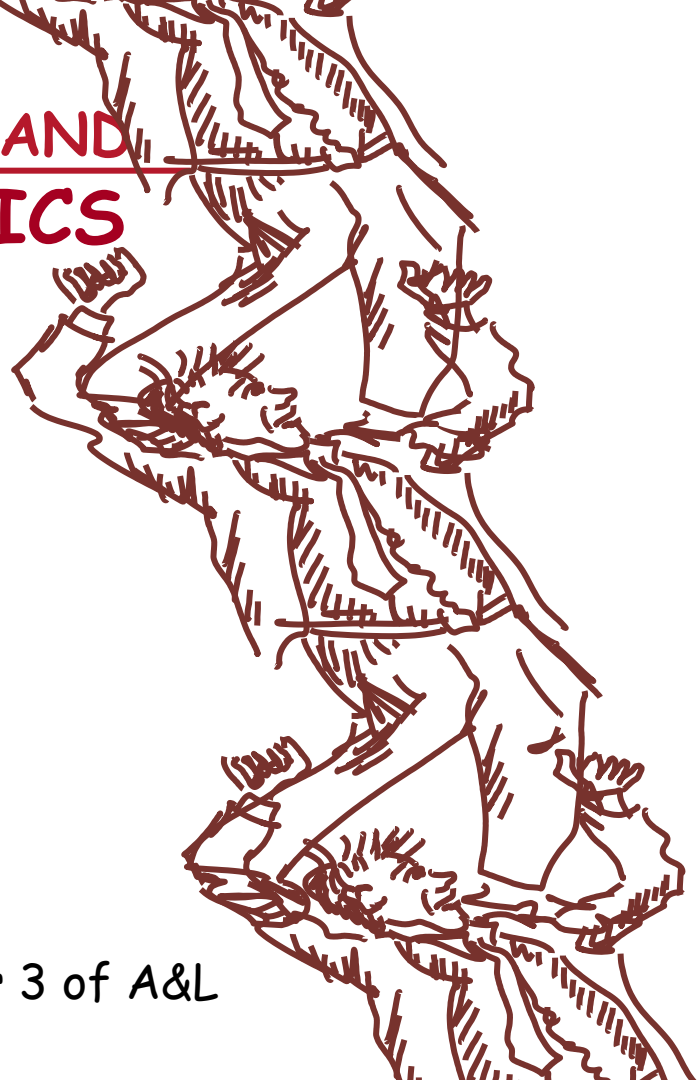
6.002 CIRCUITS AND ELECTRONICS

Superposition, Thévenin and Norton



Reading: Chapter 3 of A&L

Review

Circuit Analysis Methods

- KVL: KCL: VI

- Circuit composition rules
- Node method – the workhorse of 6.002
  KCL at nodes using V's referenced from ground
  KVL implicit in pattern

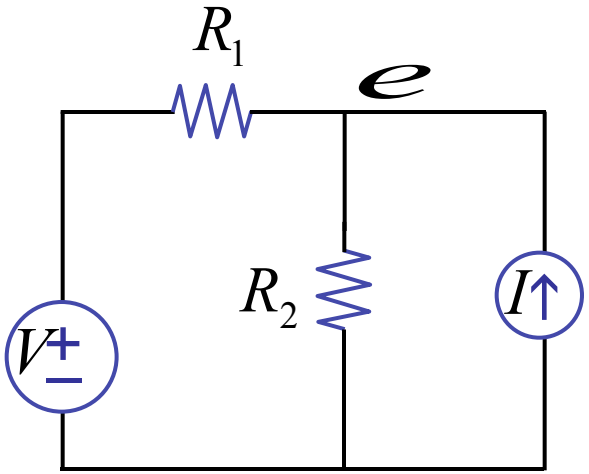
Overview

- Introduction to linear circuits
- Properties of linearity
- The superposition tool for your toolkit
- The Thévenin method
- The Norton method

Let's start by introducing linearity

Linearity

Consider

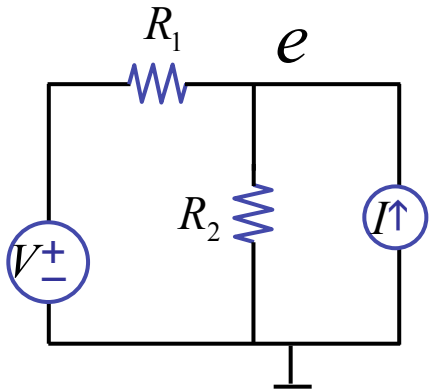


Write node equations –

Linearity

Write node equations --

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$ linear in e, V, I

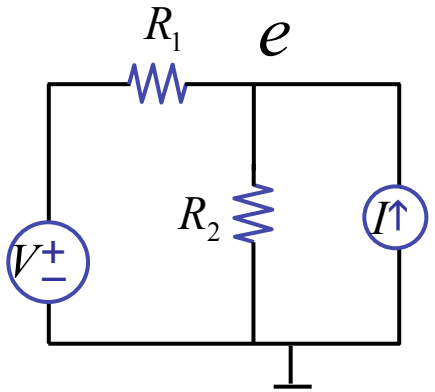


Rearrange --

Linearity

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

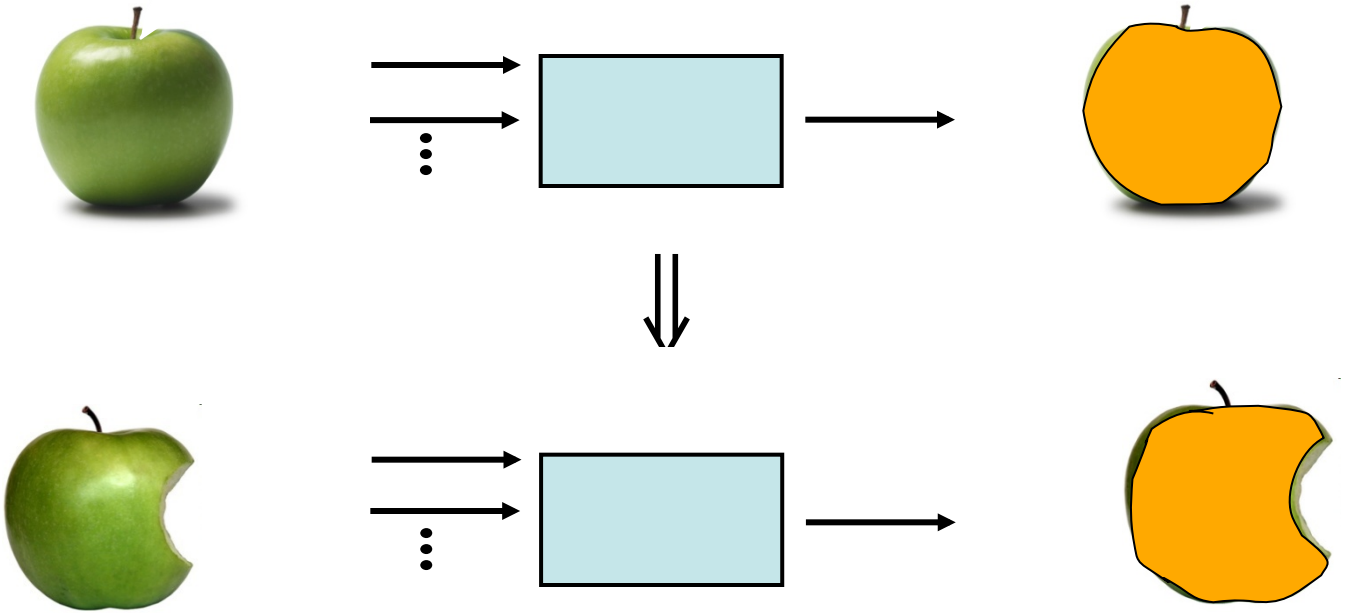
$$\left[\frac{1}{R_1}+\frac{1}{R_2}\right]e=\frac{V}{R_1}+I$$



Linearity $\Longrightarrow$ Homogeneity
Superposition

Linearity $\Rightarrow$ Homogeneity
Superposition

Homogeneity



Linearity

Superposition

$\Rightarrow$

Homogeneity

Superposition









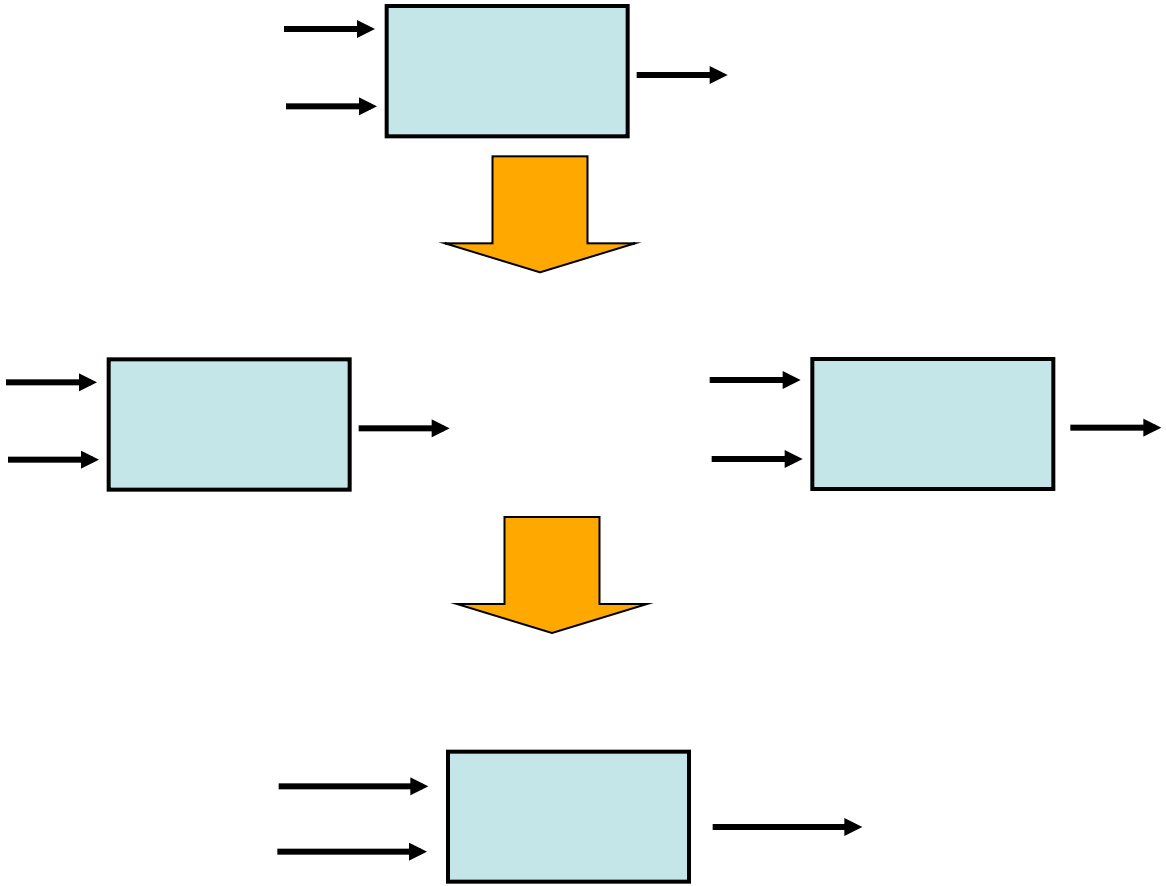




$\Downarrow$



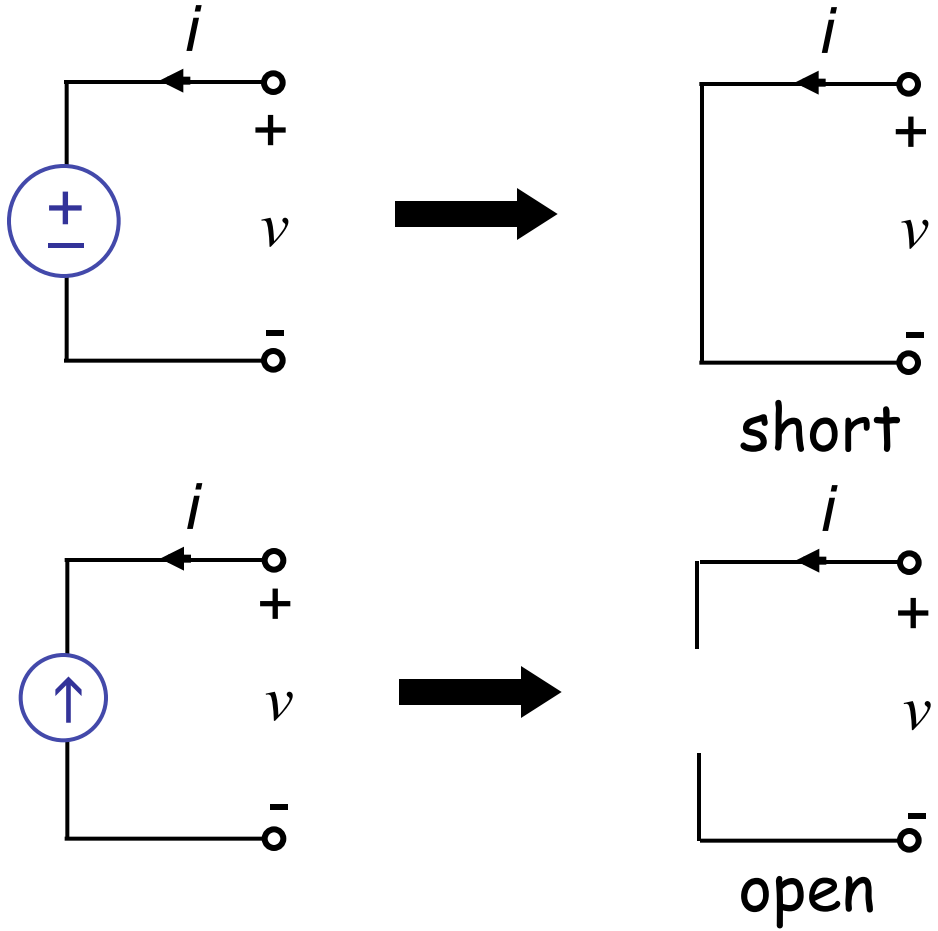
Specific superposition example:



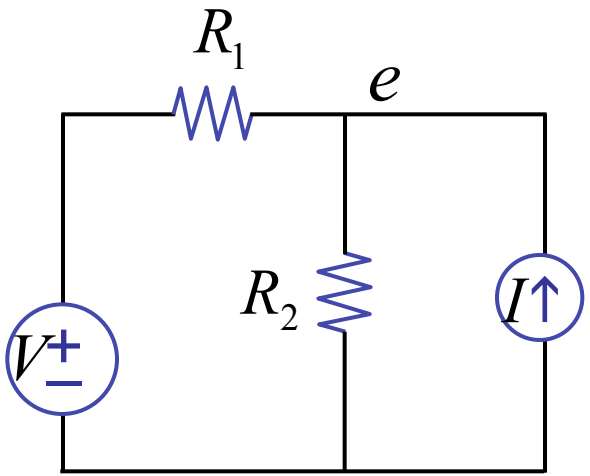
Method 4: Superposition method

1. Find the responses of the circuit to each source acting alone
2. Sum the individual respones

Each source acting alone means this



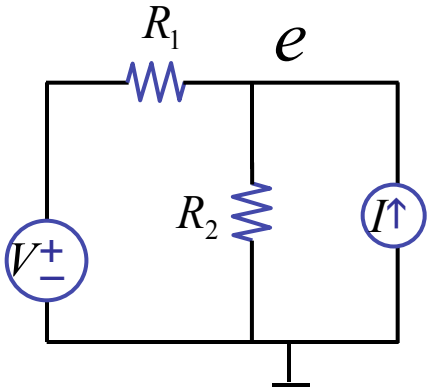
Back to the example



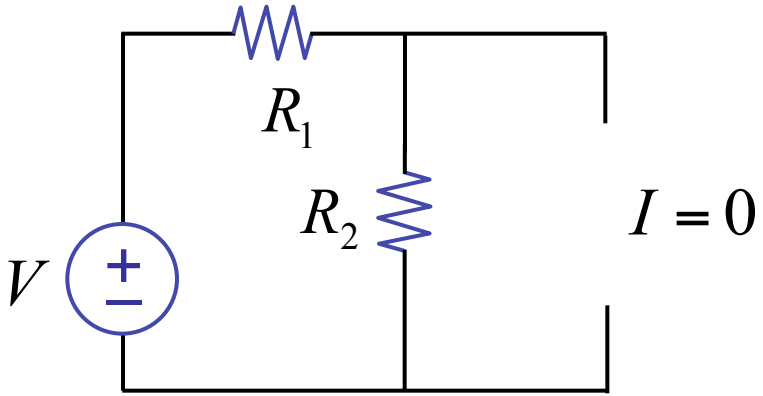
Use superposition method

Back to the example

Use superposition method



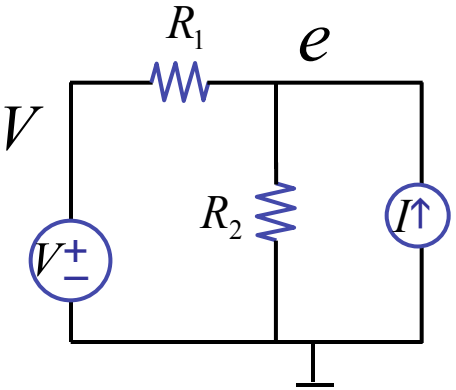
V acting alone



Back to the example

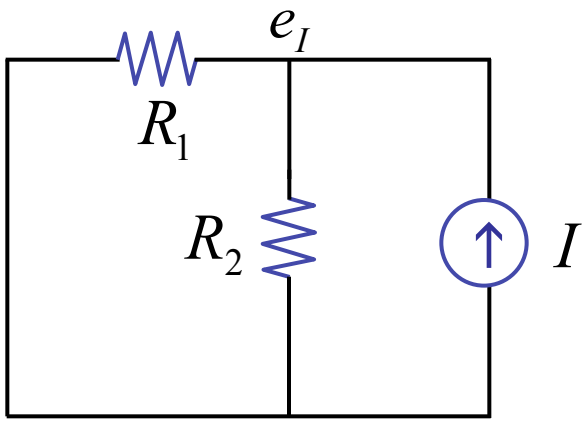
Use superposition method

$$e_V = \frac{R_2}{R_1 + R_2} V$$



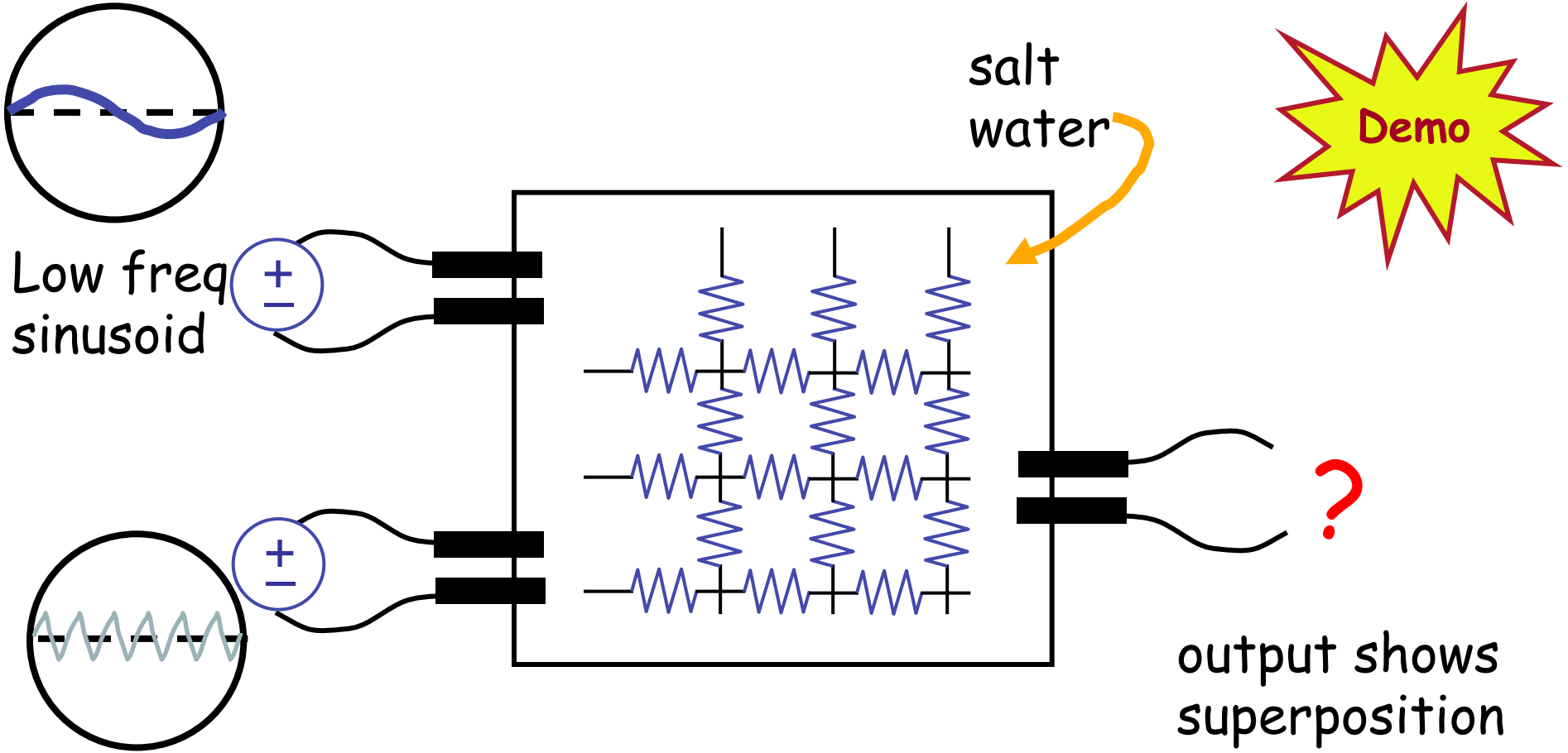
I acting alone

$V = 0$



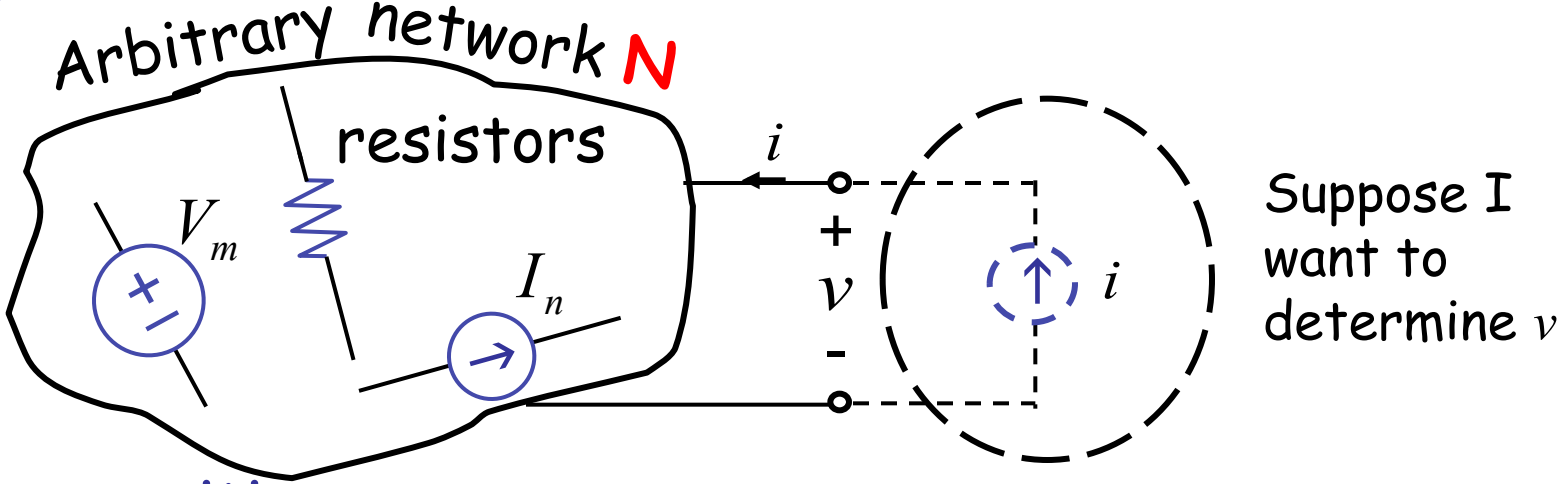
By superposition, sum the two partial voltages

Voilà !


salt
water
Demo
Low freq
sinusoid
+
−
+
−
?
output shows
superposition
High freq
triangular wave

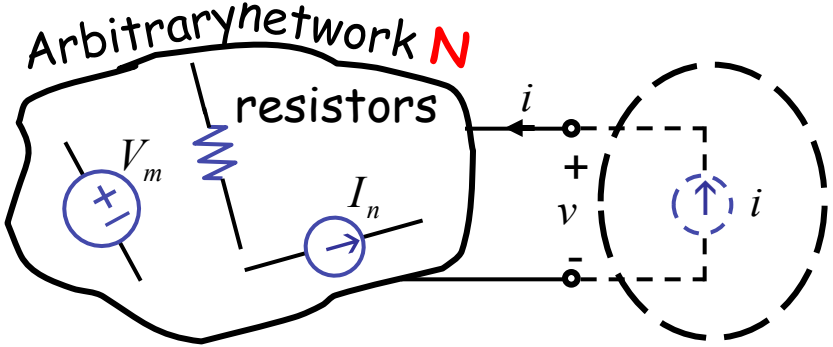
Yet another method…

Consider



By superposition

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

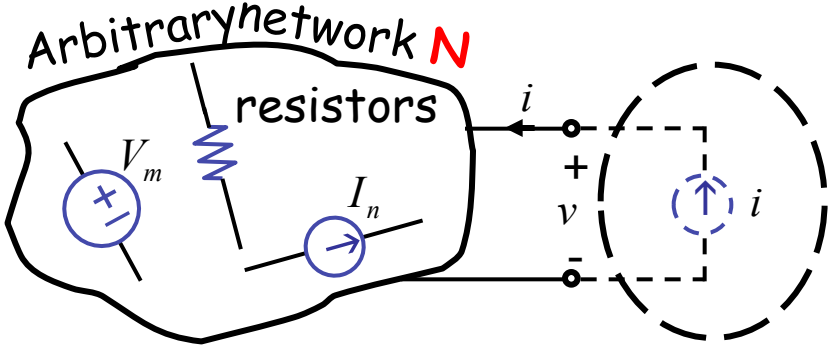


also independent of external excitement and behaves like a resistor. Let's call it "R_{TH}"

Independent of external excitation and behaves like a voltage.

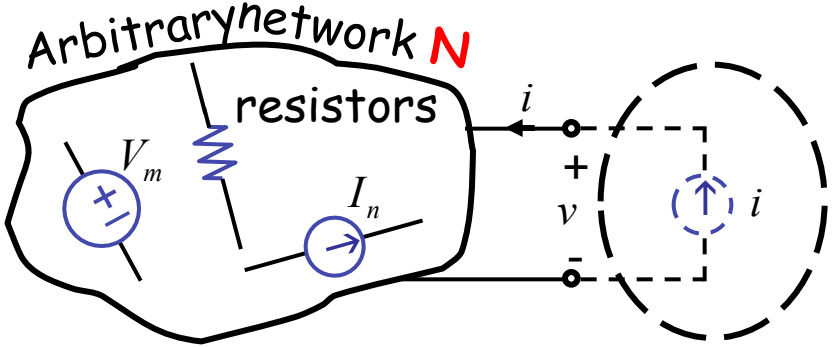
Let's call it "v_{TH}"

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

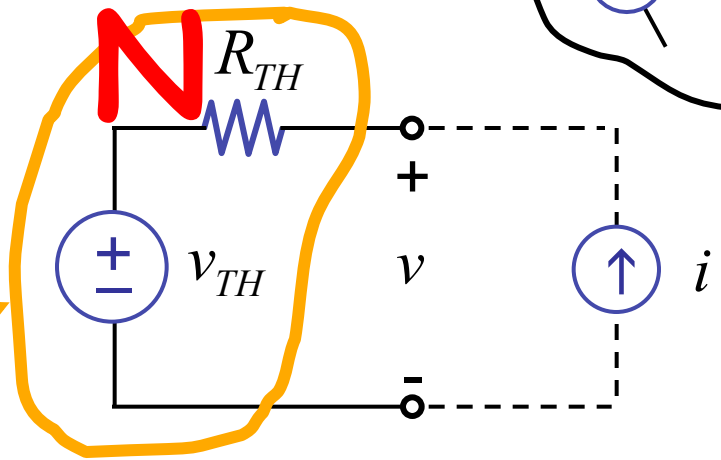


In other words, as far as the external world is concerned (for the purpose of the i-v relation), "arbitrary network N" is indistinguishable from:

$$v = v_{TH} + R_{TH} i$$



Thévenin equivalent network

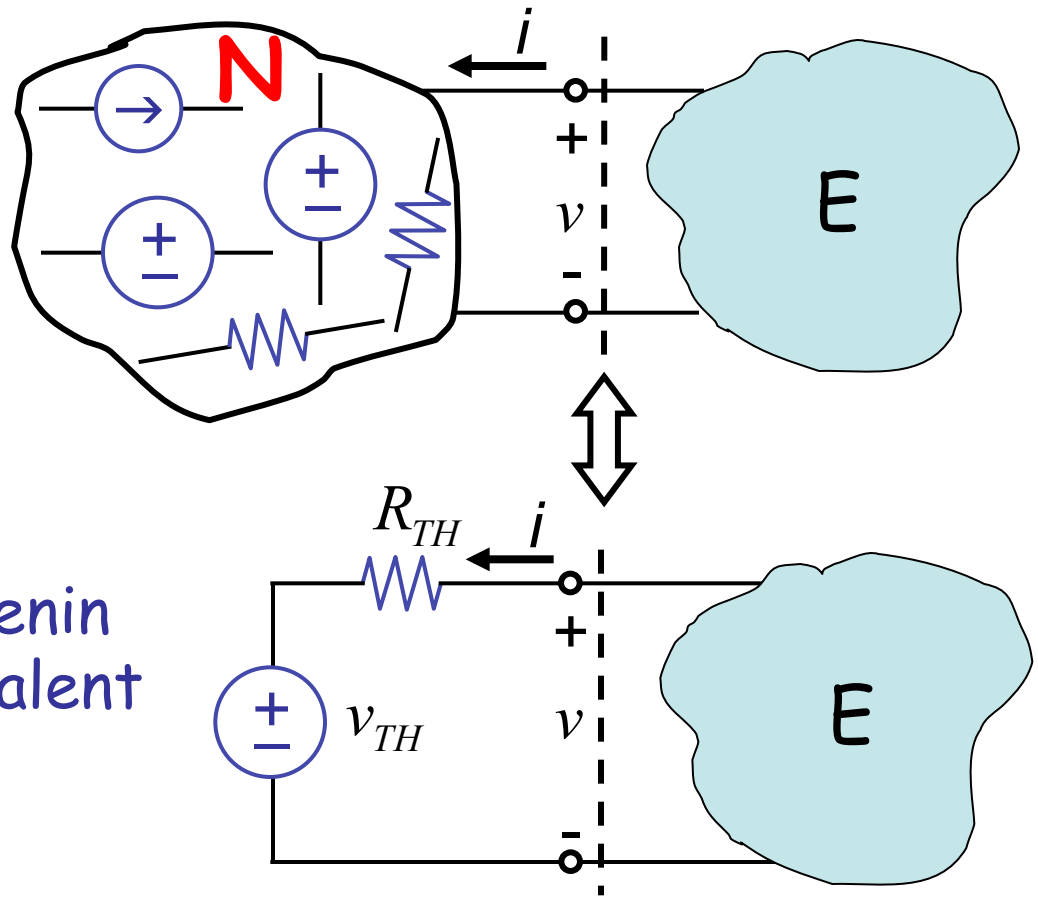


v_{TH} ⟶ Open circuit voltage seen at terminal pair (aka port)

R_{TH} ⟶ Resistance of network seen from port

(V_m's, I_n's set to 0)

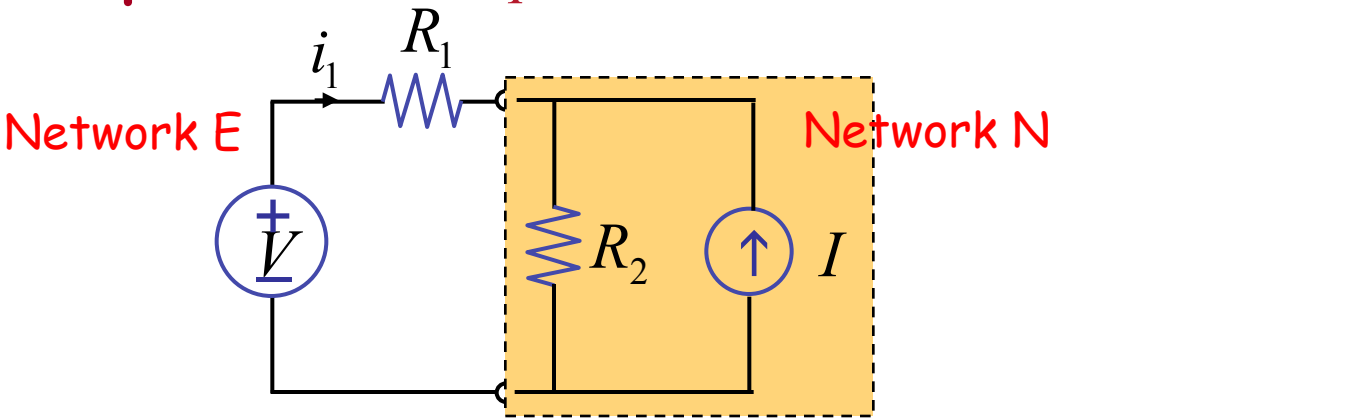
Method 4: The Thévenin Method



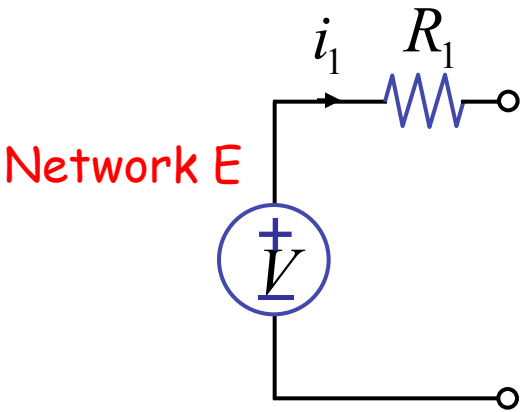
Thévenin equivalent

1. Replace network N with its Thévenin equivalent
2. Then solve with external network E.

Example: Find i_1

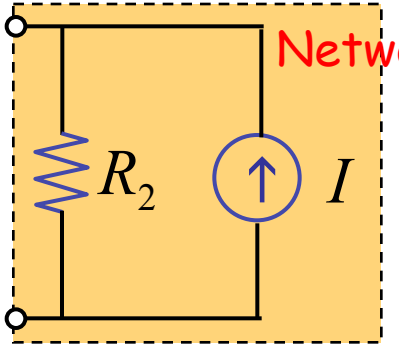


Step 1: Replace N with Thévenin equivalent



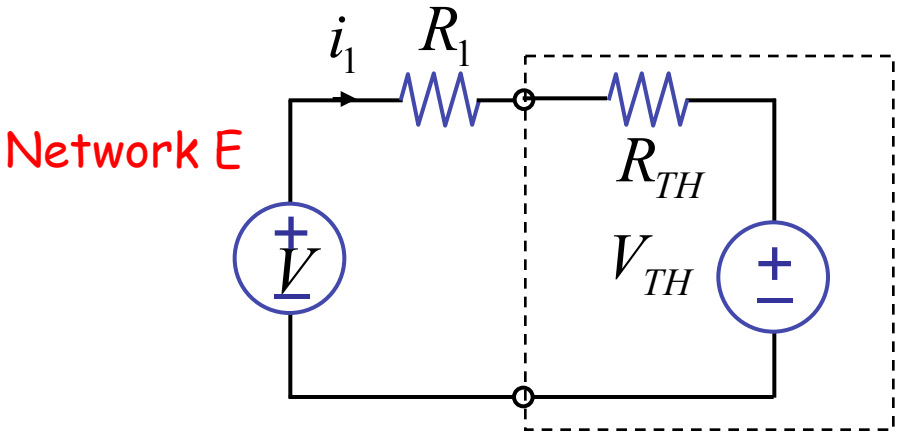
Step 1: Replace with Thévenin equivalent

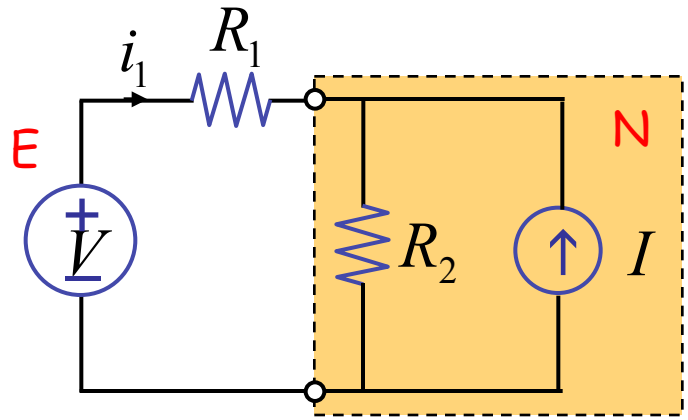
V_{TH} : open circuit voltage of network N



R_{TH} : turn off all independent sources and measure resistance of N

Step 2: Solve with external network E

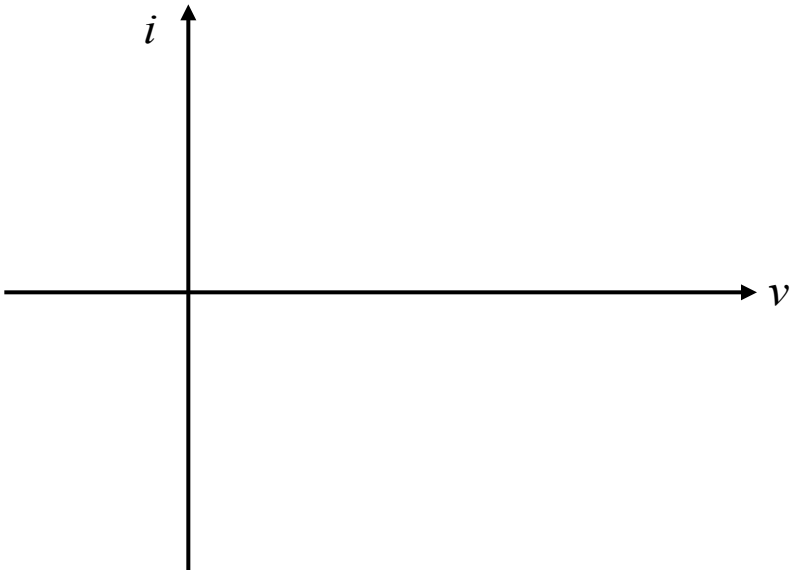


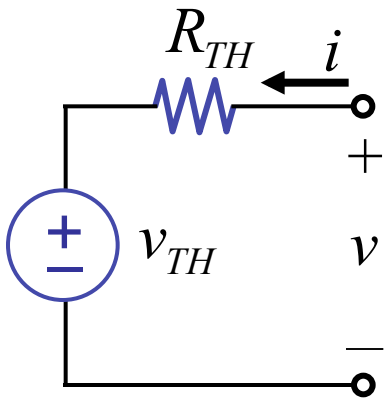


$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

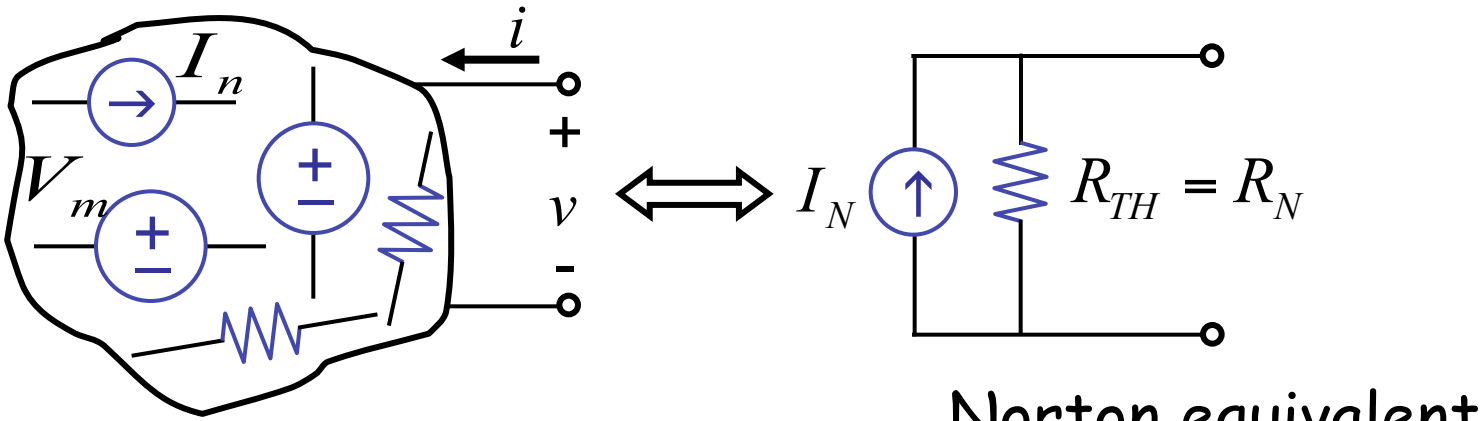
Graphically,

$$v = v_{TH} + R_{TH} i$$





Method 5: The Norton Method

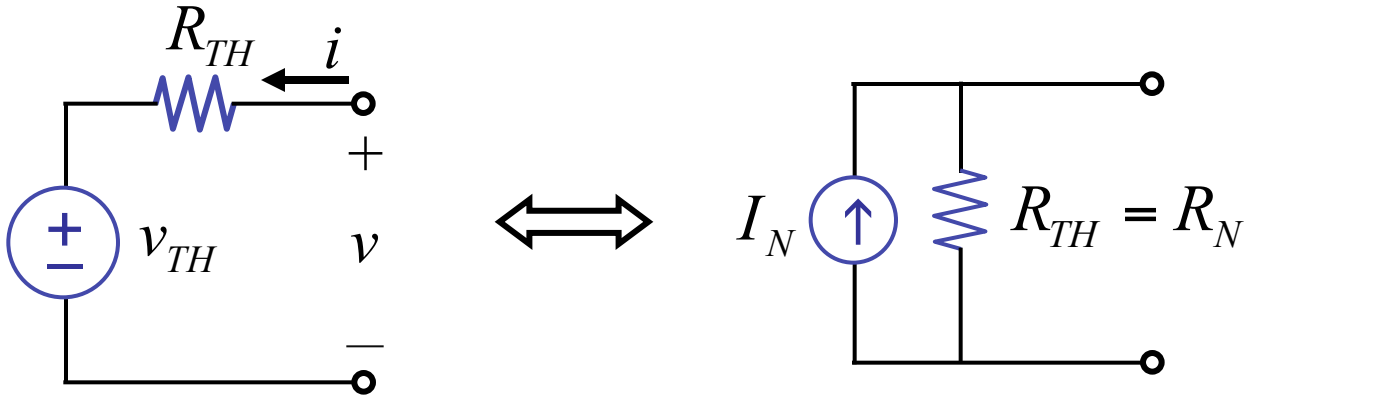


Norton equivalent

I_N ⟶ Short circuit current seen at port

R_{TH} ⟶ Resistance of network seen from port

(V_m's, I_n's set to 0)



Thevenin equivalent

Norton equivalent

Summary

Discretize matter by agreeing to observe the lumped matter discipline

Summary

Circuit analysis methods

- KVL, KCL, I — V
- Combination rules
- Node method
- Superposition
- Thévenin
- Norton

Next – Nonlinear networks