ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 5: Algorithms on Graphs 1
Lecture 2: Graphs: Representations in memory

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Two main ways to store a graph in computer memory are:

- Adjacency matrix
- Adjacency list

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These ways are different in the following aspects:

- Space complexity (expressed in $|V|$, $|E|$)
- Running time of various operations
  - Vertex insertion
  - Edge insertion, edge deletion
  - Edge existence test
  - Iteration over edges adjacent to a vertex

The graph $G = (V, E)$ without multiedges with weight function F is represented as the matrix A of size $|V| \times |V|$ in the following manner. For each ordered pair of vertices u and v with $(u, v) \in E$, the matrix stores $A[u][v] = F((u, v))$.
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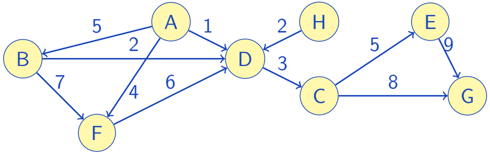

	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

- Space – $\Theta(|V|^2)$
- Vertex insertion – $\Theta(|V|)$
- Edge insertion, deletion, testing – $\Theta(1)$
- Adjacent edge iteration – $\Theta(n)$

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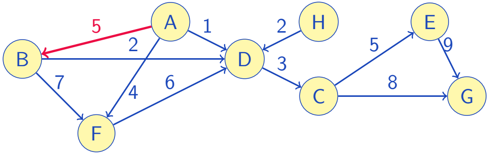

	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
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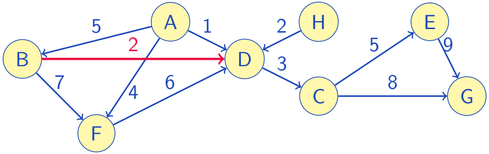

	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
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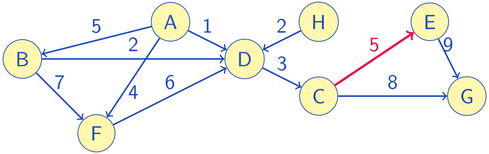

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Example: Check if there exists a cycle of length 3 in the given undirected graph

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A simple straightforward algorithm:

```
function TriangleExistence(A)
    n ← Rows(A)
    for u from 1 to n do
        for v from u + 1 to n do
            if A[u][v] = 1 then continue end if
            for w from v + 1 to n do
                if A[u][w] = 1 and A[v][w] = 1 then return TRUE end if
            end for
        end for
    end for
end function
```

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function $\textsc{TriangleExistence}(A)$
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  for u **from** 1 **to** n **do** ▷ Checking all u
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        end for
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```
function TriangleExistence(A)
    n ← Rows(A)
    for u from 1 to n do                                        ▷ Checking all u
        for v from u + 1 to n do                                ▷ Checking all v
            if A[u][v] ≠ 1 then continue end if
            for w from v + 1 to n do                            ▷ Checking all w
                if A[u][w] = 1 and A[v][w] = 1 then return TRUE end if
            end for
        end for
    end for
end function
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Running time: $O(|V|^3)$.

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                if $A[u][w] = 1$ **and** $A[v][w] = 1$ **then return** TRUE **end if**
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        end for
    end for
end function

Running time: $O(|V|^3)$. Can we make it faster?

Improvement idea: Do things “in parallel” using bitwise operations!

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0	1	0	1	1	1	0	1	0	0	1	1	0	0	1	0
0	0	1	1	0	1	0	1	0	1	1	0	1	0	1	0
0	0	0	0	1	1	1	0	1	1	0	0	0	0	0	1
1	1	0	1	1	0	0	1	1	0	1	1	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	0
0	1	1	0	0	1	0	0	0	0	1	1	0	1	1	1
1	1	1	1	1	1	1	0	1	1	0	0	1	0	0	1
1	1	0	1	0	1	0	0	0	1	1	0	0	1	0	0
0	1	0	1	1	0	1	1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	1	0	1	0	0	0	1	0	1	0
0	0	1	0	0	1	1	0	1	1	1	0	0	0	1	0
1	1	0	0	1	1	0	1	0	0	1	0	0	0	0	1
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
1	1	1	1	0	0	1	0	0	0	1	0	1	1	0	0
1	0	0	1	1	0	1	1	1	0	0	1	0	0	0	1
0	1	0	1	0	1	1	1	0	0	0	0	1	1	0	0

186	76
172	86
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1	1	0	1	1	0	0	1	1	0	1	1	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	0
0	1	1	0	0	1	0	0	0	0	1	1	0	1	1	1
1	1	1	1	1	1	1	0	1	1	0	0	1	0	0	1
1	1	0	1	0	1	0	0	0	1	1	0	0	1	0	0
0	1	0	1	1	0	1	1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	1	0	1	0	0	0	1	0	1	0
0	0	1	0	0	1	1	0	1	1	1	0	0	0	1	0
1	1	0	0	1	1	0	1	0	0	1	0	0	0	0	1
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
1	1	1	1	0	0	1	0	0	0	1	0	1	1	0	0
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0	0	0	0	1	1	1	0	1	1	0	0	0	0	0	1
1	1	0	1	1	0	0	1	1	0	1	1	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	0
0	1	1	0	0	1	0	0	0	0	1	1	0	1	1	1
1	1	1	1	1	1	1	0	1	1	0	0	1	0	0	1
1	1	0	1	0	1	0	0	0	1	1	0	0	1	0	0
0	1	0	1	1	0	1	1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	1	0	1	0	0	0	1	0	1	0
0	0	1	0	0	1	1	0	1	1	1	0	0	0	1	0
1	1	0	0	1	1	0	1	0	0	1	0	0	0	0	1
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
1	1	1	1	0	0	1	0	0	0	1	0	1	1	0	0
1	0	0	1	1	0	1	1	1	0	0	1	0	0	0	1
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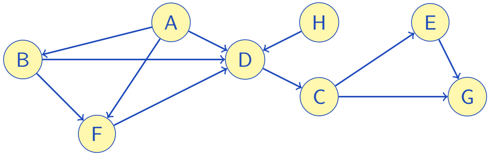
A (slightly simplified) bitmask-optimized version which works 32 times faster!

function $\textsc{TriangleExistence}(A)$
- $n \leftarrow \textsc{Rows}(A)$
- $C \leftarrow \textsc{BitmaskCompress}(A)$
- **for** u **from** 1 **to** n **do**
  - **for** v **from** $u + 1$ **to** n **do**
    - **if** $A[u][v] = 1$ **then continue end if**
    - **for** w **from** $(v + 1)/32$ **to** $(n + 31)/32$ **do**
      - **if** $(C[u][w]$ **bitwise and** $C[v][w]) \neq 0$ **then return** $\textsc{true}$ **end if**
    - **end for**
  - **end for**
- **end for**

end function

Given a graph G, find the number of paths of length k.



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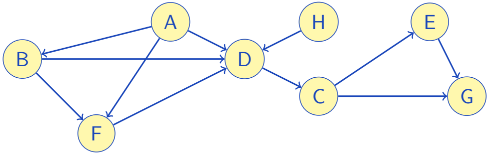

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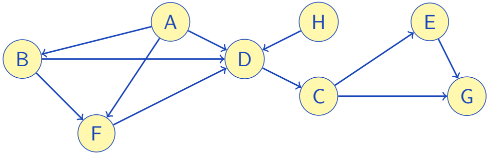

- Hint 1: Adjacency matrix $=$ paths of length 1

$k = 1$

	A	B	C	D	E	F	G	H
A	0	1	0	1	0	1	0	0
B	0	0	0	1	0	1	0	0
C	0	0	0	0	1	0	1	0
D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
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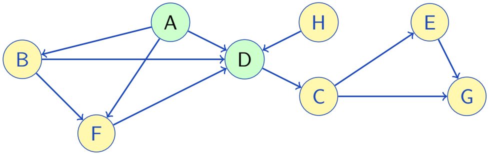

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	A	B	C	D	E	F	G	H
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D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

- Hint 1: Adjacency matrix $=$ paths of length 1
- Hint 2: What is 2-path between A and D?

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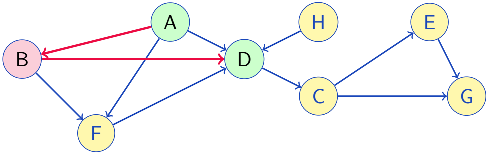

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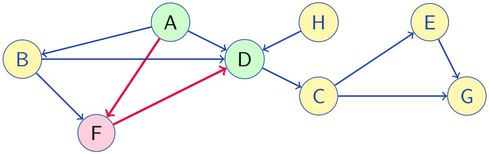

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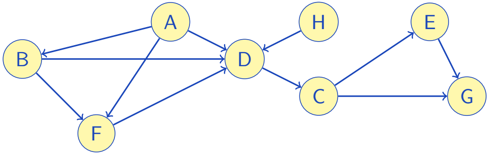

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E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

$k = 2$

	A	B	C	D	E	F	G	H
A	?	?	?	2	?	?	?	?
B	?	?	?	?	?	?	?	?
C	?	?	?	?	?	?	?	?
D	?	?	?	?	?	?	?	?
E	?	?	?	?	?	?	?	?
F	?	?	?	?	?	?	?	?
G	?	?	?	?	?	?	?	?
H	?	?	?	?	?	?	?	?

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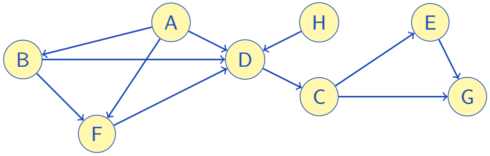

- Hint 1: Adjacency matrix = paths of length 1
- Hint 2: What is 2-path between A and D?
- Hint 3: $A_2[i][j] = \sum_k A_1[i][k] \cdot A_1[k][j]$

$k = 1$

	A	B	C	D	E	F	G	H
A	0	1	0	1	0	1	0	0
B	0	0	0	1	0	1	0	0
C	0	0	0	0	1	0	1	0
D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

$k = 2$

	A	B	C	D	E	F	G	H
A	?	?	?	2	?	?	?	?
B	?	?	?	?	?	?	?	?
C	?	?	?	?	?	?	?	?
D	?	?	?	?	?	?	?	?
E	?	?	?	?	?	?	?	?
F	?	?	?	?	?	?	?	?
G	?	?	?	?	?	?	?	?
H	?	?	?	?	?	?	?	?

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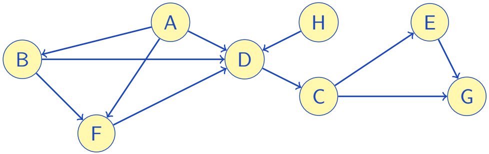


- Hint 1: Adjacency matrix = paths of length 1
- Hint 2: What is 2-path between A and D?
- Hint 3: $A_2[i][j] = \sum_k A_1[i][k] \cdot A_1[k][j]$
  - or simply $A_2 = A_1 \cdot A_1 = (A_1)^2$

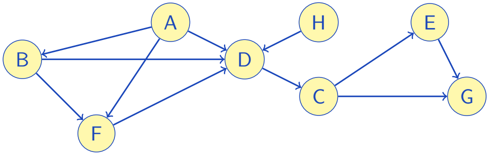
$k = 1$

	A	B	C	D	E	F	G	H
A	0	1	0	1	0	1	0	0
B	0	0	0	1	0	1	0	0
C	0	0	0	0	1	0	1	0
D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

$k = 2$

	A	B	C	D	E	F	G	H
A	?	?	?	2	?	?	?	?
B	?	?	?	?	?	?	?	?
C	?	?	?	?	?	?	?	?
D	?	?	?	?	?	?	?	?
E	?	?	?	?	?	?	?	?
F	?	?	?	?	?	?	?	?
G	?	?	?	?	?	?	?	?
H	?	?	?	?	?	?	?	?

Given a graph G, find the number of paths of length k.



- Hint 1: Adjacency matrix = paths of length 1
- Hint 2: What is 2-path between A and D?
- Hint 3: $A_2[i][j] = \sum_k A_1[i][k] \cdot A_1[k][j]$
  - or simply $A_2 = A_1 \cdot A_1 = (A_1)^2$

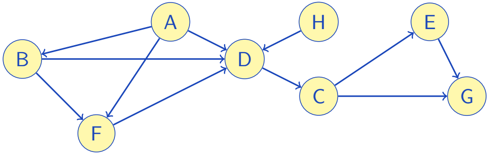
$k = 1$

	A	B	C	D	E	F	G	H
A	0	1	0	1	0	1	0	0
B	0	0	0	1	0	1	0	0
C	0	0	0	0	1	0	1	0
D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

$k = 2$

	A	B	C	D	E	F	G	H
A	0	0	1	2	0	1	0	0
B	0	0	1	1	0	0	0	0
C	0	0	0	0	0	0	1	0
D	0	0	0	0	1	0	1	0
E	0	0	0	0	0	0	0	0
F	0	0	1	0	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	1	0	0	0	0	0

Given a graph G, find the number of paths of length k.



$k = 1$

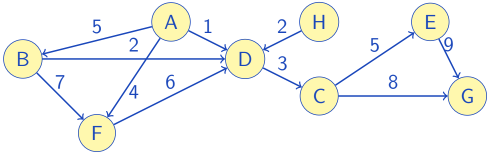
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	1	0	0
B	0	0	0	1	0	1	0	0
C	0	0	0	0	1	0	1	0
D	0	0	1	0	0	0	0	0
E	0	0	0	0	0	0	1	0
F	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	0	1	0	0	0	0

$k = 2$

	A	B	C	D	E	F	G	H
A	0	0	1	2	0	1	0	0
B	0	0	1	1	0	0	0	0
C	0	0	0	0	0	0	1	0
D	0	0	0	0	1	0	1	0
E	0	0	0	0	0	0	0	0
F	0	0	1	0	0	0	0	0
G	0	0	0	0	0	0	0	0
H	0	0	1	0	0	0	0	0

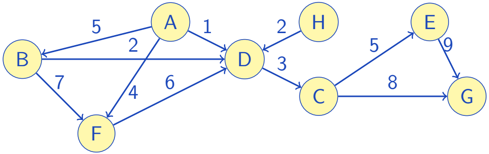
- Hint 1: Adjacency matrix $=$ paths of length 1
- Hint 2: What is 2-path between A and D?
- Hint 3: $A_2[i][j] = \sum_k A_1[i][k] \cdot A_1[k][j]$
  - or simply $A_2 = A_1 \cdot A_1 = (A_1)^2$
- $A_k = (A_1)^k$, can be evaluated in $O(|V|^3 \log k)$
  - $O(|V|^3)$ (or faster): matrix multiplication

A compact storage for sparse graphs.
For every vertex, store outgoing edges.



	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store outgoing edges.

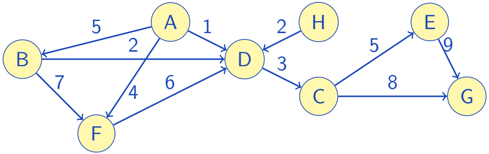


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$
B	$(D;2)$	$(F;7)$	
C	$(E;5)$	$(G;8)$	
D	$(C;3)$		
E	$(G;9)$		
F	$(D;6)$		
G			
H	$(D;2)$		

A compact storage for sparse graphs.

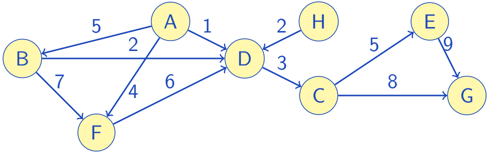
For every vertex, store incoming and outgoing edges.



	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

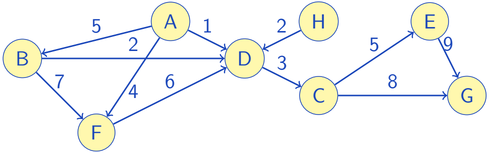
A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.



	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.

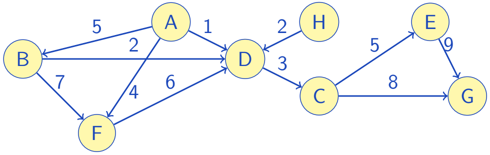


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

- Space requirements: $\Theta(|V| + |E|)$

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.

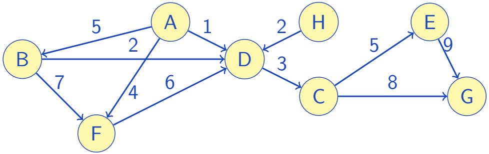


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.

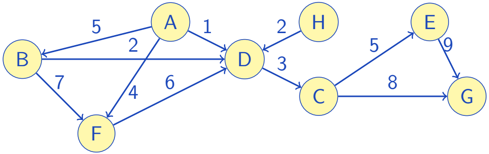


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.

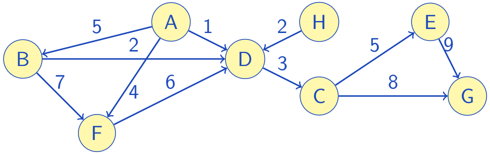


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.

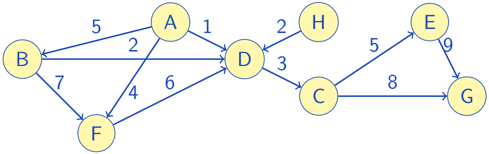


	A	B	C	D	E	F	G	H
A	–	5	–	1	–	2	–	–
B	–	–	–	2	–	7	–	–
C	–	–	–	–	5	–	8	–
D	–	–	3	–	–	–	–	–
E	–	–	–	–	–	–	9	–
F	–	–	–	6	–	–	–	–
G	–	–	–	–	–	–	–	–
H	–	–	–	2	–	–	–	–

A	$(B;5)$	$(D;1)$	$(F;2)$			A
B	$(D;2)$	$(F;7)$			$(A;5)$	B
C	$(E;5)$	$(G;8)$			$(D;3)$	C
D	$(C;3)$	$(H;2)$	$(F;6)$	$(B;2)$	$(A;1)$	D
E	$(G;9)$				$(C;5)$	E
F	$(D;6)$			$(B;7)$	$(A;4)$	F
G				$(E;9)$	$(C;8)$	G
H	$(D;2)$					H

- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$
  - $O(\log(deg(v)))$ if balanced search trees are used

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant’s way: $O(1)$ dynamic data structures (outgoing only edges shown)

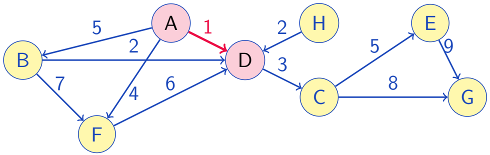


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	–	–	–	–	–	–	–	–

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	–	–	–	–	–	–	–	–	–	–	–
Value	–	–	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant's way: $O(1)$ dynamic data structures (outgoing only edges shown)

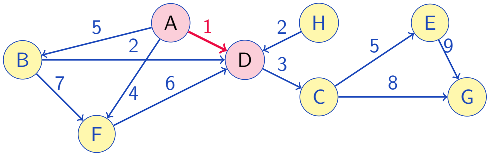


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	–	–	–	–	–	–	–	–

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	–	–	–	–	–	–	–	–	–	–	–
Value	–	–	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant's way: $O(1)$ dynamic data structures (outgoing only edges shown)

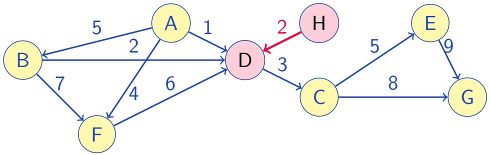


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	–	–	–	–	–

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	–	–	–	–	–	–	–	–	–	–
Value	1	–	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant’s way: $O(1)$ dynamic data structures (outgoing only edges shown)

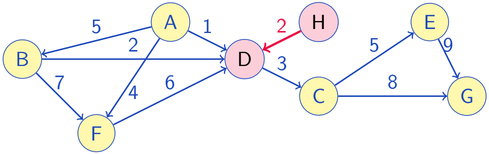


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	–	–	–	–	–

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	–	–	–	–	–	–	–	–	–	–
Value	1	–	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant’s way: $O(1)$ dynamic data structures (outgoing only edges shown)

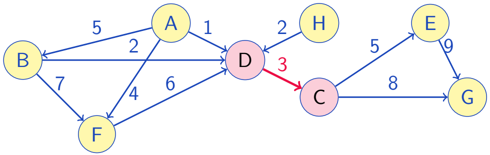


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	–	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	–	–	–	–	–	–	–	–	–
Value	1	2	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant’s way: $O(1)$ dynamic data structures (outgoing only edges shown)

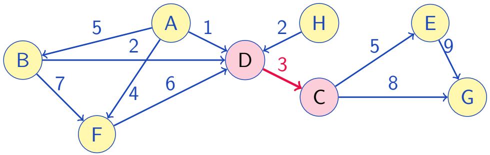


- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	–	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	–	–	–	–	–	–	–	–	–
Value	1	2	–	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant's way: $O(1)$ dynamic data structures (outgoing only edges shown)



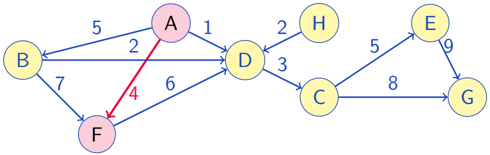
- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	–	–	–	–	–	–	–	–
Value	1	2	3	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

Adjacency list: Alternative "old school" implementation

A compact storage for sparse graphs.
For every vertex, store incoming and outgoing edges.
The old contestant's way: $O(1)$ dynamic data structures (outgoing only edges shown)



- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	1	–	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	–	–	–	–	–	–	–	–
Value	1	2	3	–	–	–	–	–	–	–	–
Next	–	–	–	–	–	–	–	–	–	–	–

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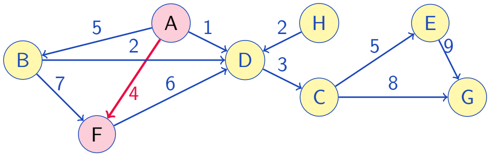

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Vertex	A	B	C	D	E	F	G	H
Next	4	–	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	–	–	–	–	–	–	–
Value	1	2	3	4	–	–	–	–	–	–	–
Next	–	–	–	1	–	–	–	–	–	–	–

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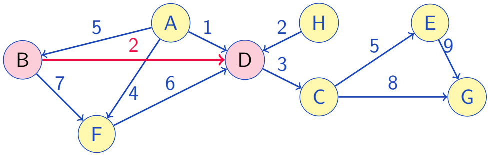

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Vertex	A	B	C	D	E	F	G	H
Next	4	–	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	–	–	–	–	–	–	–
Value	1	2	3	4	–	–	–	–	–	–	–
Next	–	–	–	1	–	–	–	–	–	–	–

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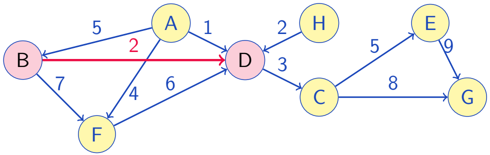

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Vertex	A	B	C	D	E	F	G	H
Next	4	5	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	–	–	–	–	–	–
Value	1	2	3	4	2	–	–	–	–	–	–
Next	–	–	–	1	–	–	–	–	–	–	–

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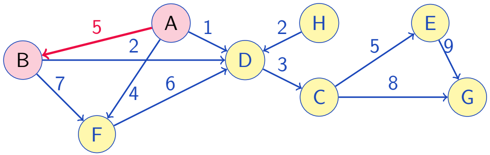

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Vertex	A	B	C	D	E	F	G	H
Next	4	5	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	–	–	–	–	–	–
Value	1	2	3	4	2	–	–	–	–	–	–
Next	–	–	–	1	–	–	–	–	–	–	–

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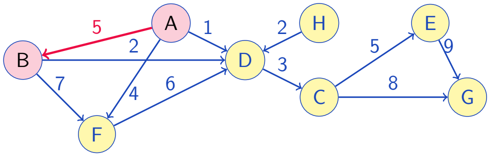

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Vertex	A	B	C	D	E	F	G	H
Next	6	5	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	–	–	–	–	–
Value	1	2	3	4	2	5	–	–	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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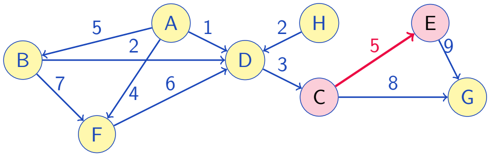

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Next	6	5	–	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	–	–	–	–	–
Value	1	2	3	4	2	5	–	–	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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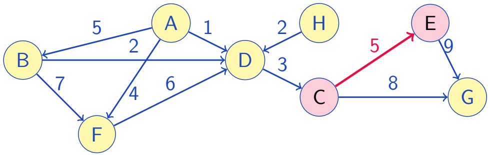

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Vertex	A	B	C	D	E	F	G	H
Next	6	5	7	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	–	–	–	–
Value	1	2	3	4	2	5	5	–	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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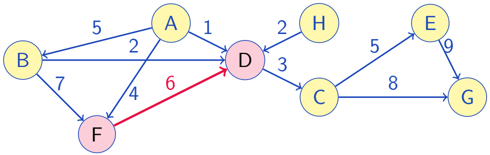

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Vertex	A	B	C	D	E	F	G	H
Next	6	5	7	3	–	–	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	–	–	–	–
Value	1	2	3	4	2	5	5	–	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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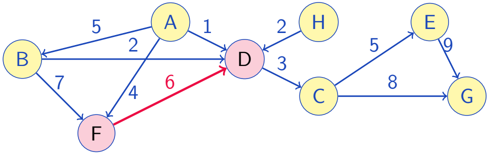

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Vertex	A	B	C	D	E	F	G	H
Next	6	5	7	3	–	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	–	–	–
Value	1	2	3	4	2	5	5	6	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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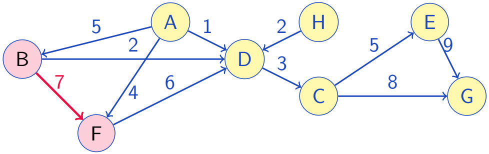

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Next	6	5	7	3	–	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	–	–	–
Value	1	2	3	4	2	5	5	6	–	–	–
Next	–	–	–	1	–	4	–	–	–	–	–

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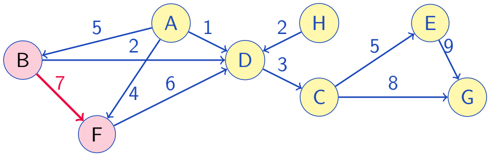

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Vertex	A	B	C	D	E	F	G	H
Next	6	9	7	3	–	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	–	–
Value	1	2	3	4	2	5	5	6	7	–	–
Next	–	–	–	1	–	4	–	–	5	–	–

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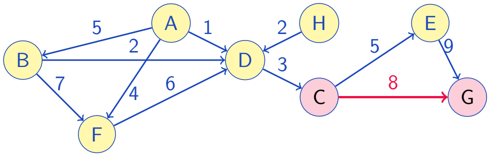

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Next	6	9	7	3	–	8	–	2

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Vertex	D	D	C	F	D	B	E	D	F	–	–
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Next	–	–	–	1	–	4	–	–	5	–	–

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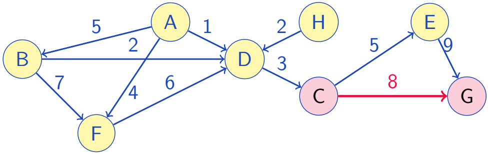

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Vertex	D	D	C	F	D	B	E	D	F	G	–
Value	1	2	3	4	2	5	5	6	7	8	–
Next	–	–	–	1	–	4	–	–	5	7	–

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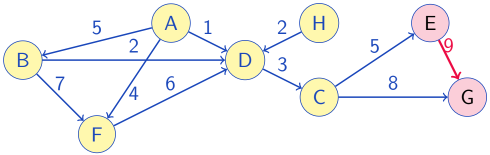

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Next	6	9	10	3	–	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	–
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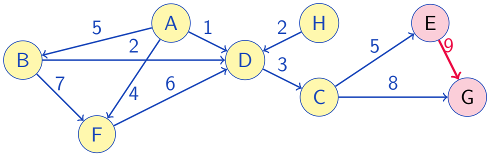

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Next	6	9	10	3	11	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	G
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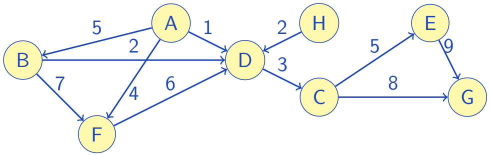

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Next	6	9	10	3	11	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	G
Value	1	2	3	4	2	5	5	6	7	8	9
Next	–	–	–	1	–	4	–	–	5	7	–

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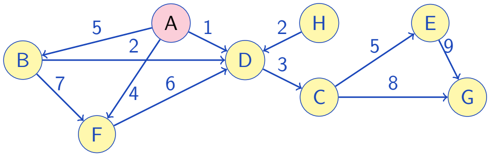

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Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	G
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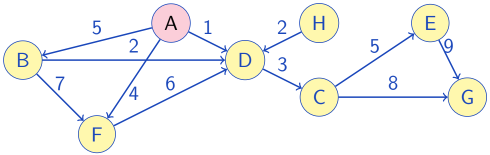

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Vertex	D	D	C	F	D	B	E	D	F	G	G
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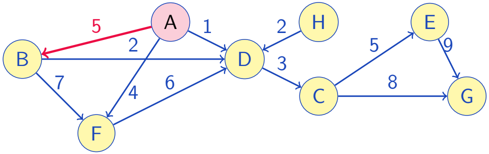

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Vertex	D	D	C	F	D	B	E	D	F	G	G
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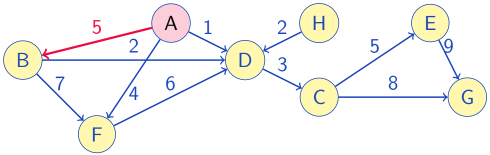

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Next	6	9	10	3	11	8	–	2

Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	G
Value	1	2	3	4	2	5	5	6	7	8	9
Next	–	–	–	1	–	4	–	–	5	7	–

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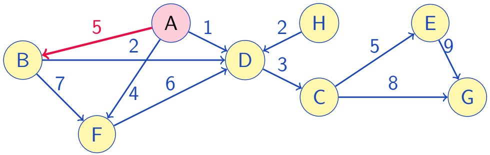

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Vertex	D	D	C	F	D	B	E	D	F	G	G
Value	1	2	3	4	2	5	5	6	7	8	9
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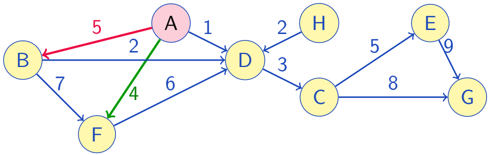

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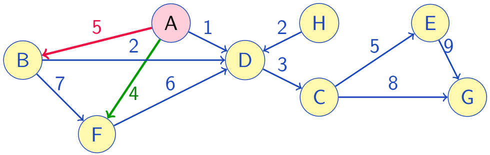
Index	1	2	3	4	5	6	7	8	9	10	11
Vertex	D	D	C	F	D	B	E	D	F	G	G
Value	1	2	3	4	2	5	5	6	7	8	9
Next	–	–	–	1	–	4	–	–	5	7	–

A compact storage for sparse graphs.

For every vertex, store incoming and outgoing edges.

The old contestant's way: $O(1)$ dynamic data structures (outgoing only edges shown)



- Space requirements: $\Theta(|V| + |E|)$
- Edge addition: $\Theta(1)$ (amortized)
- Vertex addition: $\Theta(1)$ (amortized)
- Edge lookup/removal: $O(deg(v))$

Vertex	A	B	C	D	E	F	G	H
Next	6	9	10	3	11	8	–	2

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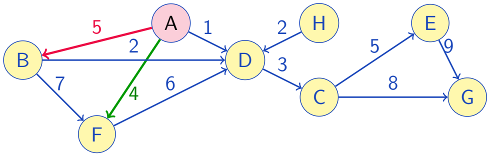

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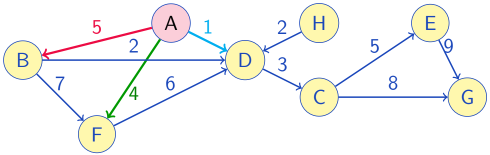

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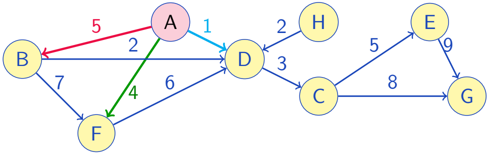

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  - Space complexity: $\Theta(|V|^2)$
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  - Good for storing dense graphs (say $|V| \approx 5000$, $|E| \approx 10\,000\,000$)
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- Choose between them wisely!