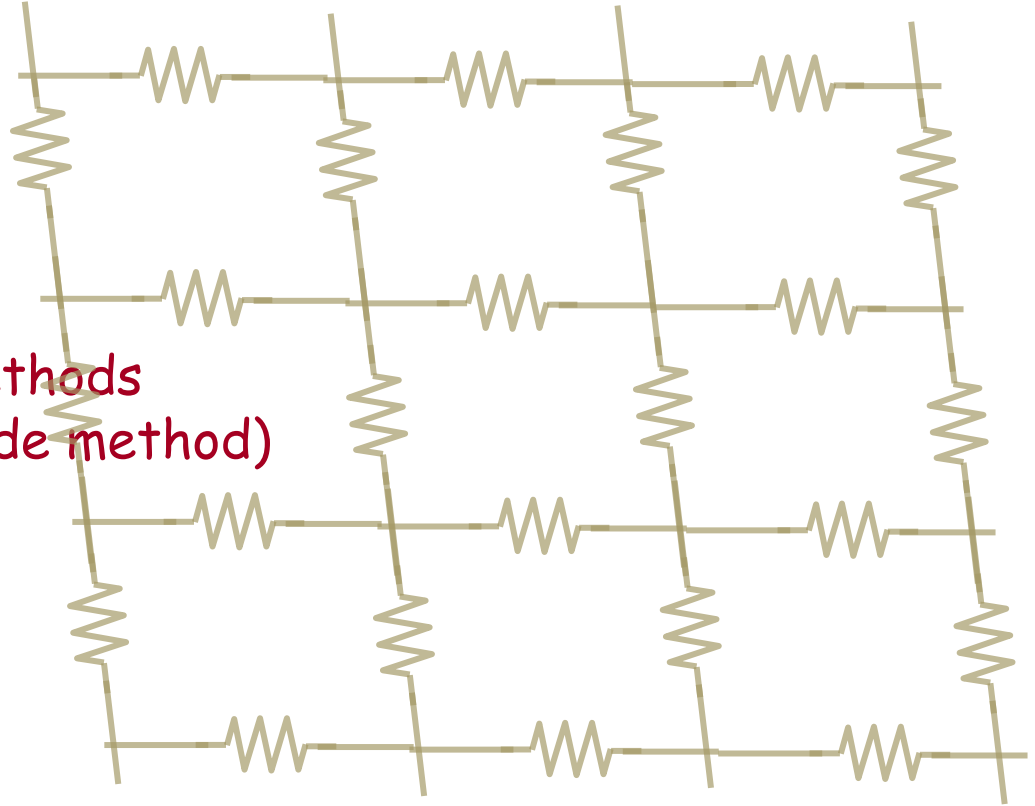
6.002x CIRCUITS AND ELECTRONICS

Basic Circuit Analysis Methods
(KVL and KCL method, Node method)

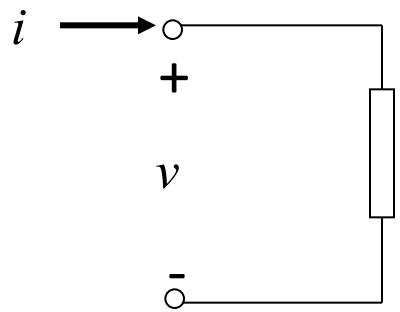


Review

Remember, our EECS playground

Observe the lumped matter discipline LMD

Review

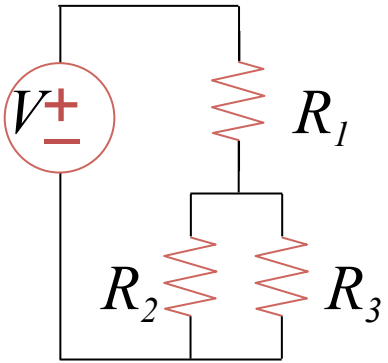


Lumped circuit element

power consumed by element = vi

Review

LMD allows us to create the lumped circuit abstraction

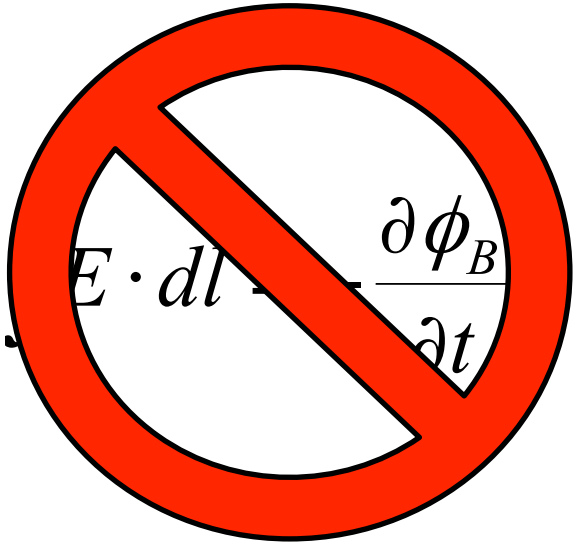


Review

Maxwell's equations simplify to algebraic KVL and KCL under LMD!

KVL:

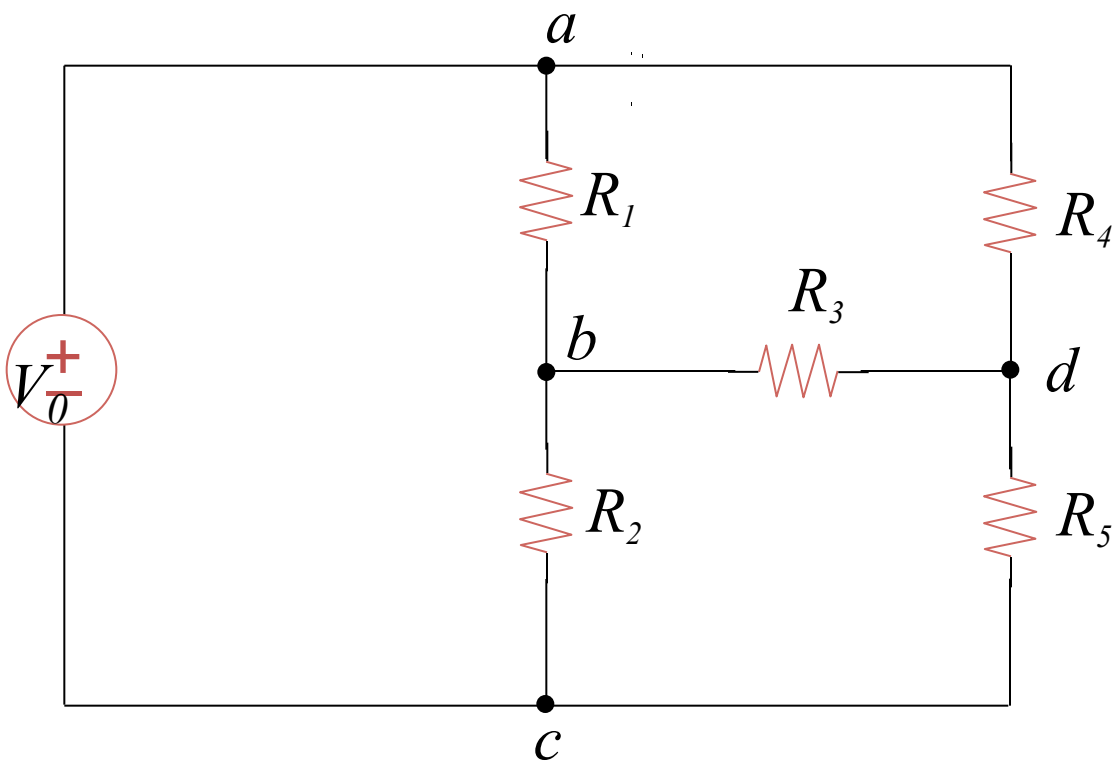
$$\sum_j v_j = 0$$ For all loops



KCL:

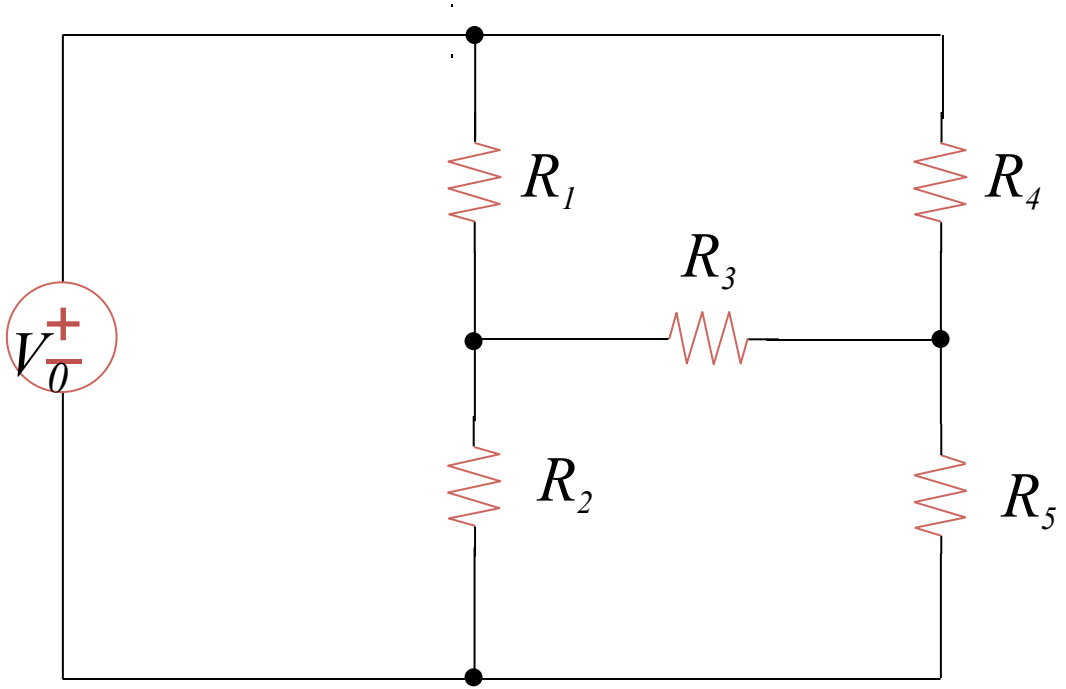
$$\sum_j i_j = 0$$ For all nodes

Review



DEMO

Let's Begin by Building aToolchest of Analysis Techniques



Analyzing a circuit means:

Find all the element v's and i's

Method 1: Basic KVL, KCL method of Circuit analysis

Goal: Find all element v's and i's

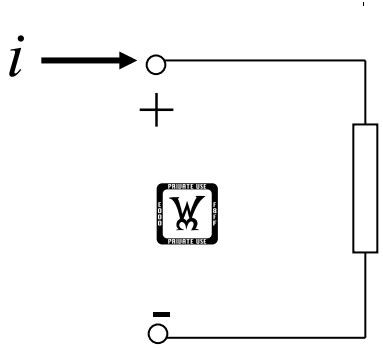
1. write element v-i relationships
   (from lumped circuit abstraction)
2. write KCL for all nodes
3. write KVL for all loops

lots of unknowns
lots of equations
lots of fun
solve

Method 1: Basic KVL, KCL method of Circuit analysis

Goal: Find all element v's and i's

Labeling element v's and i's



Element e

This convention is called: Associated variables discipline

Current is taken to be positive going into the positive voltage terminal

Then power consumed by element e

$= vi$ is positive

Method 1: Basic KVL, KCL method of Circuit analysis

You will need this for step 1: Element Relationships

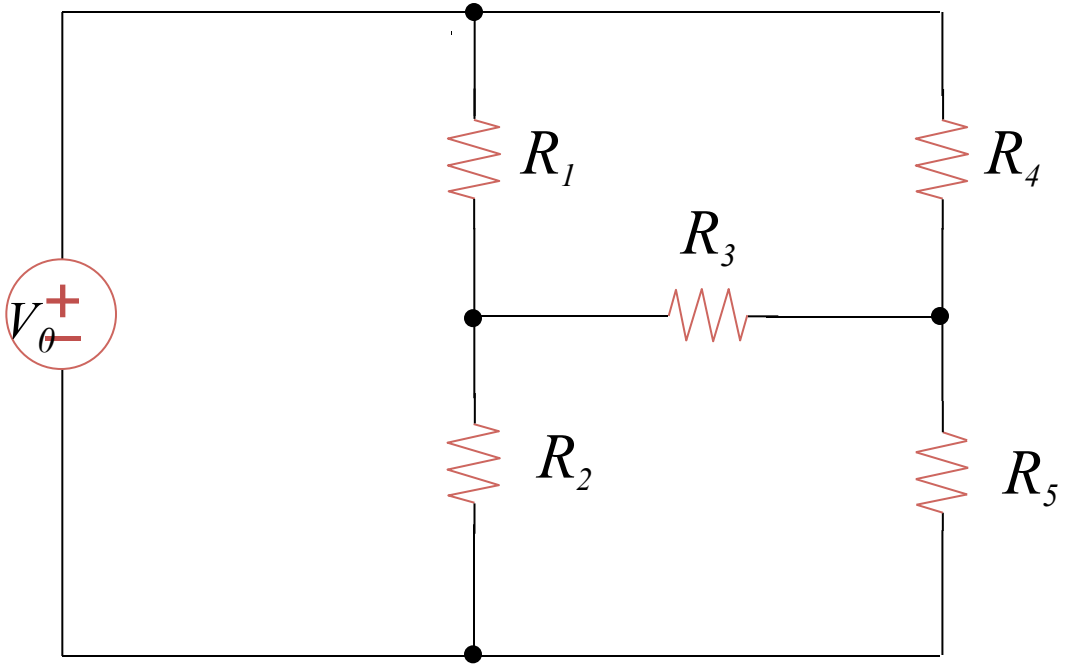
For R

For voltage source

For current source

Let's Apply KVL, KCL Method to this Example

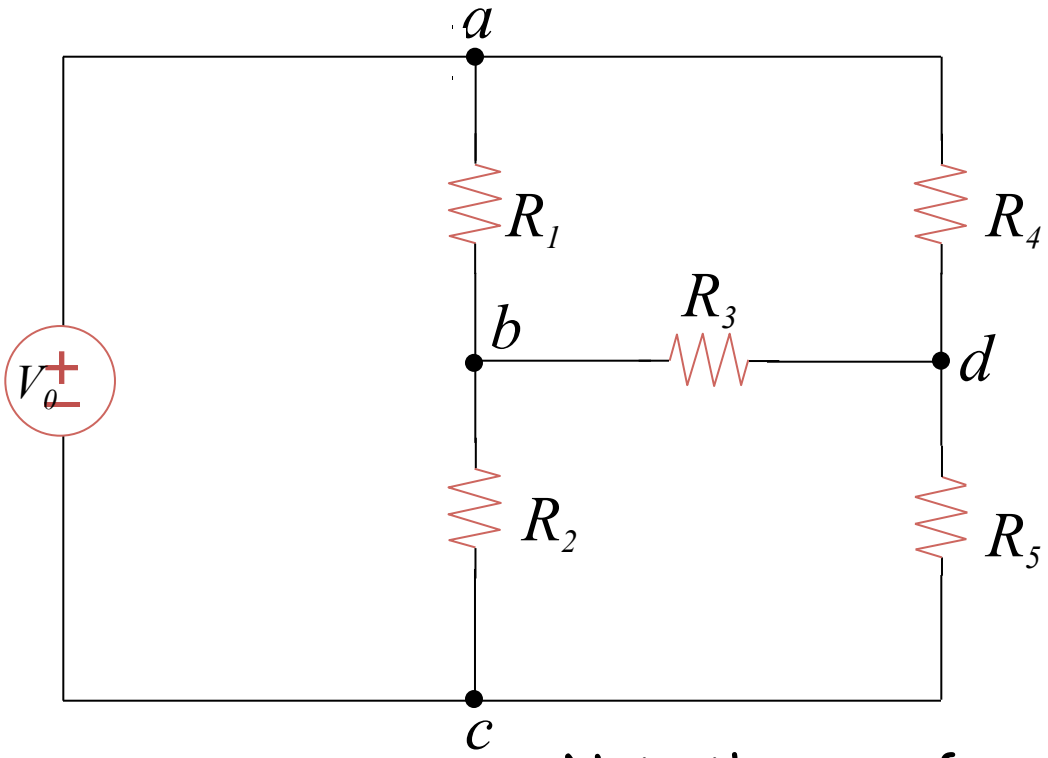
Goal: Find all element v's and i's



The Demo Circuit

KVL, KCL Example

Label all v's and i's



Goal: Find all element v's and i's

12 unknowns

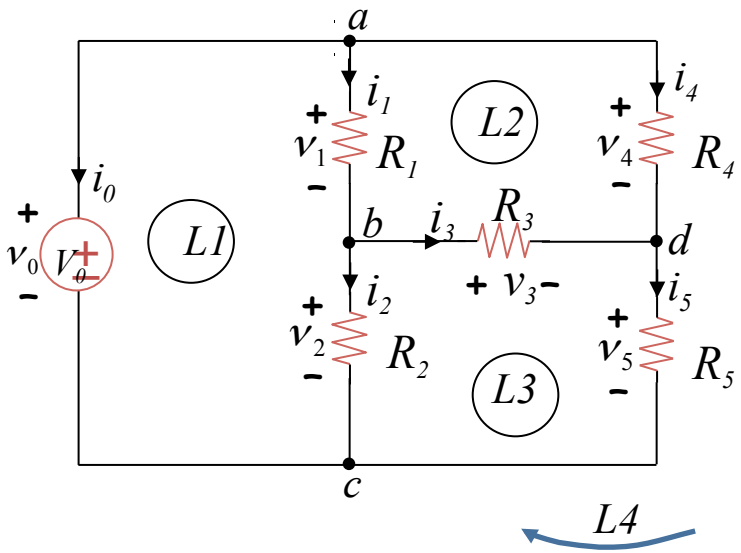
$v_0 \ldots v_5, i_0 \ldots i_5$

Note the use of associated variables…

Step 1 of KVL, KCL Method

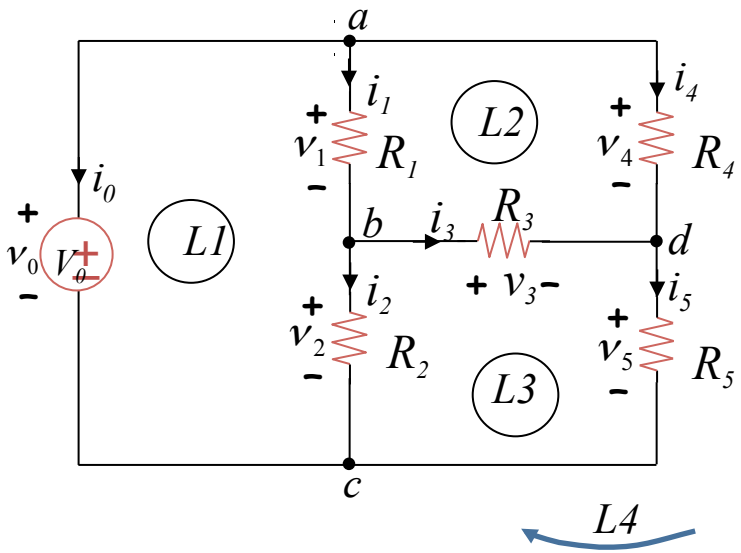
$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns

1. Element relationships (v, i)



Step 2 of KVL, KCL Method

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns

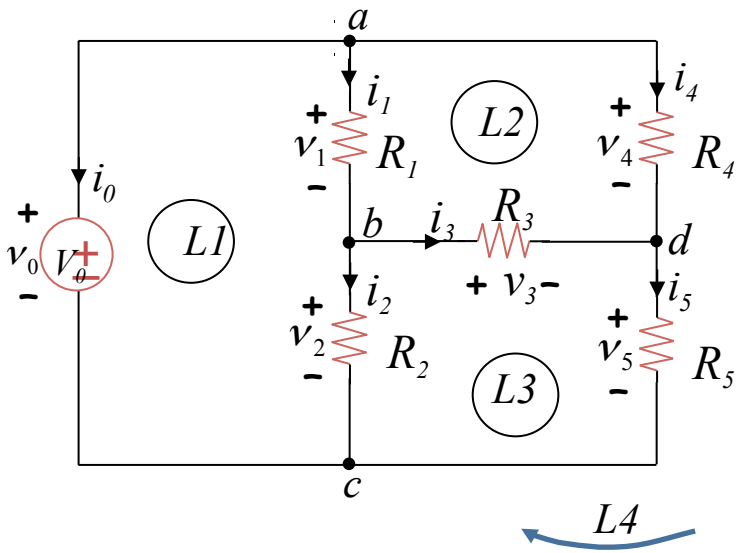


2. KCL at the nodes

(use convention, e.g., sum currents leaving the node)

Step 3 of KVL, KCL Method

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns



3. KVL for loops

(use convention, e.g., as you go around loop, assign first encountered sign to each voltage)

KVL, KCL Method

1. Element v, i relationships

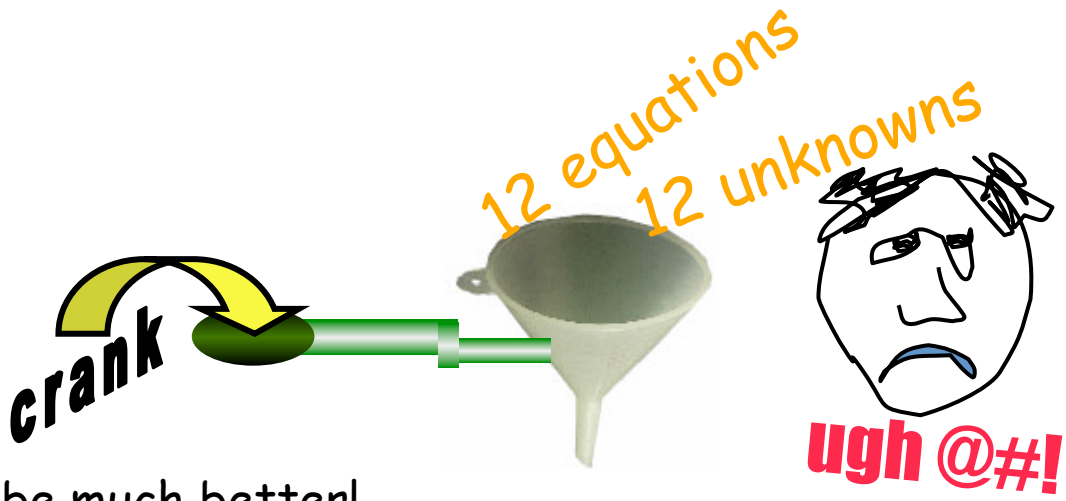
$$v_0 = V_0 \qquad v_3 = i_3 R_3$$
$$v_1 = i_1 R_1 \qquad v_4 = i_4 R_4$$
$$v_2 = i_2 R_2 \qquad v_5 = i_5 R_5$$

2. KCL at the nodes

$$a: \quad i_0 + i_1 + i_4 = 0$$
$$b: \quad i_2 + i_3 - i_1 = 0$$
$$d: \quad i_5 - i_3 - i_4 = 0$$
$$c: \quad -i_0 - i_2 - i_5 = 0 \text{ redundant}$$

3. KVL for loops

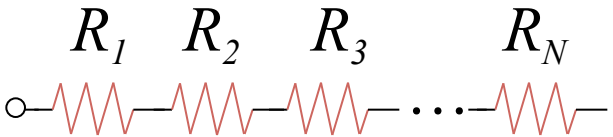
$$L1: \quad -v_0 + v_1 + v_2 = 0$$
$$L2: \quad v_1 + v_3 - v_4 = 0$$
$$L3: \quad v_3 + v_5 - v_2 = 0$$
$$L4: \quad -v_0 + v_4 + v_5 = 0 \text{ redundant}$$



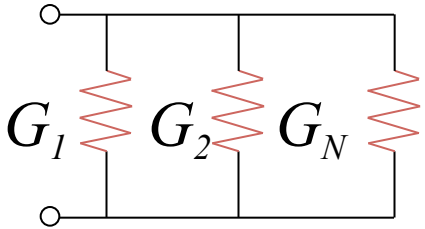
Method 3 – the node method will be much better!

Other Analysis Methods
Method 2— Apply element combination rules

A



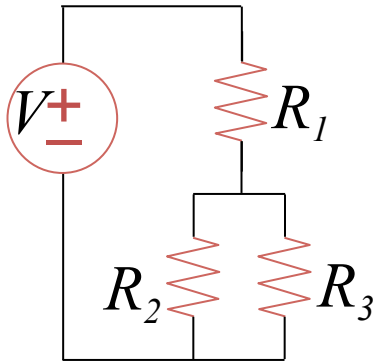
B



Method 2 — Apply element combination rules

Method 2— Apply element combination rules

Example



Method 3 — Node analysis

1. Select reference node (ground) from which voltages are measured.

2. Label voltages of remaining nodes with respect to ground. These are the primary unknowns.

3. Write KCL for all but the ground node, substituting device laws and KVL.

4. Solve for node voltages.

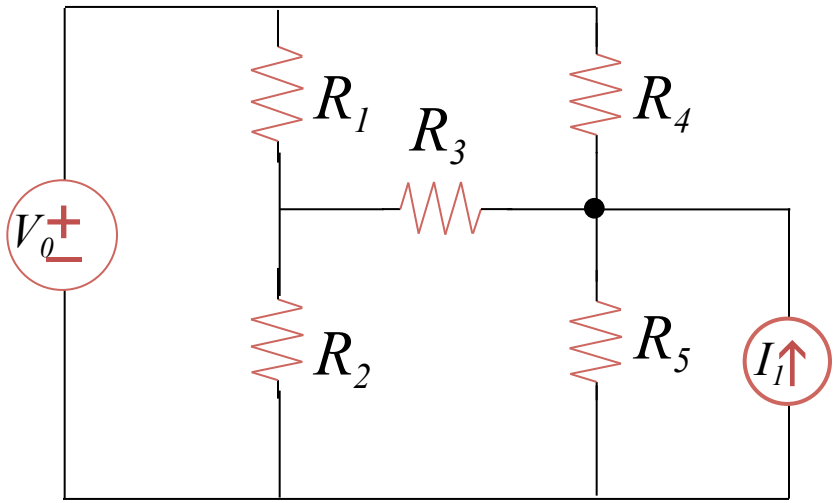
5. Back solve for branch voltages and currents (i.e., the secondary unknowns).

Particular application of KVL, KCL method

6.002x workhorse!

Method 3 — Node analysis

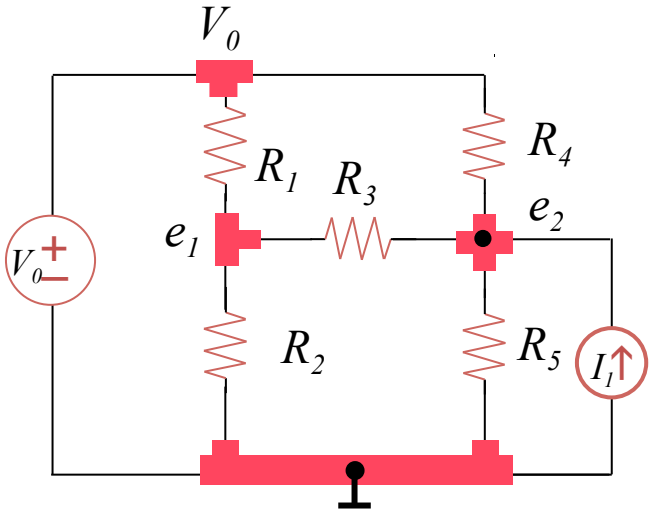
Example: Old Faithful, plus current source



1. Select reference ground node
2. Label node voltages with respect to ground.

Step 3 of Node Method

For convenience, write $G_i = \dfrac{1}{R_i}$



3. Write KCL for nodes, substituting device laws and KVL.

To avoid mistakes, use convention –
E.g., always sum the currents leaving a node

Step 4 of Node Method

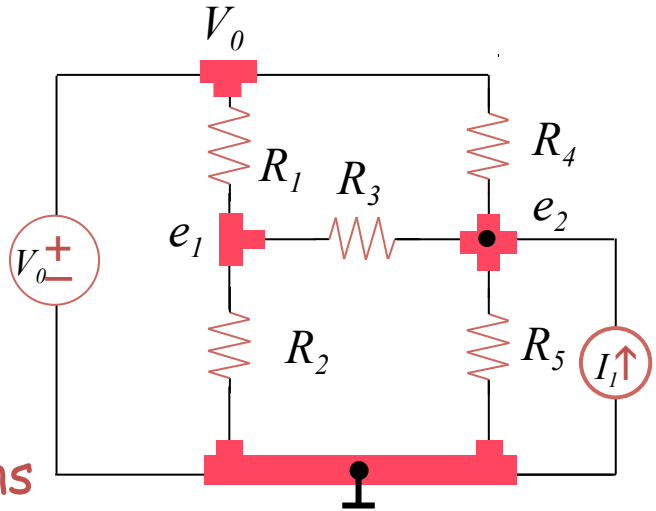
KCL at e_1

$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0$$

KCL at e_2

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0$$

Move constant terms to RHS & collect unknowns



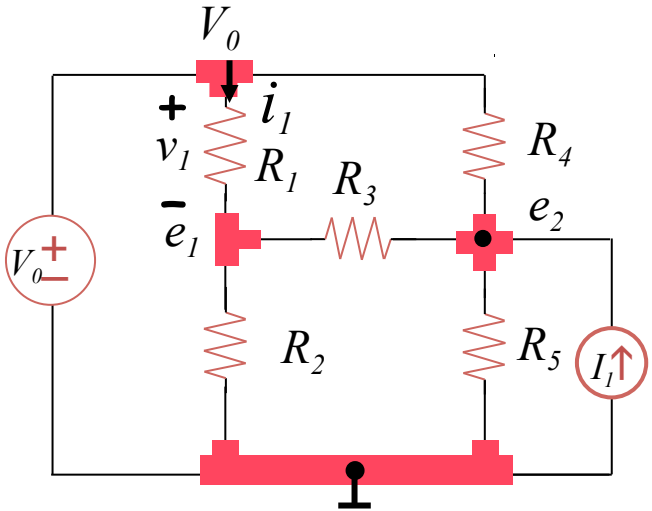
4. Solve for node voltages

2 equations, 2 unknowns ⟶ Solve for e's (compare units)

Step 5 of Node Method

Once you have solved for e_1 and e_2, easy to find branch v's and i's

For example:



5. Back solve for branch voltages and currents

Revisit Step 4 of Node Method for Cultural Interest

$$e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0(G_1)$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0(G_4) + I_1$$

4. Solve for node voltages

In matrix form:

$$\left[\begin{array}{c|c} G_1 + G_2 + G_3 & -G_3 \\ \hline -G_3 & G_3 + G_4 + G_5 \end{array}\right] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1 V_0 \\ G_4 V_0 + I_1 \end{bmatrix}$$

conductivity matrix

unknown node voltages

sources

Step 4 of Node Method

4. Solve for node voltages

$$\left[\begin{array}{c|c} G_1+G_2+G_3 & -G_3 \\ \hline -G_3 & G_3+G_4+G_5 \end{array}\right] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

Solve

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\left[\begin{array}{c|c} G_3+G_4+G_5 & G_3 \\ \hline G_3 & G_1+G_2+G_3 \end{array}\right] \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}}{(G_1+G_2+G_3)(G_3+G_4+G_5)-G_3^2}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^2+G_3G_4+G_3G_5}$$

$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^2+G_3G_4+G_3G_5}$$

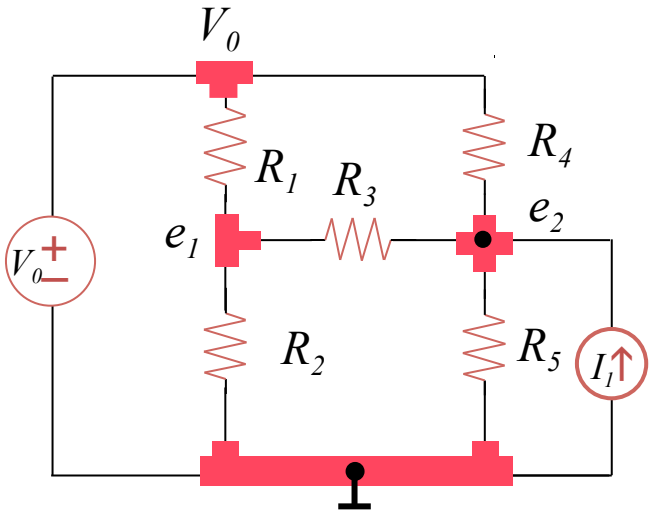
(same denominator)

Notice: linear in V_0, I_1, no negatives in denominator – we will use this later

Step 4 of Node Method

E.g., solve for e_2, given

$$\left.\begin{matrix} G_1 \\ G_5 \end{matrix}\right\} = \frac{1}{8.2K} \qquad \left.\begin{matrix} G_2 \\ G_4 \end{matrix}\right\} = \frac{1}{3.9K} \qquad G_3 = \frac{1}{1.5K} \qquad I_1 = 0$$



$$e_2 = 0.6V_0$$

If $V_0 = 3V$, then $e_2 = 1.8V_0$