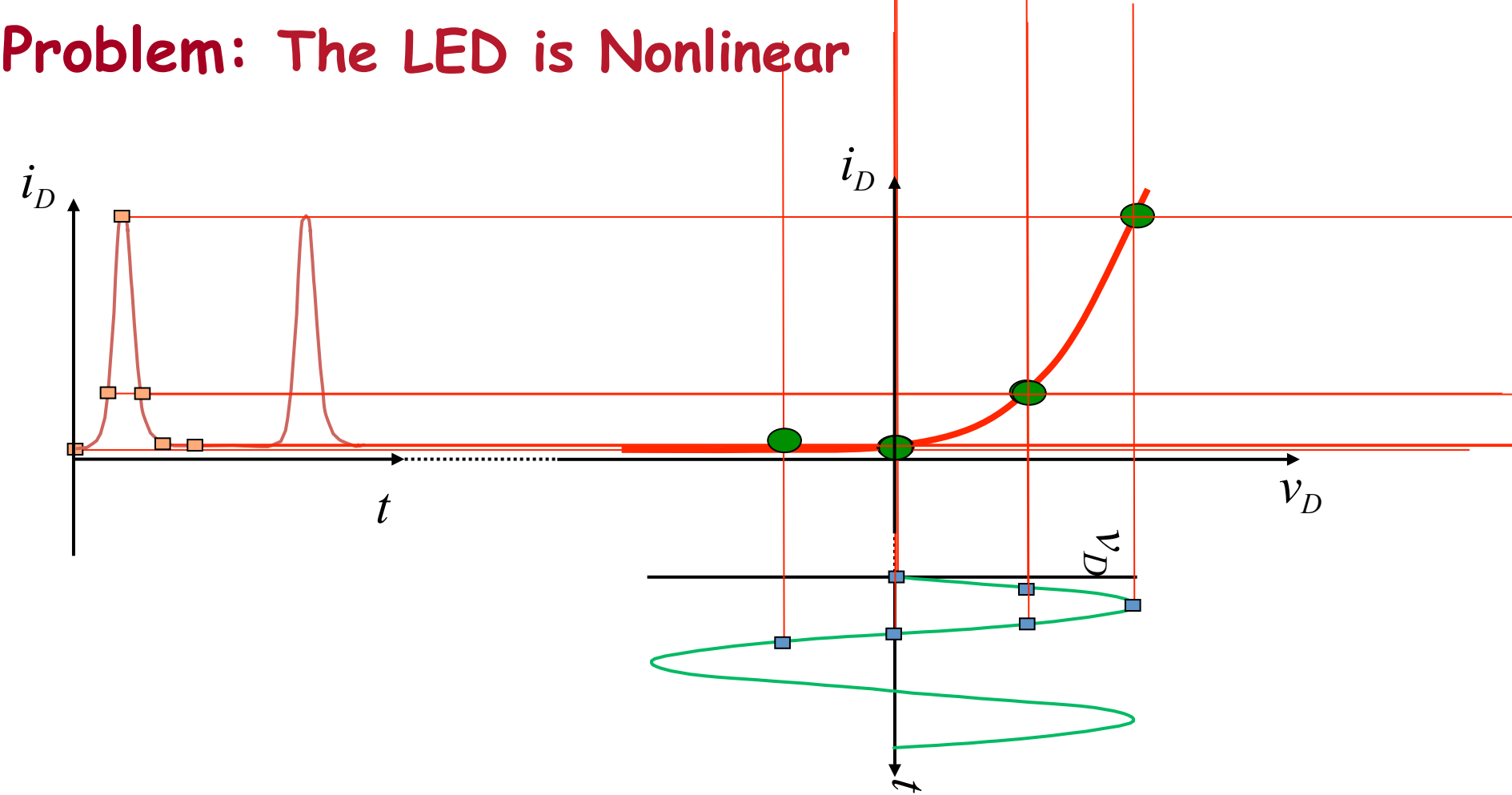
Problem: The LED is nonlinear → distortion





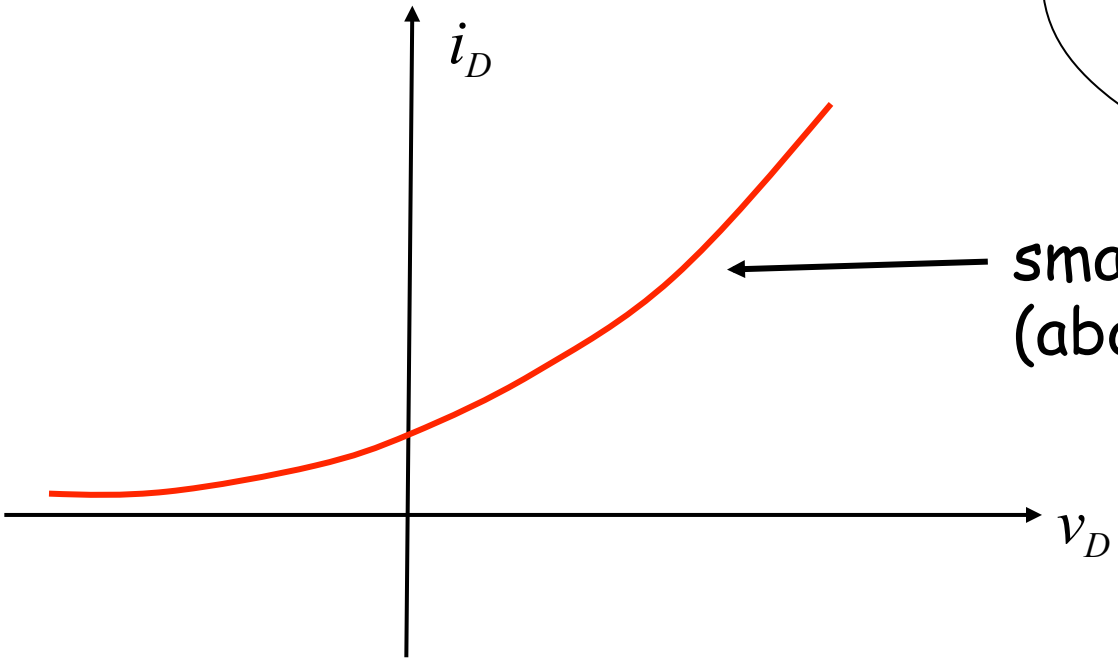


Problem: The LED is Nonlinear



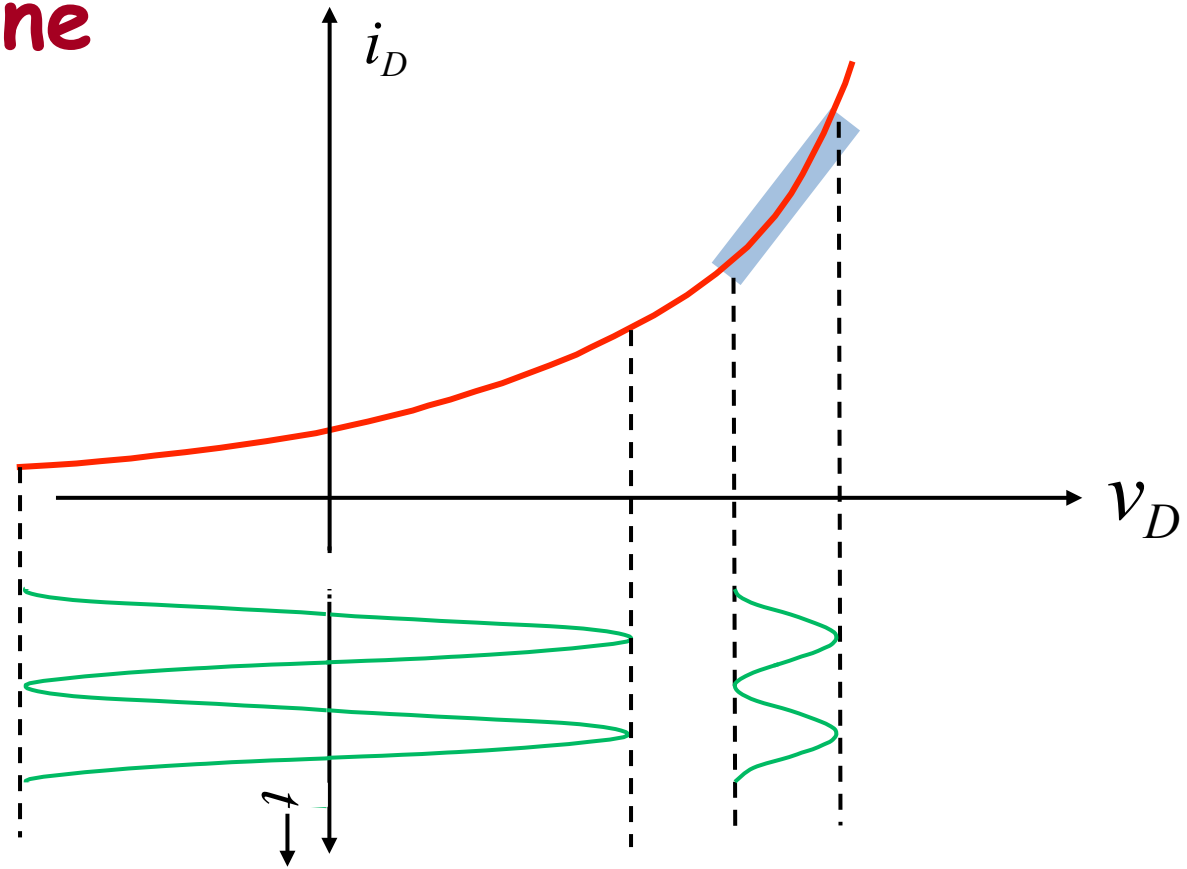
Insight

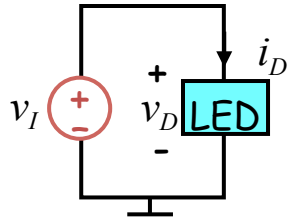




small region looks linear
(about some given V_D, I_D)

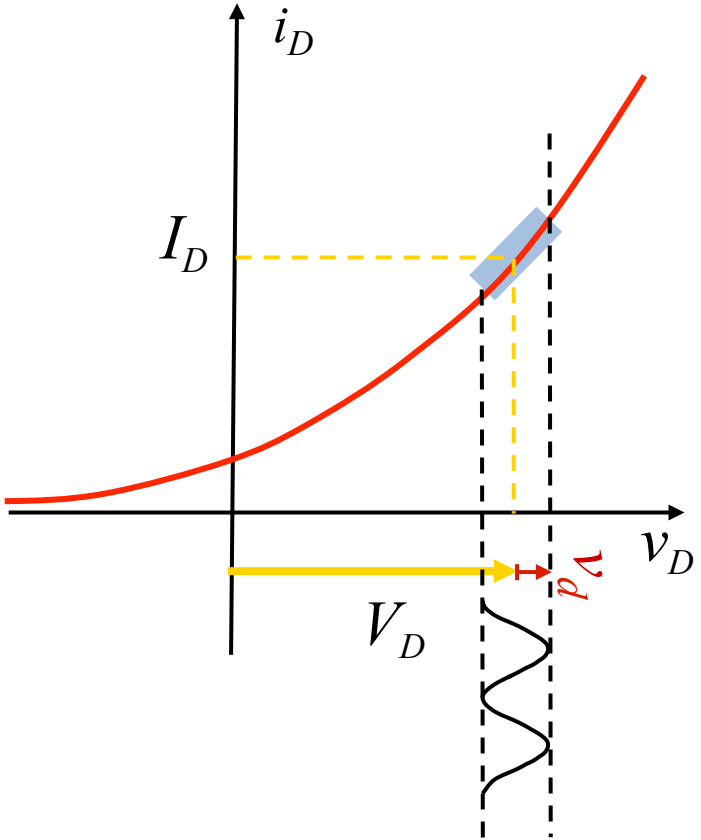
Imagine

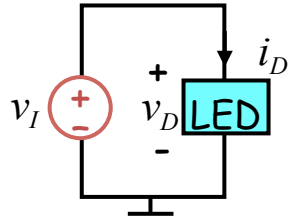




Boost and Shrink

How to implement this trick



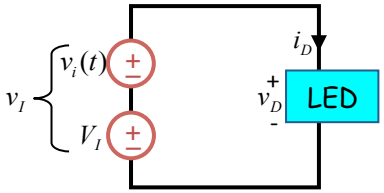


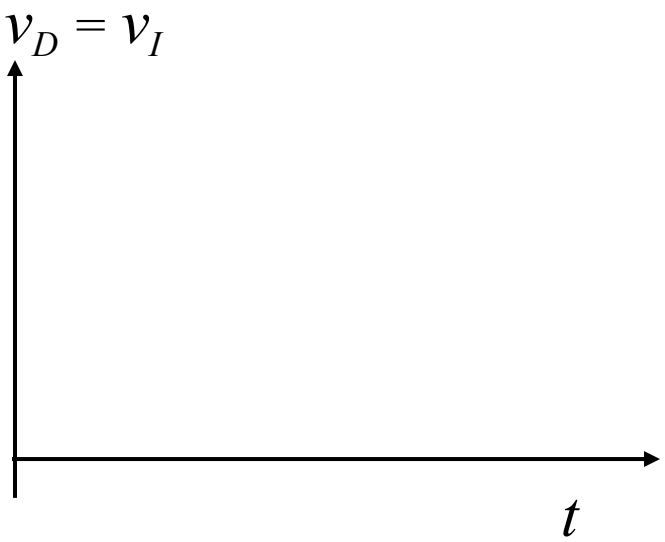
"Boost and Shrink"

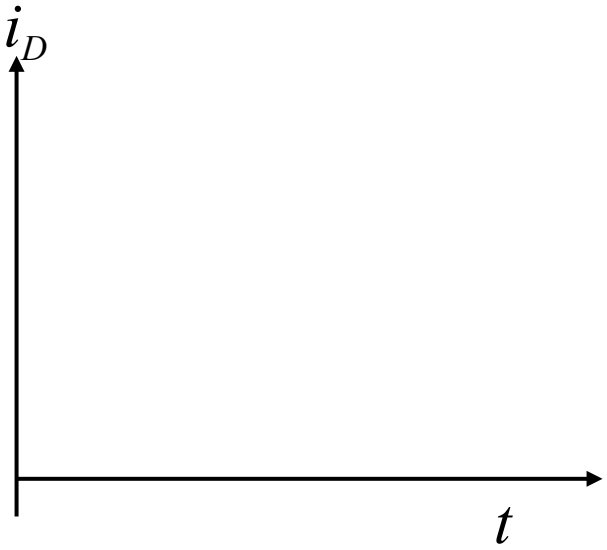
Shrink signal of interest v_i

Add DC offset to it V_I

Result







So, this is really a very clever and disciplined way of using a circuit so that we can get a more or less linear response out of a nonlinear circuit



The incremental method: (or small signal method)

1. Operate at some DC offset or bias point V_D, I_D.
2. Superimpose small signal v_d (music) on top of V_D.
3. Response i_d to small signal v_d is approximately linear.

What does this mean mathematically?

Or, why is the small signal response linear?

Using Taylor's Expansion to expand $f(v_D)$ near $v_D = V_D$:

Why is the small signal response linear?

$$i_D \approx f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

constant w.r.t. Δv_D

constant w.r.t. Δv_D
slope at V_D, I_D

Why is the small signal response linear?

✗: $I_D + \Delta i_D \approx f(V_D) + \left.\frac{d f(v_D)}{d v_D}\right|_{v_D = V_D} \cdot \Delta v_D$

slope at V_D, I_D

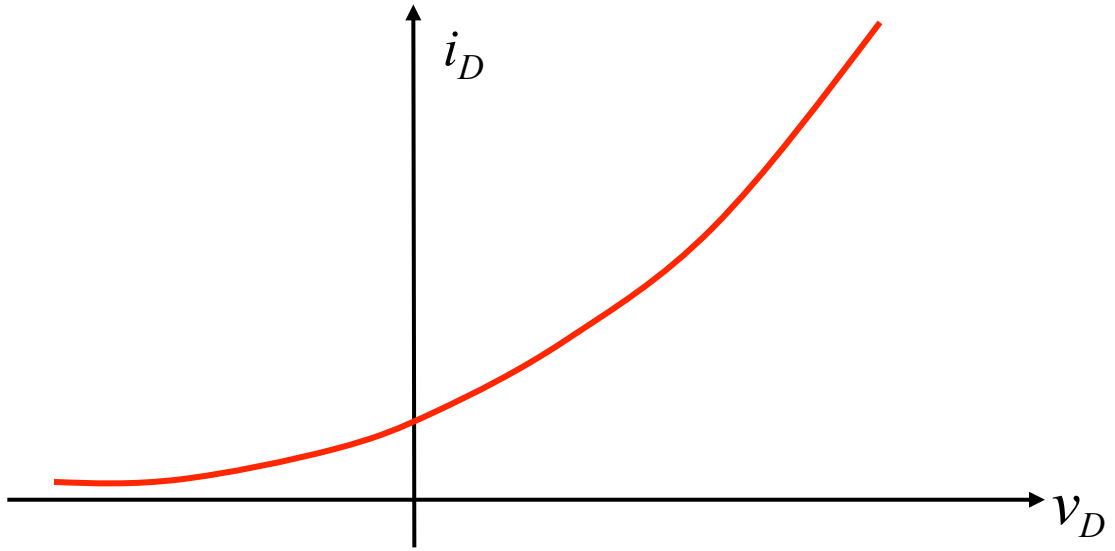
In our example

⊗ : $I_D + \Delta i_D \approx f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$

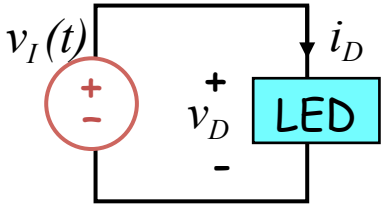
Graphical interpretation

$$I_D = a\, e^{bV_D} \longrightarrow \text{operating point}$$

$$i_d = I_D \cdot b \cdot v_d$$



we are approximating (A) with (B)



$$v_D = f(v_D) = a\, e^{bV_D}$$

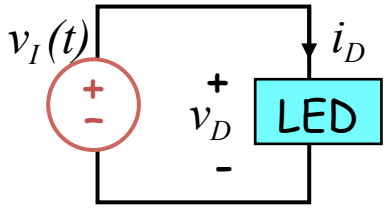
$$I_D = a\, e^{bV_D}$$

$$i_d = I_D \cdot b \cdot v_d$$

We studied the small signal → graphically
→ mathematically
→ Next, circuit view

A circuit view of the small signal model

Large signal circuit:



$$v_D = f(v_D) = a\, e^{bV_D}$$

$$I_D = a\, e^{bV_D}$$

$$i_d = I_D \cdot b \cdot v_d$$

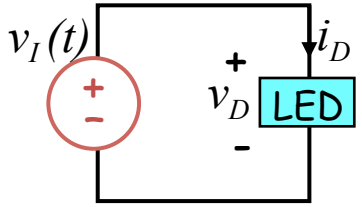
Small signal reponse:

Where did you see i = constant x v before?

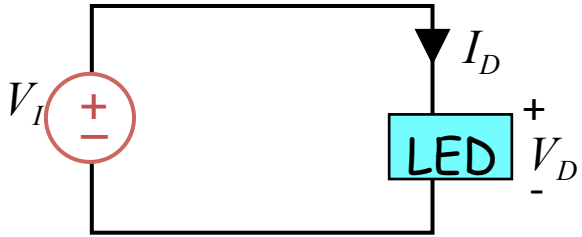
$$R = \left. \frac{1}{\frac{df(v_D)}{dv_D}} \right|_{v_D = V_D}$$

For small signals, device behaves like a resistor!

So, We Can Build a Small Signal Circuit



Large signal circuit:



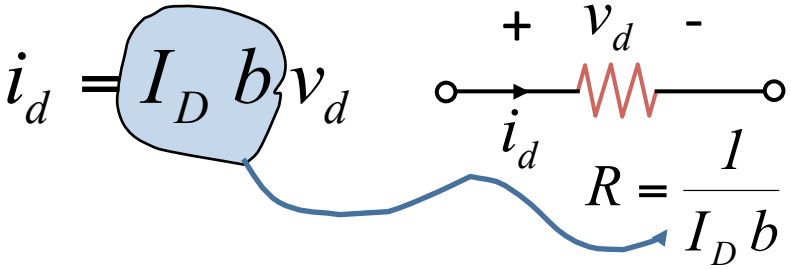
$$I_D = a\, e^{bV_D}$$

$I_D = a\, e^{bV_D}$

$i_d = I_D \cdot b \cdot v_d$

Small signal reponse:

We can build a small signal circuit:



For small signals, device behaves like a resistor!

See pages 222 – 228 of textbook

Small Signal Circuit Method

1. Find operating point using DC bias inputs from large signal circuit

2. Develop small signal (linearized) models for each of the elements around the operating point.

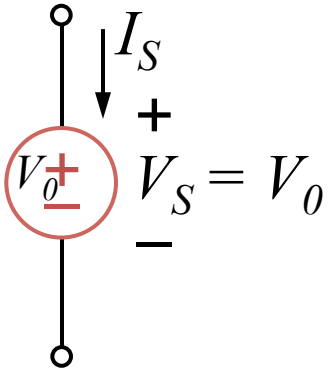
Key: we can use superposition and other linear circuit tools with linearized circuit!

Replace original elements with small signal element models.

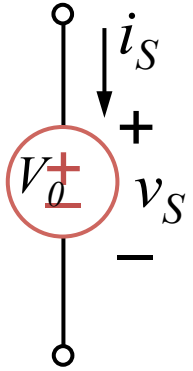
3. Analyze resulting linearized circuit to obtain small signal response…

Step 2: Voltage Sources and DC Supply V_0

large signal



Small signal



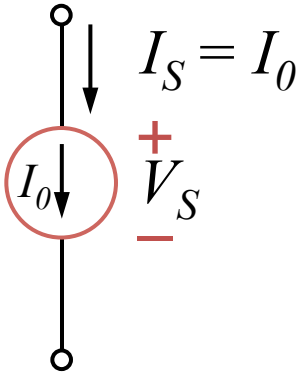
DC voltage source behaves as short to small signals.

DC current source behaves as open to small signals.

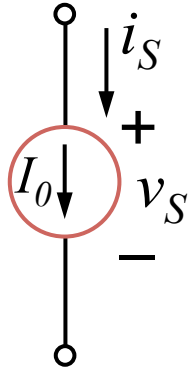
See page 416 of textbook

Current Sources

large signal



$I_S = I_0$



Small signal

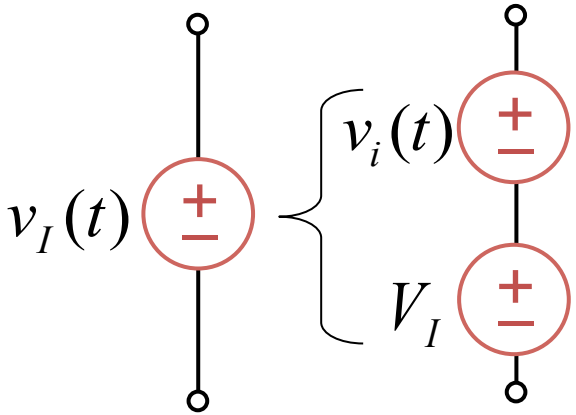
DC voltage source behaves as short to small signals.

DC current source behaves as open to small signals.

Voltage Source Containing Both DC and Small Signal

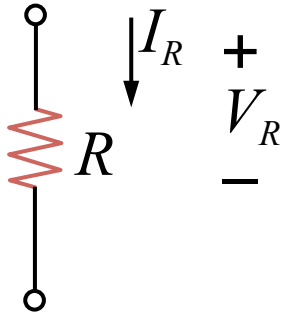
large signal

Small signal

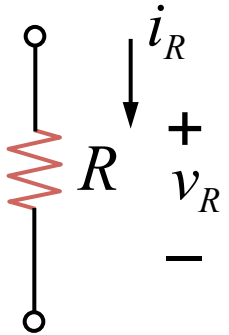


Similarly, R

large signal



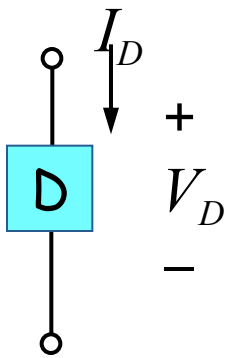
$$V_R = R\, I_R$$



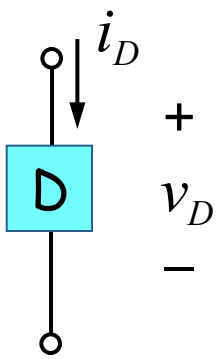
small signal

For Non-Linear Device D

large signal



$$I_D = a\, e^{bV_D}$$



$$i_D = a\, e^{bv_D}$$

small signal

We will visit small signal circuits again shortly…

Small signal circuit analysis example

Step 1

Step 2

Step 3