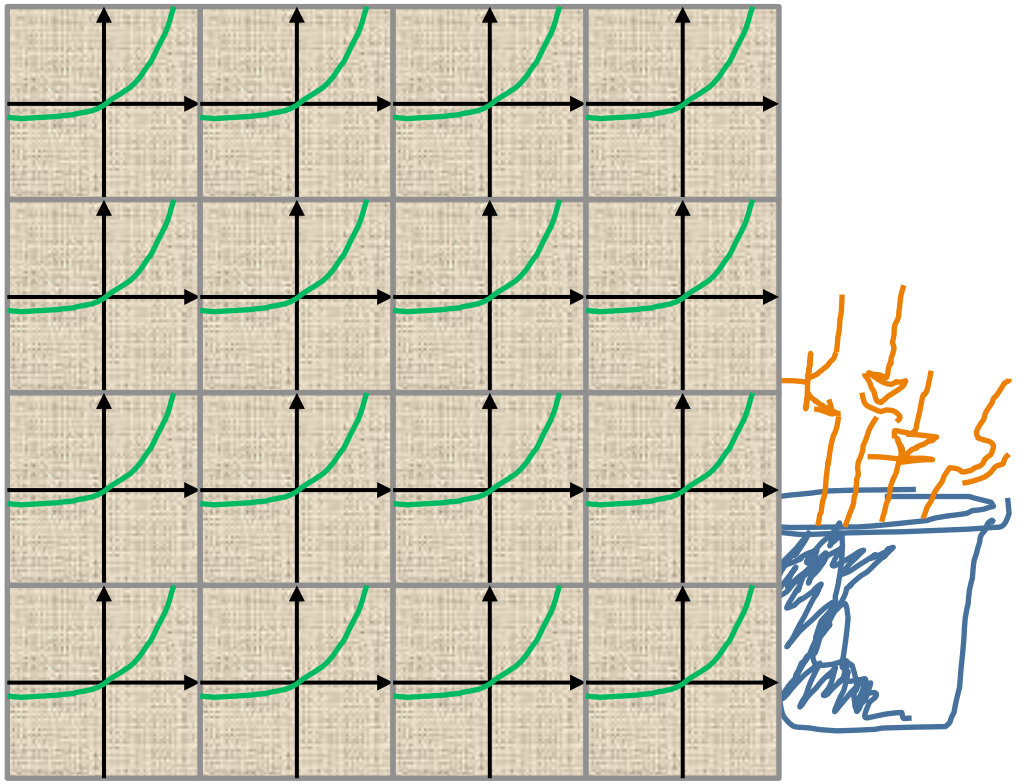
6.002x CIRCUITS AND ELECTRONICS

Nonlinear Circuits



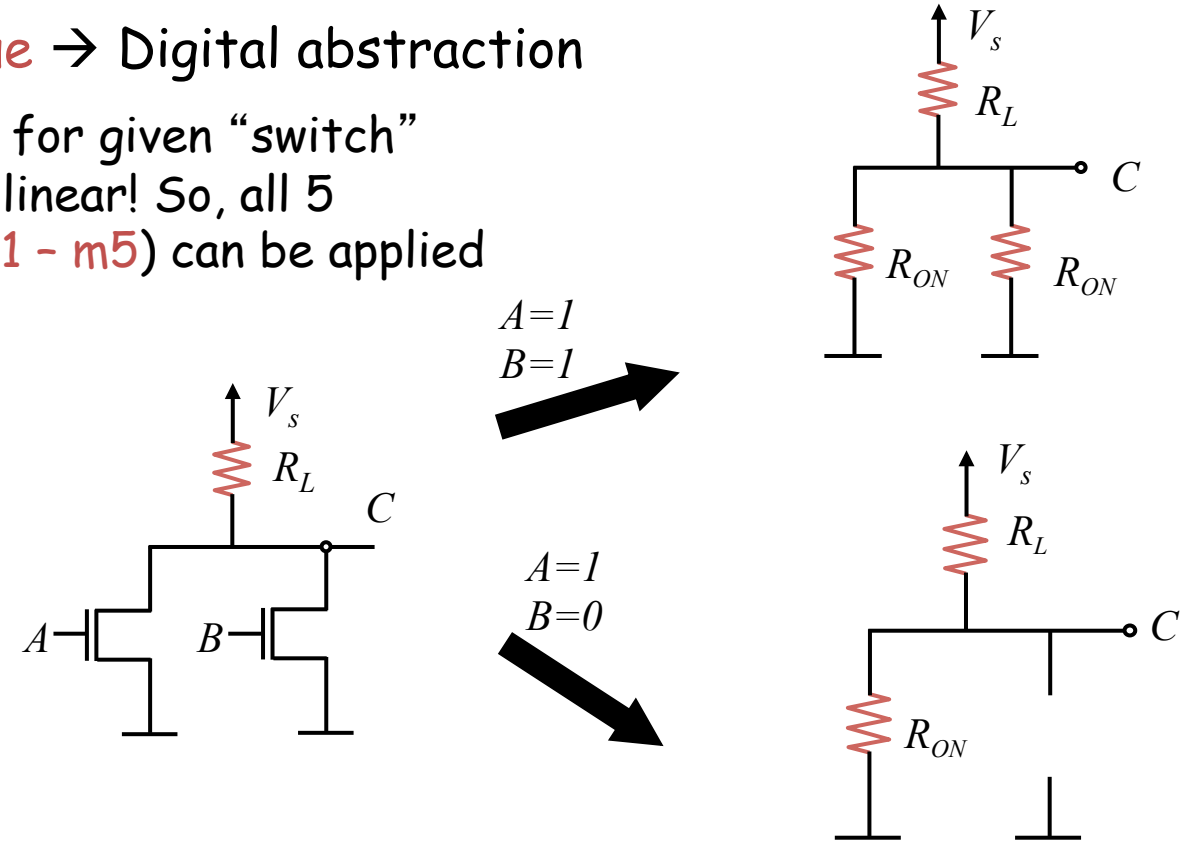
Reading Chap 4.1 – 4.3

Review

- Discretize matter → Lumped circuit abstraction
  - m1 ▶ KVL, KCL, *i-v*
  - m2 ▶ Composition rules
  - m3 ▶ Node method
  - m4 ▶ Superposition
  - m5 ▶ Thévenin, Norton

Review

- Discretize value → Digital abstraction
  - Subcircuits for given "switch" setting are linear! So, all 5 methods (m1 – m5) can be applied



Use SR MOSFET Model

Today

- Nonlinear circuits and their analysis
  - Analytical method
    based on m1, m2, m3
  - Graphical method
  - Piecewise linear method
  - Introduction to incremental analysis

Non-Linear Elements

A square law two-terminal element

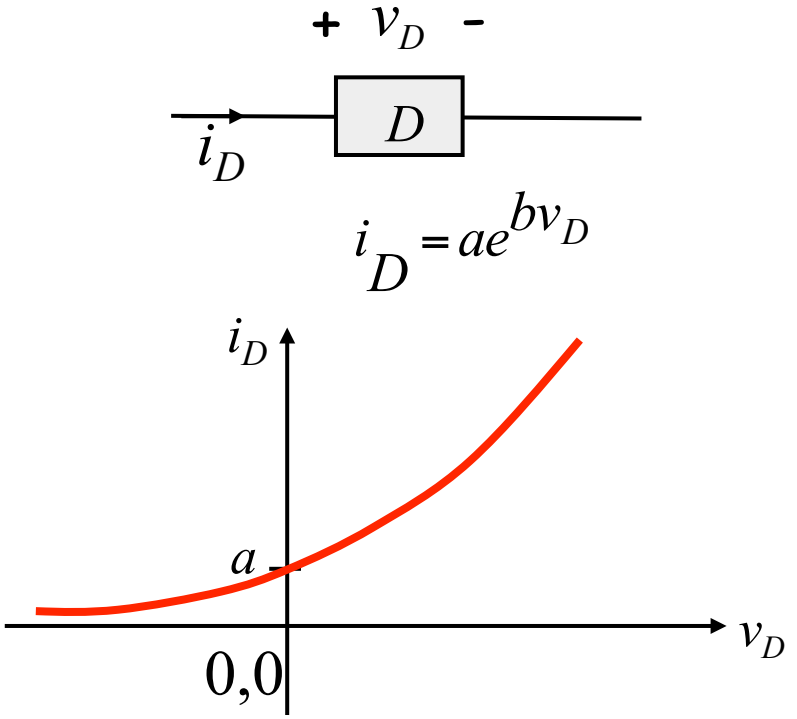
Non-Linear Elements

Hypothetical nonlinear device
(ExpoDweeb ☺)

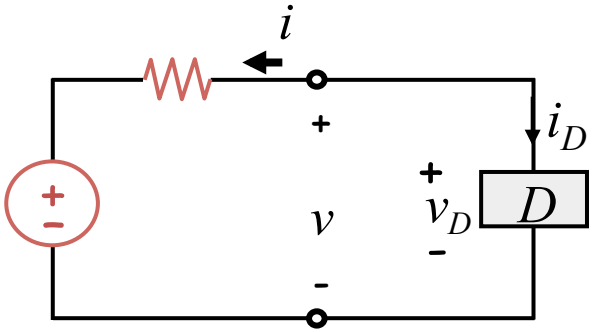
(Curiously, this funky device supplies power when v_D is negative!)

How do we analyze nonlinear circuits

For example:



Quick Aside: Note this Key Circuits Hack



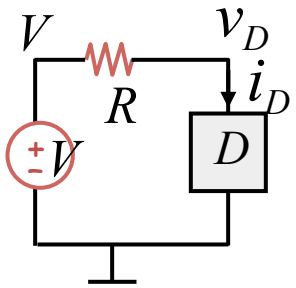
I claim this is a very important circuit pattern! Why?

If you know how to analyze the above circuit pattern, you can analyze any of these more complex circuits below

Quick Aside: Note this Key Circuits Hack Thevenin Trick

Method 1: Analytical Method

Using the node method,
(remember the node method applies for linear or nonlinear circuits)



$$i_D = ae^{bv_D}$$

Solve by trial and error

$$\frac{v_D - V}{R} + i_D = 0 \quad (1)$$

$$i_D = ae^{bv_D} \quad (2)$$

$$\frac{v_D - V}{R} + ae^{bv_D} = 0$$

E.g., for $V=1V$, $R=1\ Ohm$, $a=1/4\ A$, $b=1V^{-1}$

$$v_D = 1 - \frac{1}{4}e^{v_D}$$

1V

0.32V

0.65V

0.52V

0.58V

0.55V

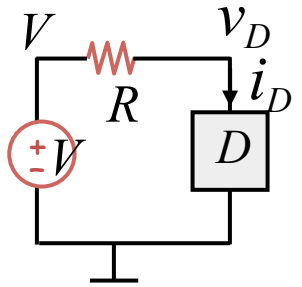
0.56V

0.56V

Corresponding i_D is 0.44A

Method 2: Graphical Method

Using the node method



$$\frac{v_D - V}{R} + i_D = 0 \qquad (1)$$

$$i_D = ae^{bv_D} \qquad (2)$$

$$i_D = ae^{bv_D}$$

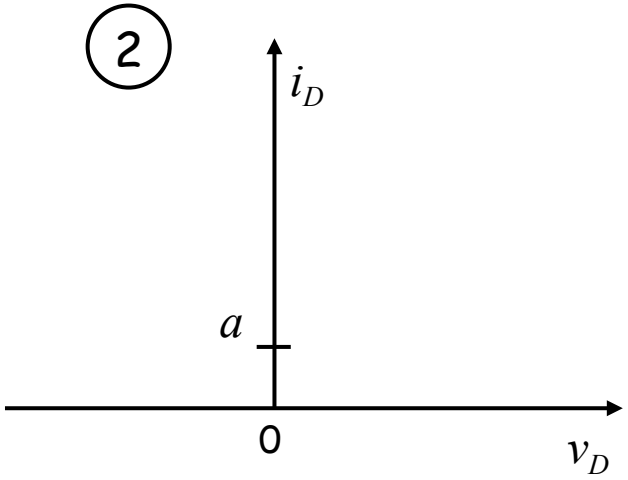
2 unknowns, 2 equations

Can also solve by the graphical method

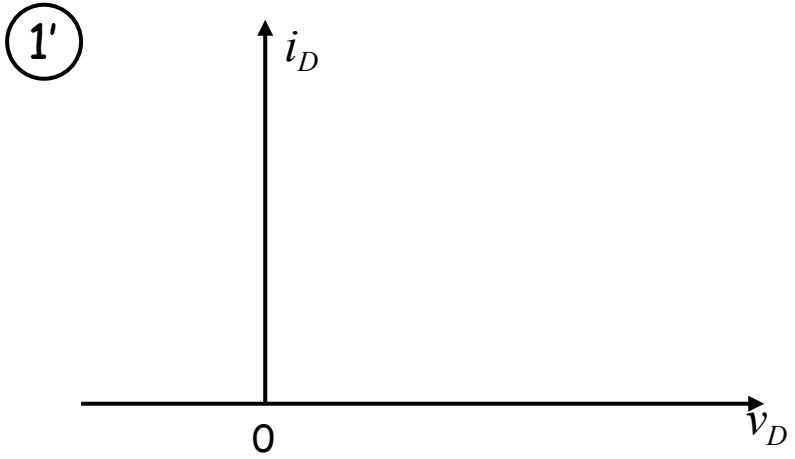
Method 2: Graphical Method

Notice: the solution satisfies equations (1′) and (2)

$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$
$$i_D = ae^{bv_D} \quad (2)$$



Constraint on i_D and v_D imposed by device



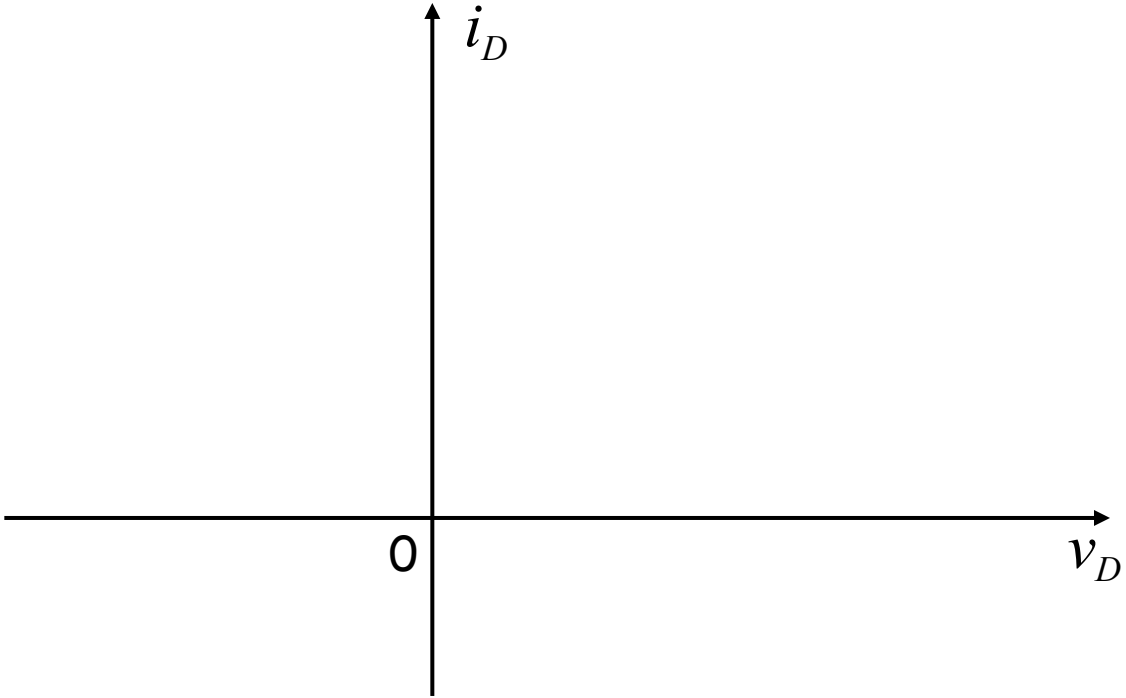
Constraint on on i_D and v_D imposed by the rest of the circuit (Thevenin equivalent circuit)

Combine the two constraints

$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$

$$i_D = ae^{bv_D} \quad (2)$$

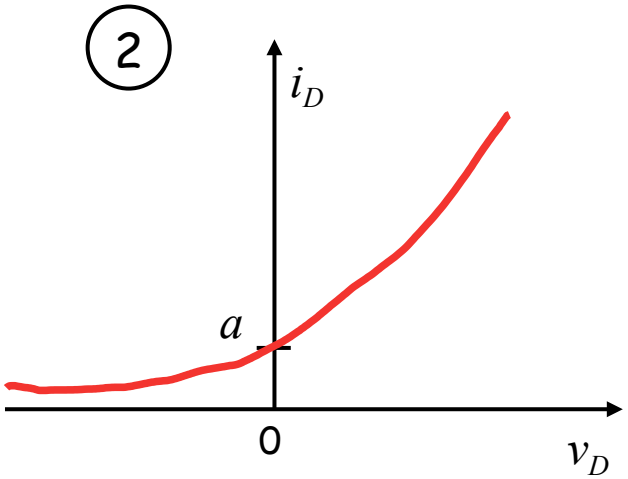
E.g., for $V=1V$, $R=1\ Ohm$, $a=1/4\ A$, $b=1V^{-1}$

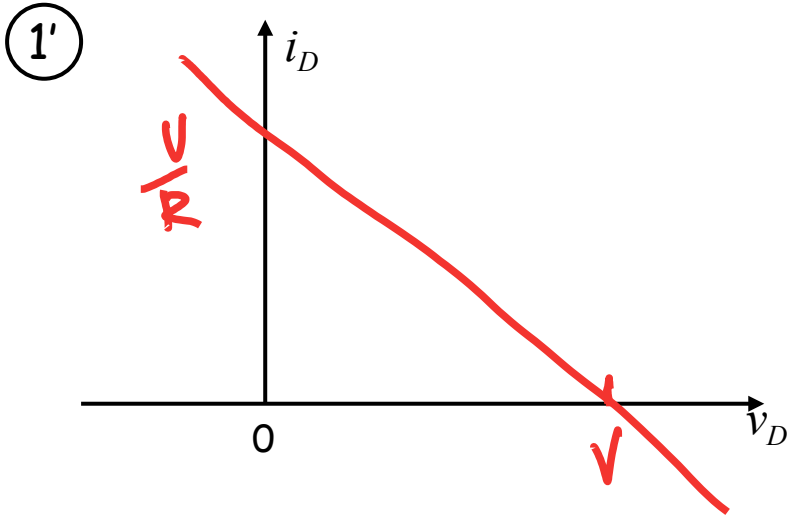


Combining the two constraints

$$i_D = \frac{V}{R} - \frac{v_D}{R} \quad (1')$$

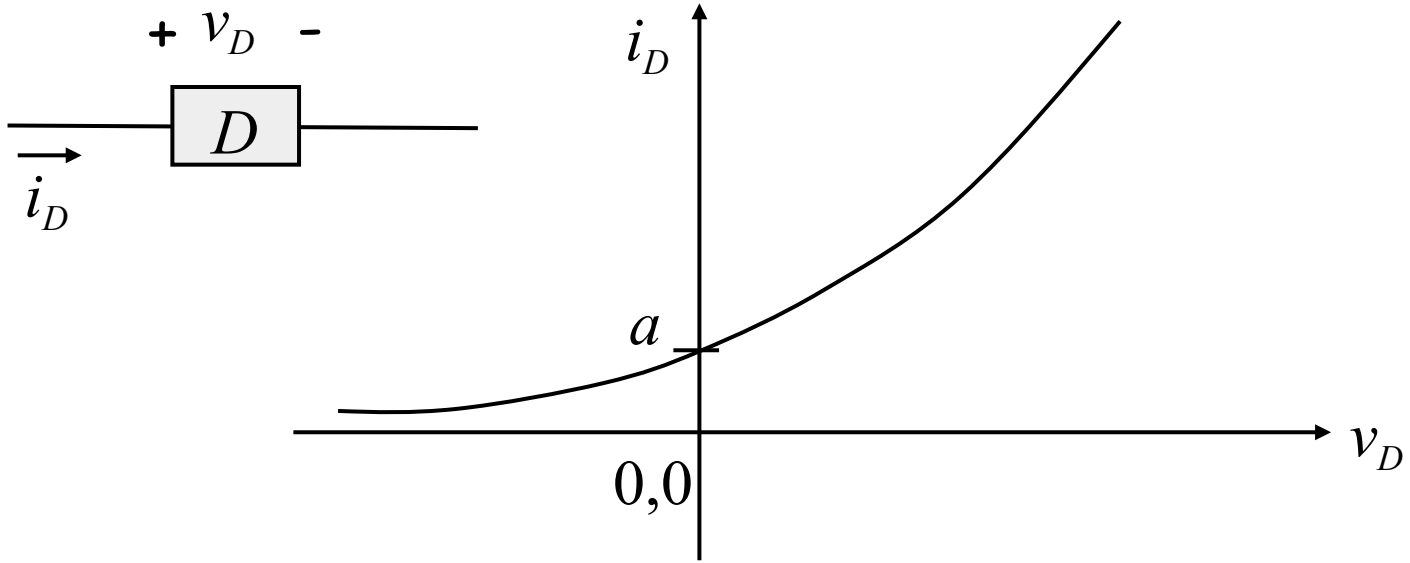
$$i_D = ae^{bv_D} \quad (2)$$





Method 3: Piecewise Linear Method

Insight:

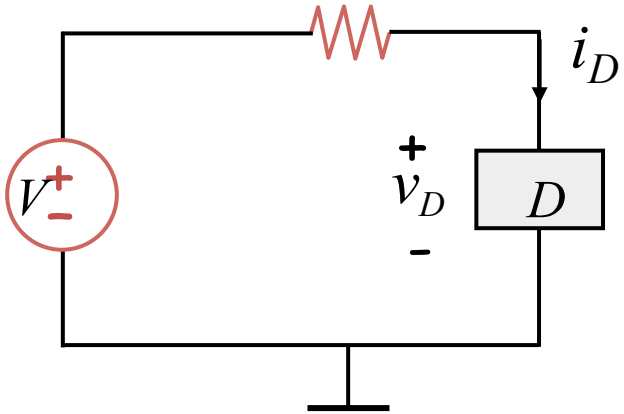


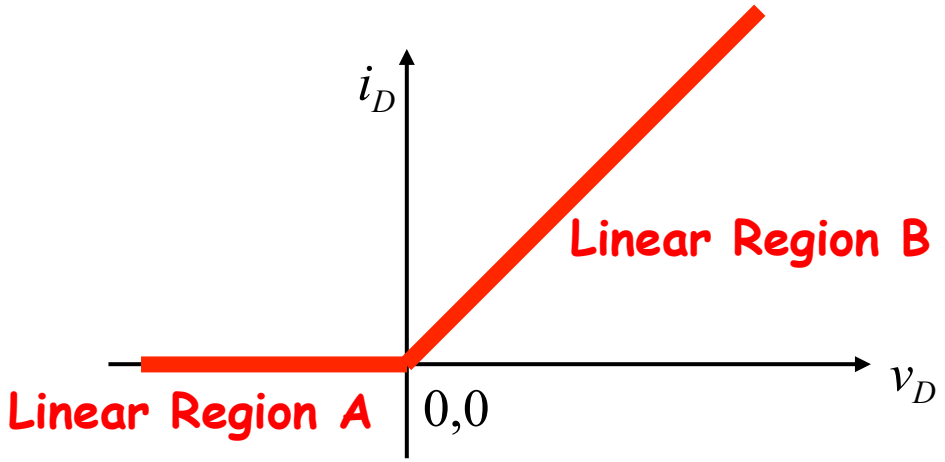
Determine which of the linear regions applies (e.g., region B)
Solve circuit assuming that linear relation (from region B)

See Sec 4.4 of the text for details and examples

Piecewise Linear Method

For example:





Suppose we are given that V is positive

Then D will operate in Linear Region B

We can replace D in the circuit with a resistance of value $1/R$ when D operates in region B

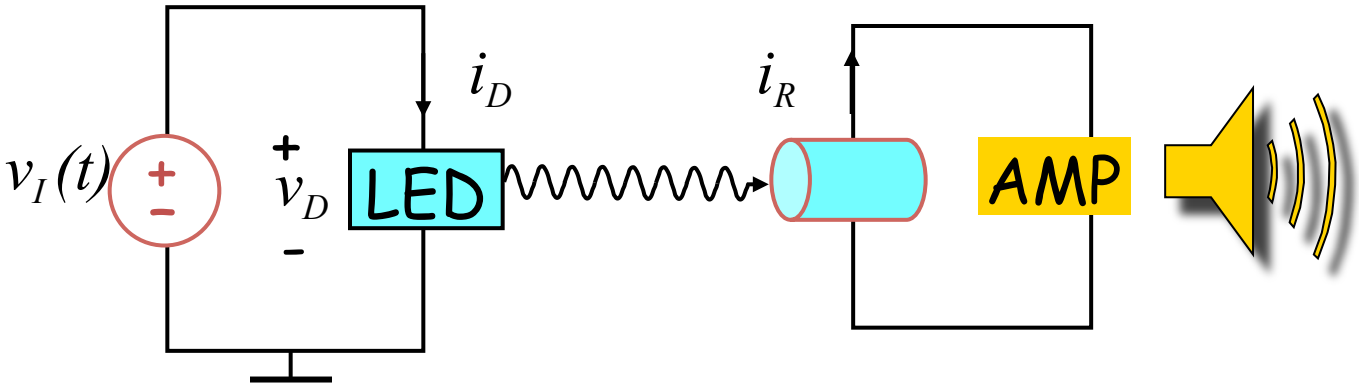
Method 3: Incremental Analysis

(Actually, a disciplined way of using a circuit called small signal method)

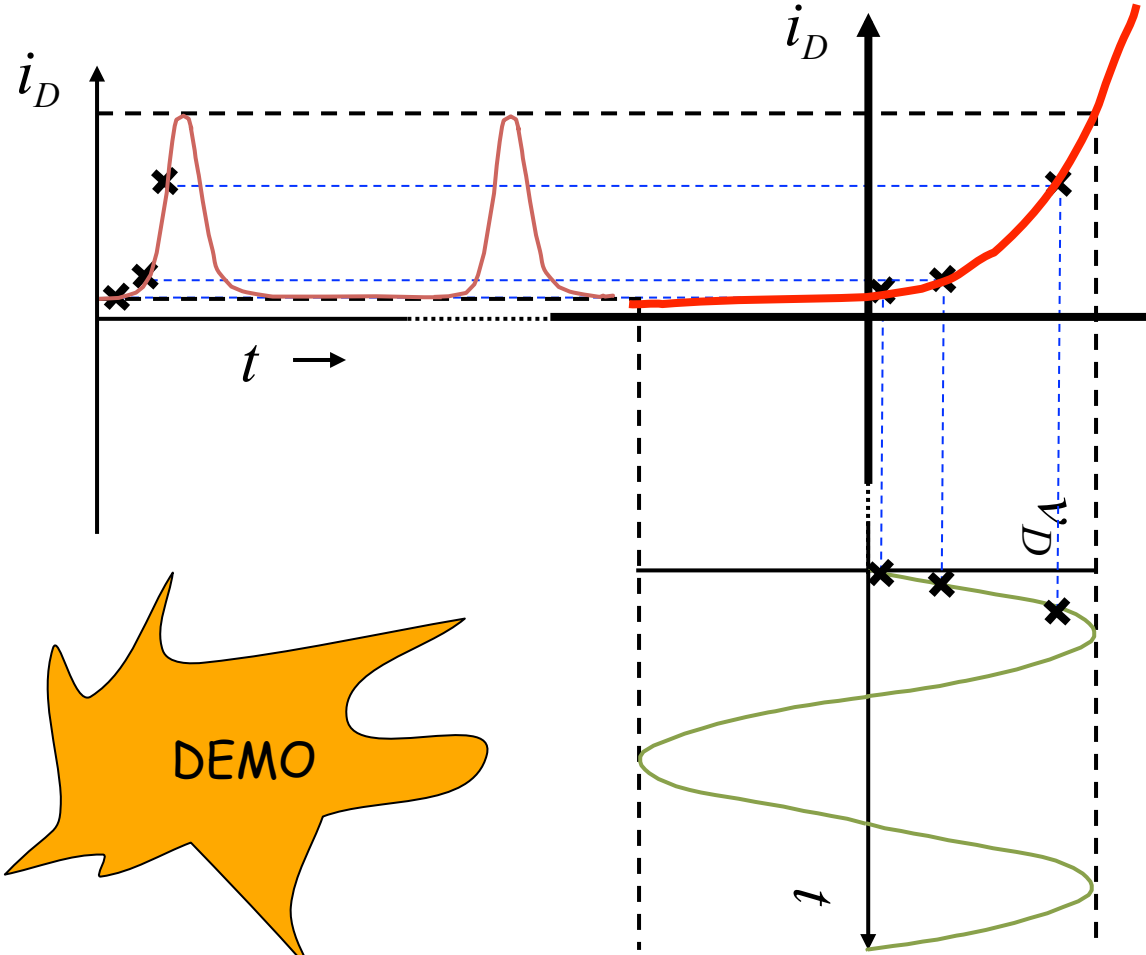
Motivation: music over a light beam. Can we pull this off?

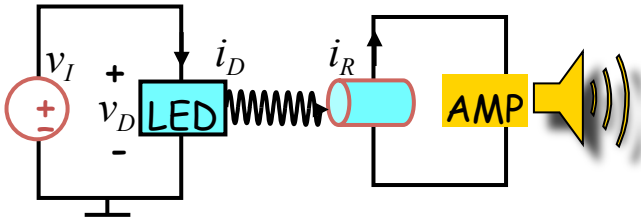
Remember
LED: Light
Emitting
expoDweep ☺

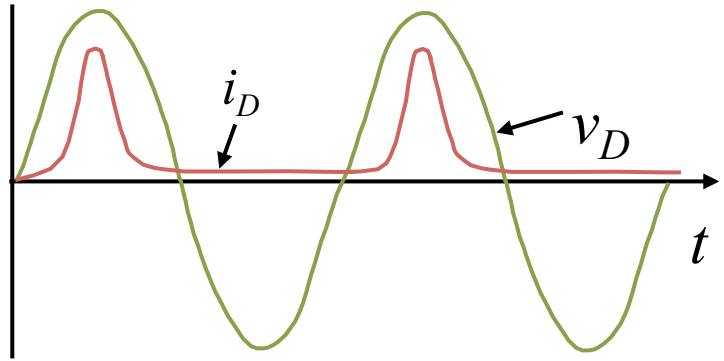
Method 3: Incremental Analysis



Problem: The LED is nonlinear → distortion

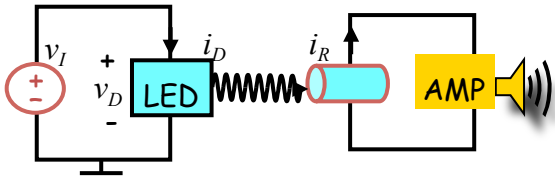


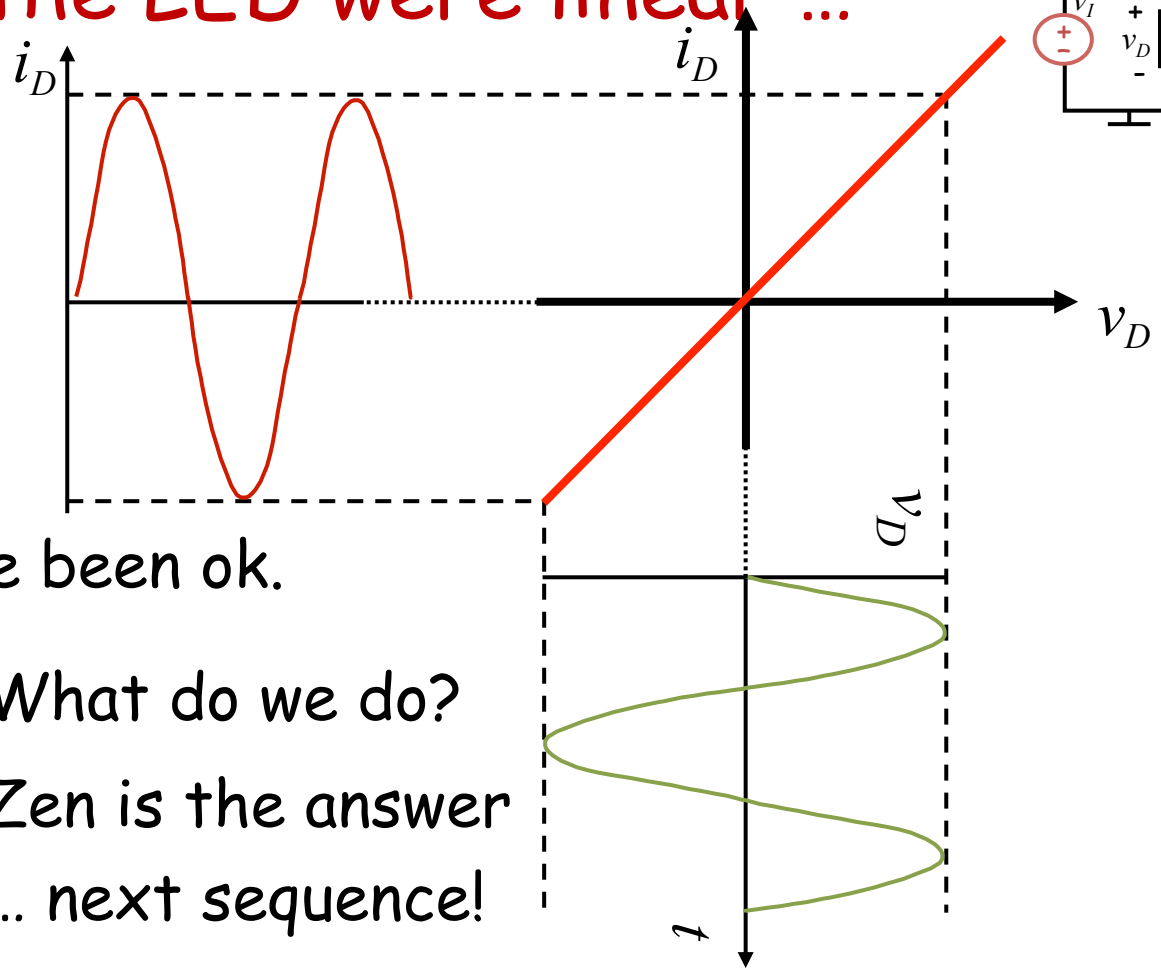




DEMO

If only the LED were linear …





it would've been ok.

What do we do?

Zen is the answer

… next sequence!