ITMO UNIVERSITY

How to Win Coding Competitions: Secrets of Champions

Week 4: Algorithms on Graphs
Lecture 5: Topological sort

Maxim Buzdalov
Saint Petersburg 2016

Assume you are solving a problem from a competition.
What should you do?

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What should you do?

Write the solution template

Get the names of I/O files

Read the statement

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Invent the algorithm

Submit the solution

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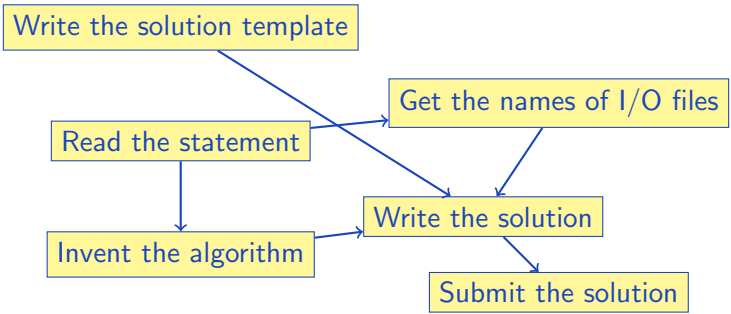

For the directed acyclic graph $G = \langle V, E \rangle$,
a topological sort is an assignment $I : V \to [1; N]$ of indices to vertices, such that:

- for every two distinct vertices u, v it holds that $I(u) \neq I(v)$
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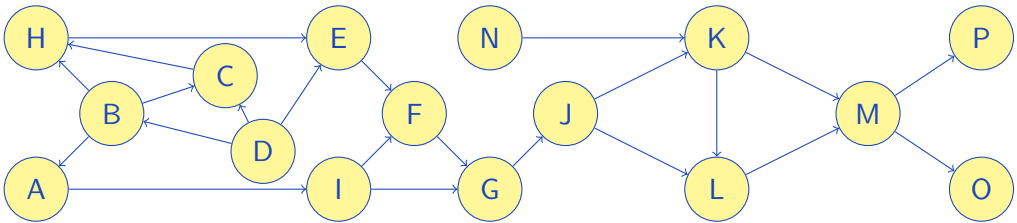

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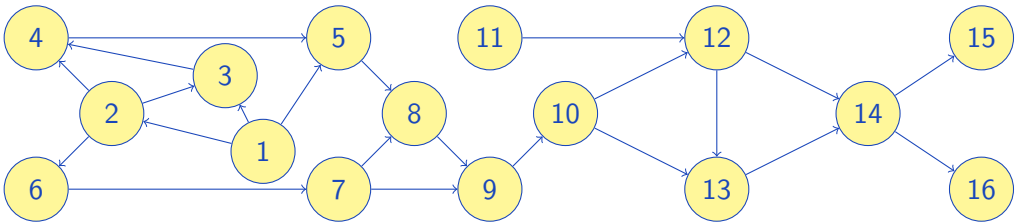

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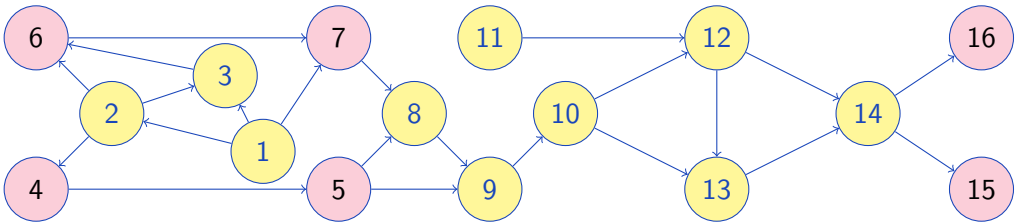

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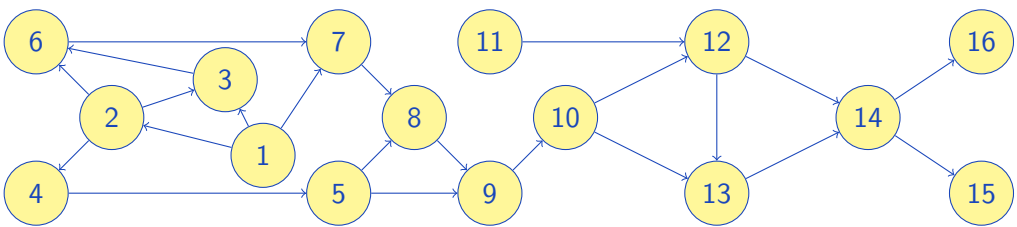
Sometimes it is convenient to have an array $o[i]$ such that:

- $o[I(v)] = v$, or, just the same, $I(o[i]) = i$



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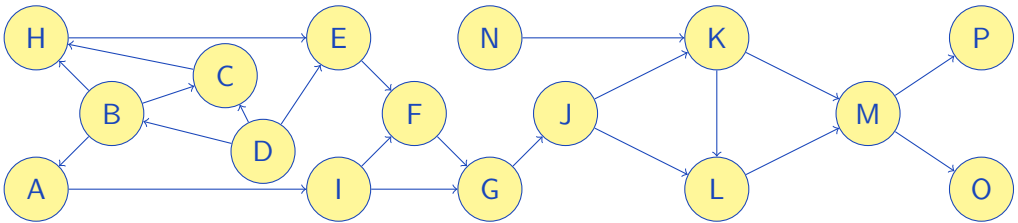

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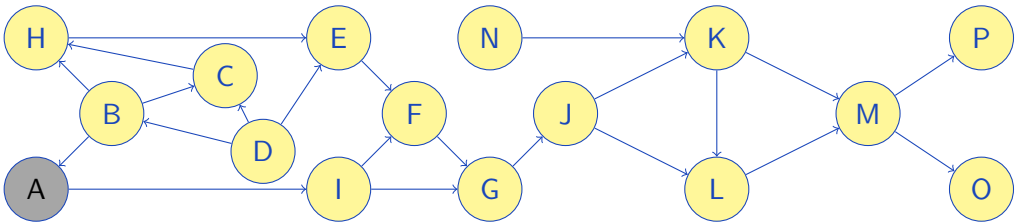

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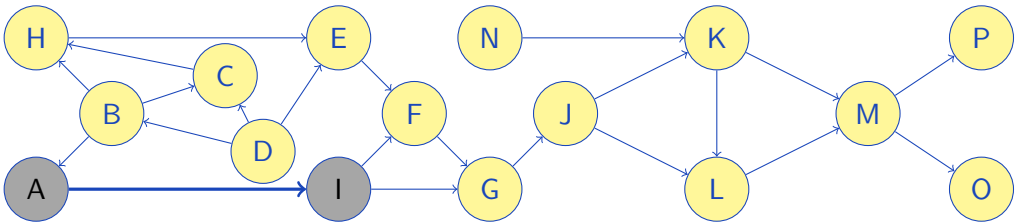

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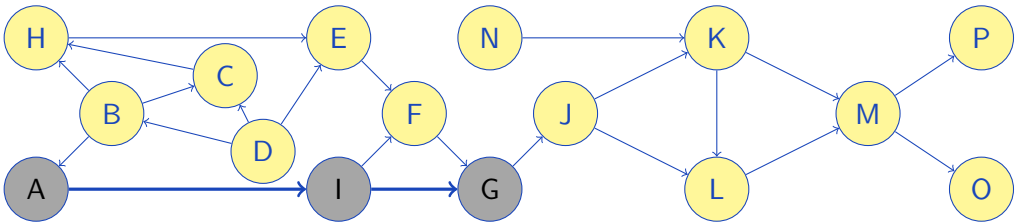

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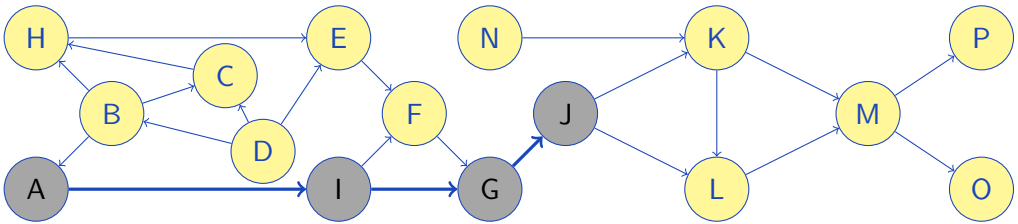

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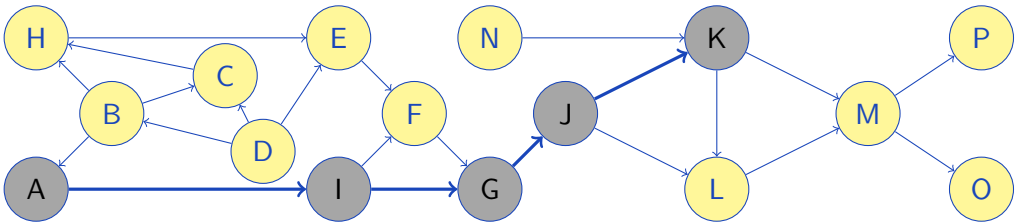

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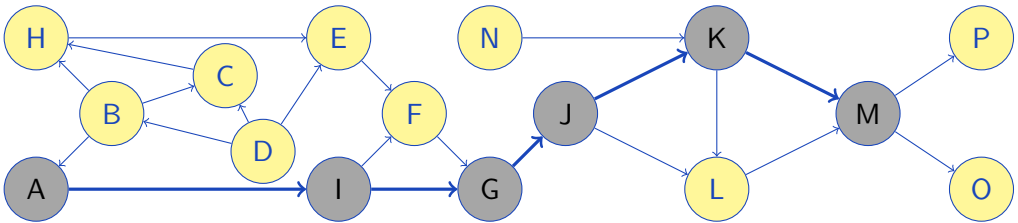

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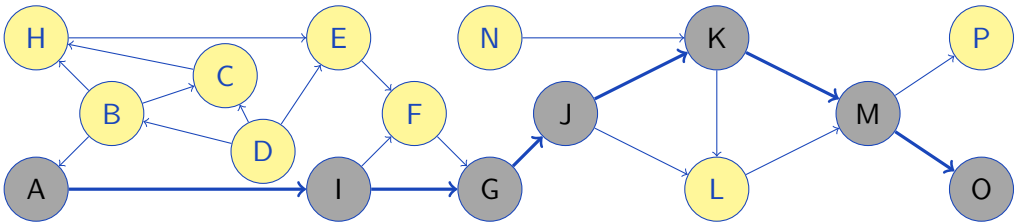

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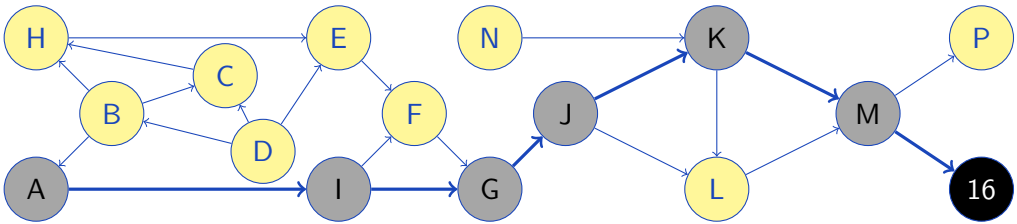

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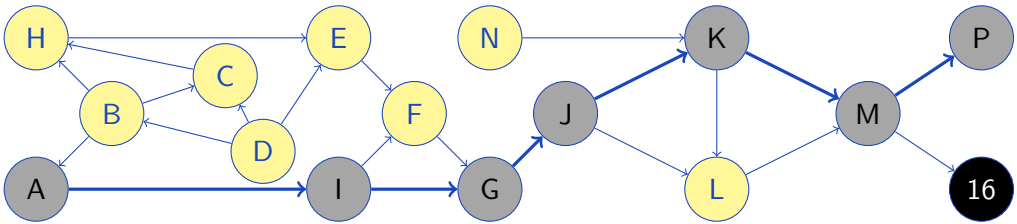

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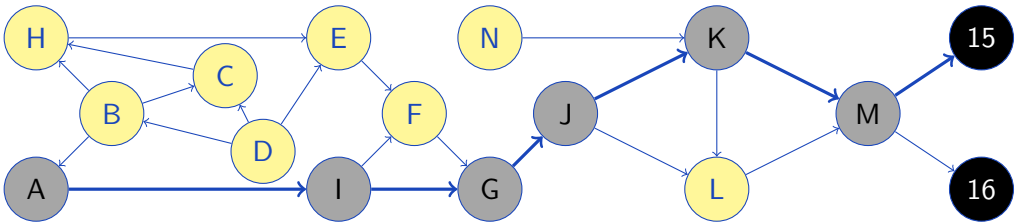

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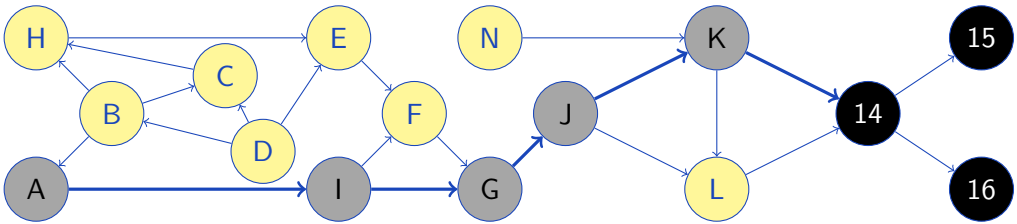

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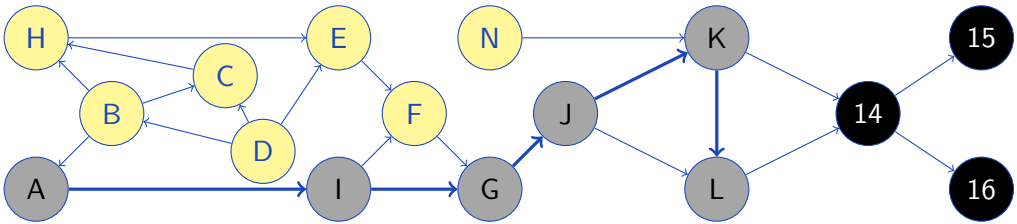

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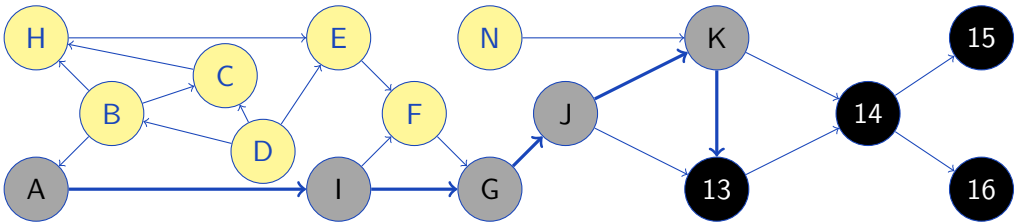

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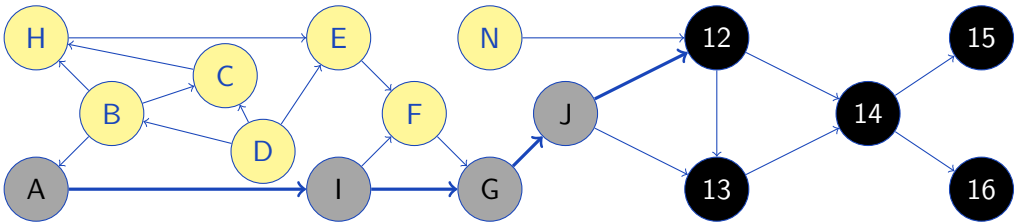

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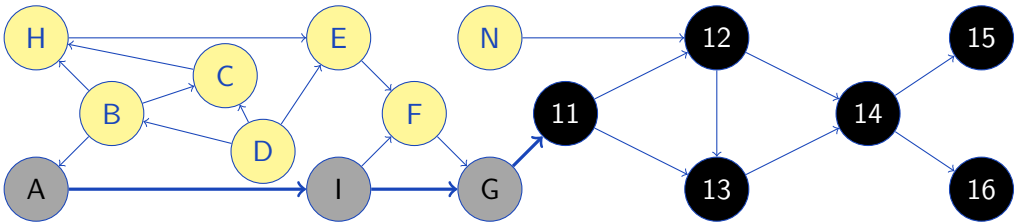

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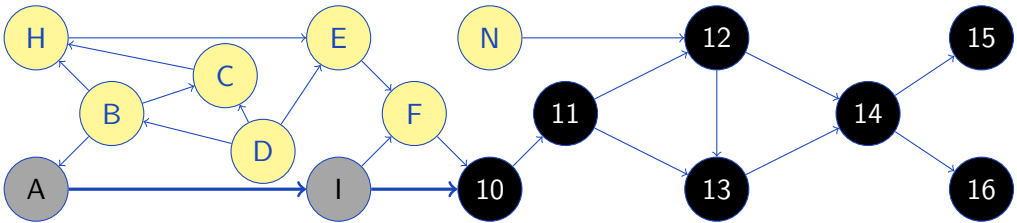

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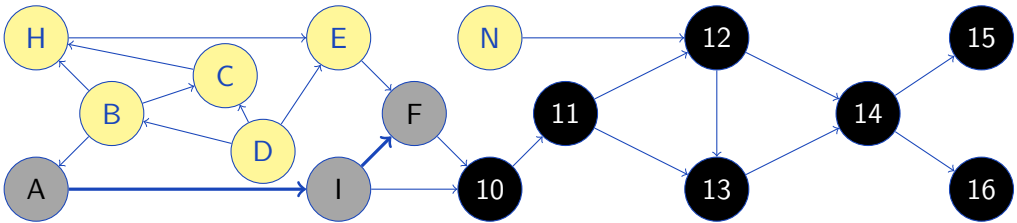

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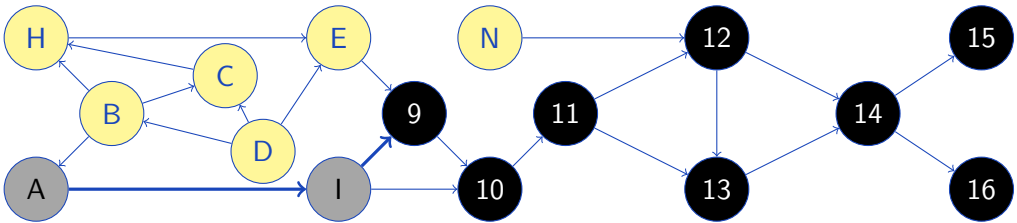

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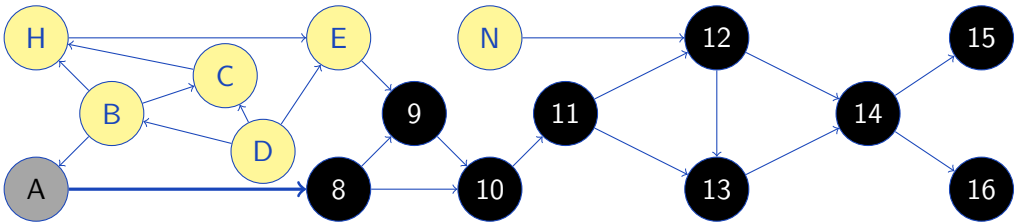

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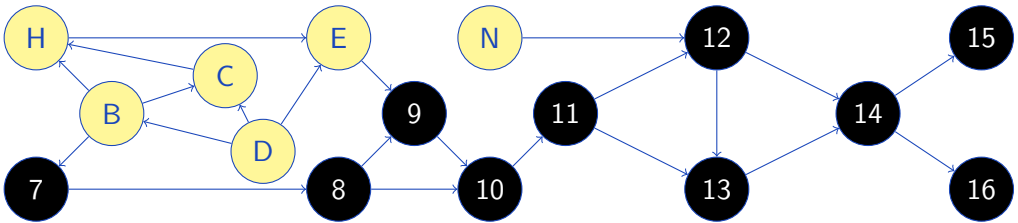

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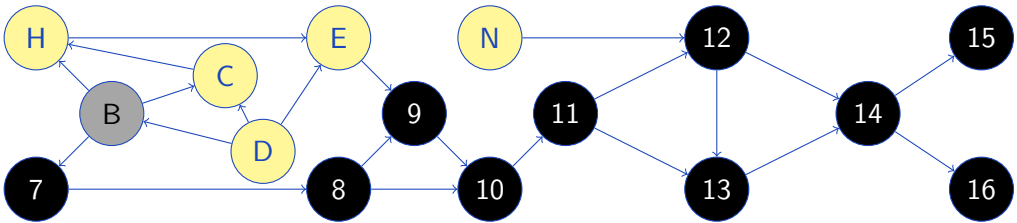

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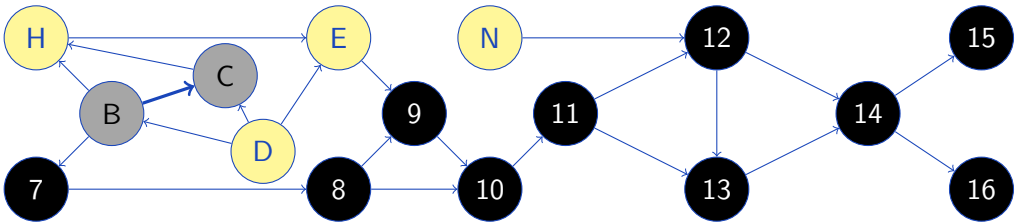

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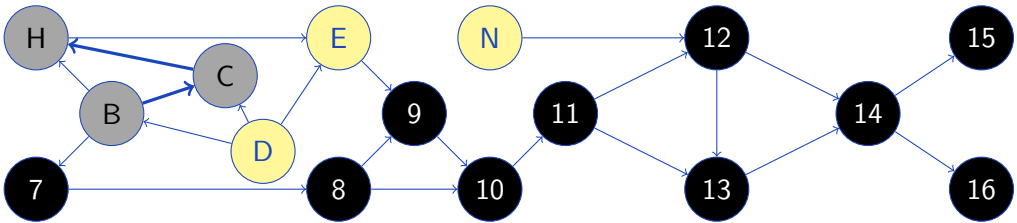

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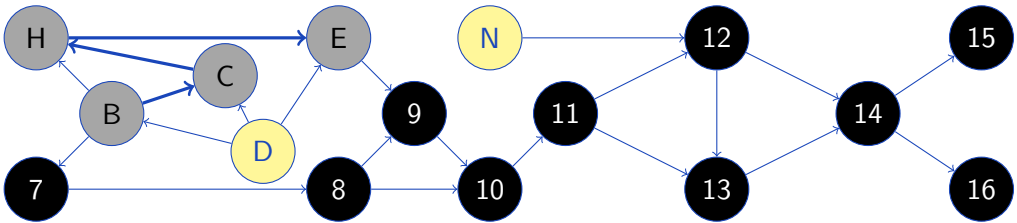

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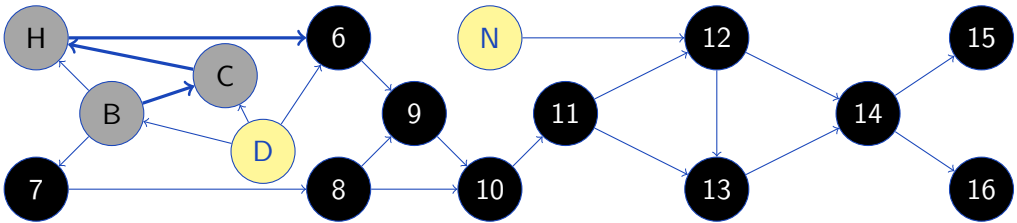

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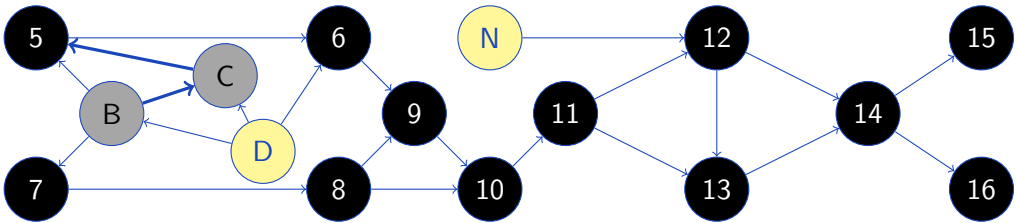

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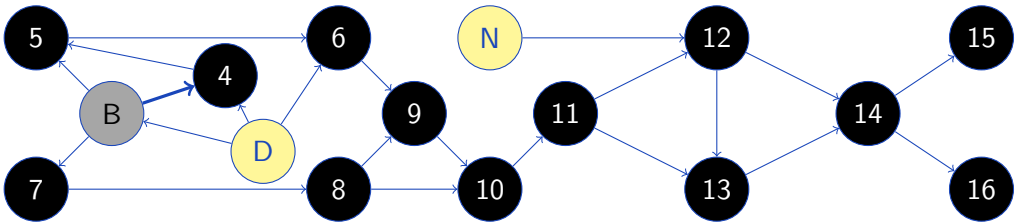

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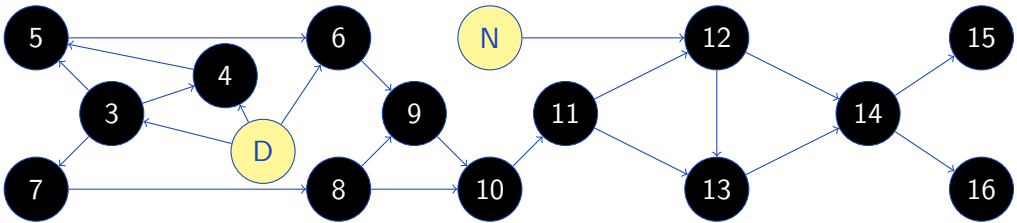

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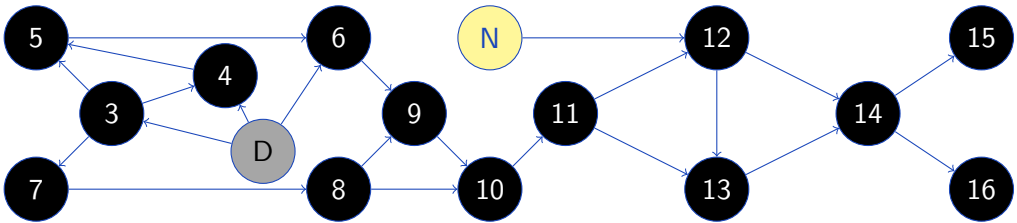

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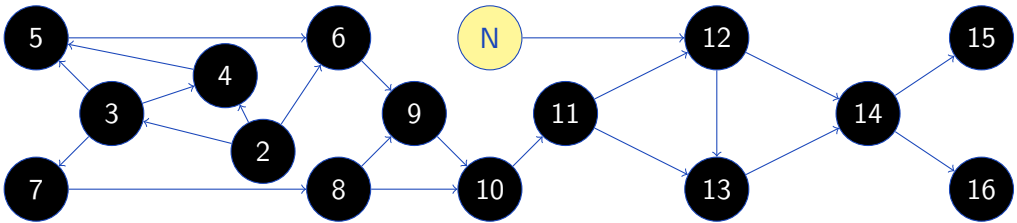

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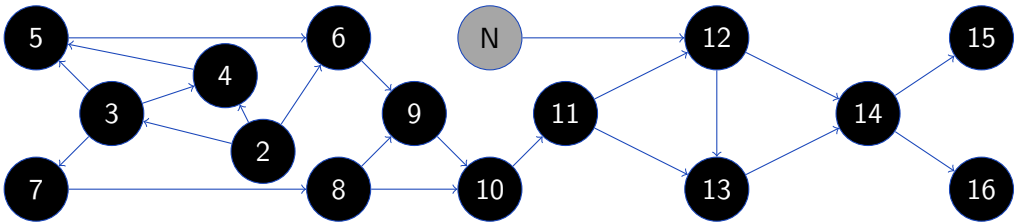

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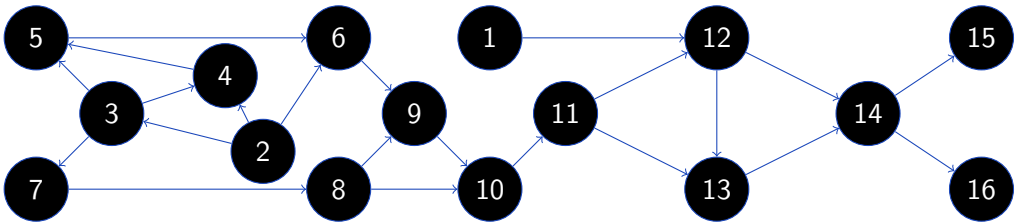

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  - Condensation is a tree
  - ... and this tree will be traversed in a correct order by topological sort!
  - So we may use topological sort to find the strong connectivity components

```
procedure CONDENSATION(V, E)
    TOPOSORT(V, E)
    A^R(v) = {u | (u, v) ∈ E}
    C ← {−1}
    K ← 1
    for i from 1 to |V| do
        if C[o[i]] = −1 then
            DFS(o[i], A^R, K)
            K ← K + 1
        end if
    end for
end procedure
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procedure DFS(v, A, K)
    C[v] ← K
    for u ∈ A(v) do
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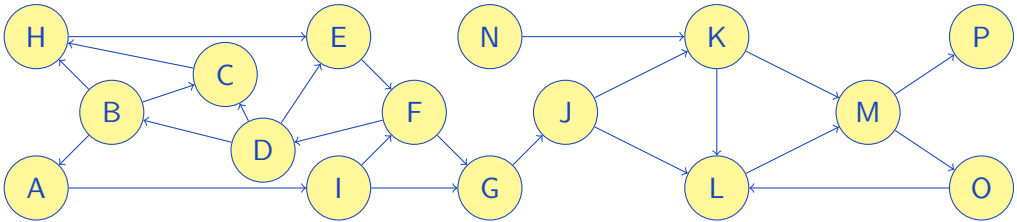
- Second DFS traverses vertices in the order of topological sort
- ... and does it using reversed edges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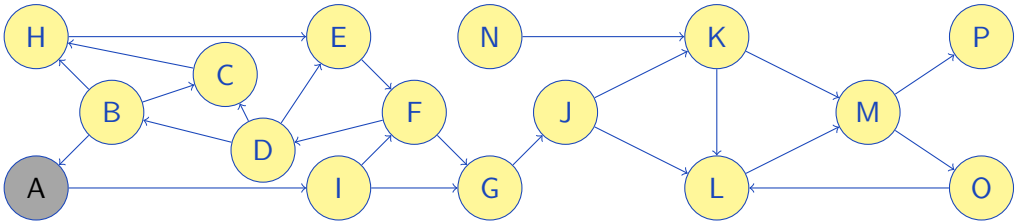
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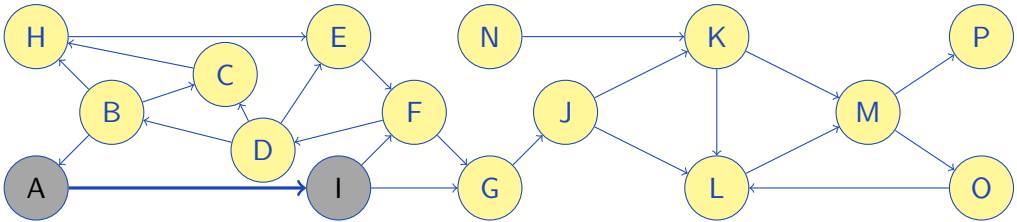
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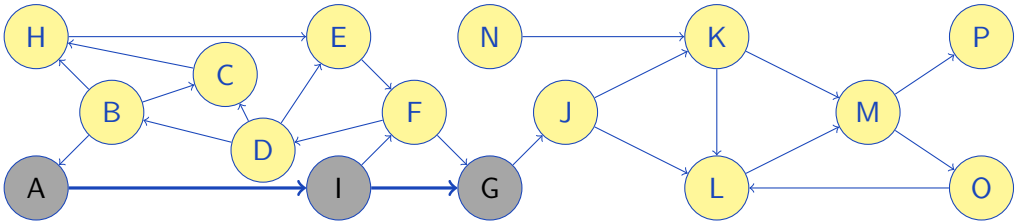
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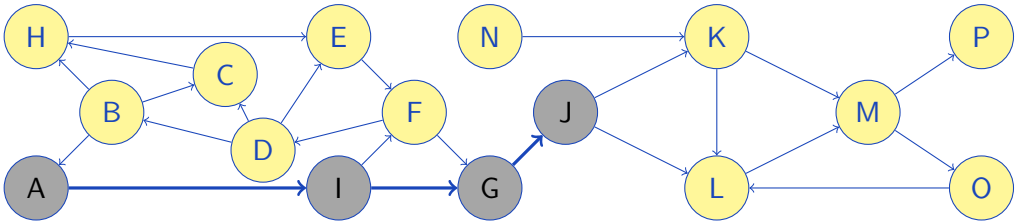
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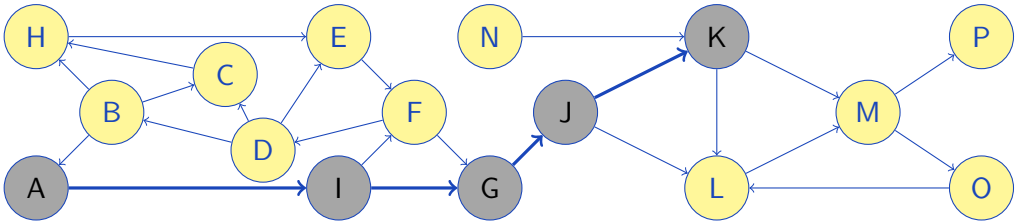
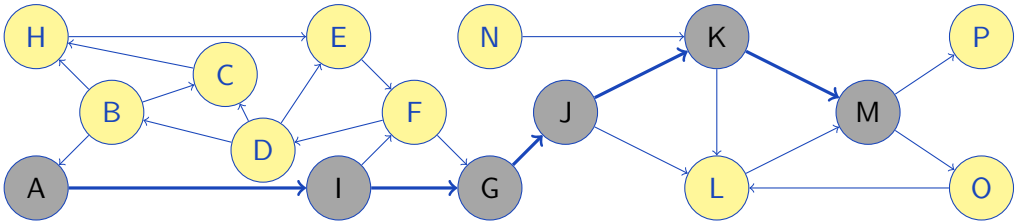
Example of condensation

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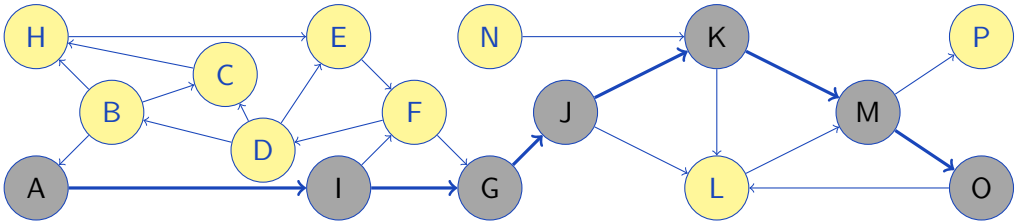

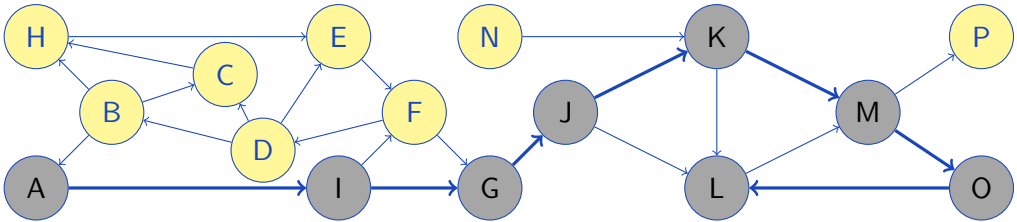
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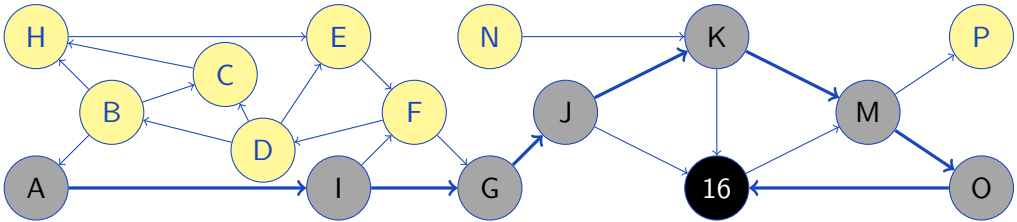
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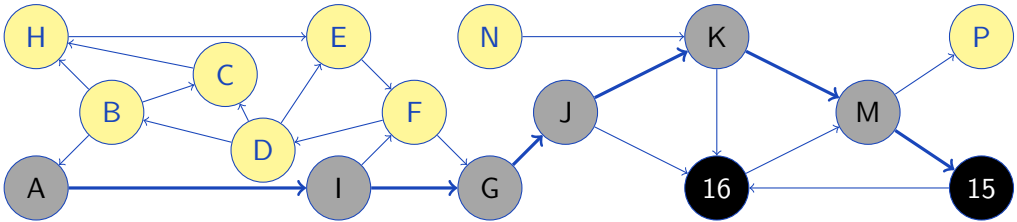
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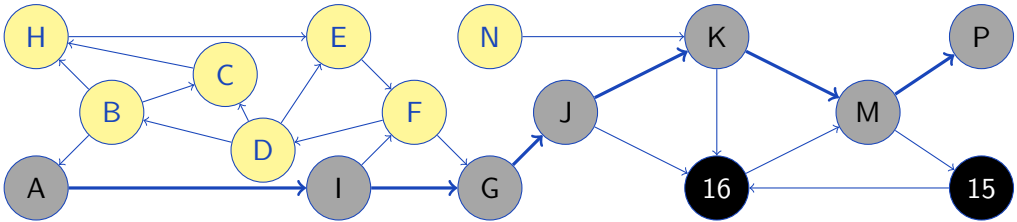
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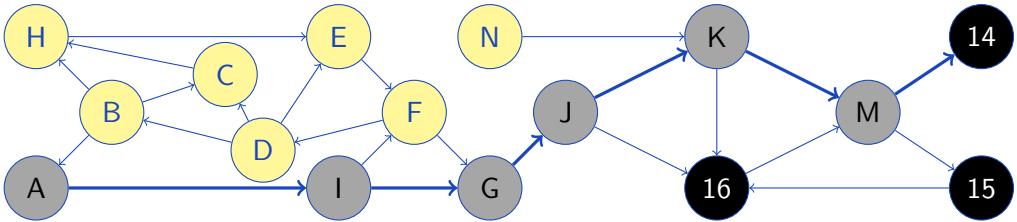
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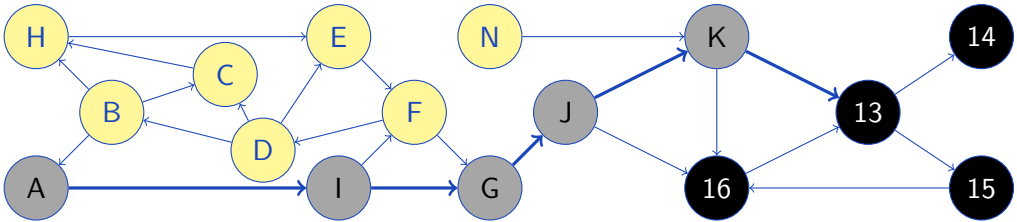
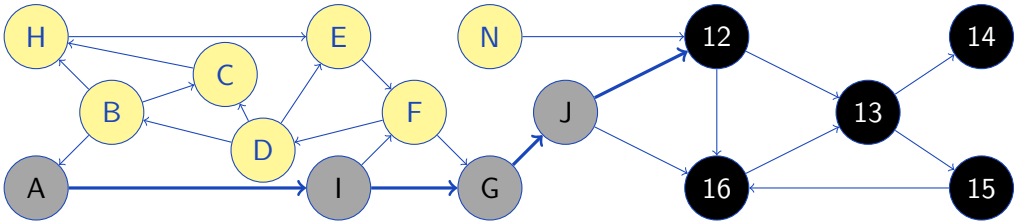
Example of condensation

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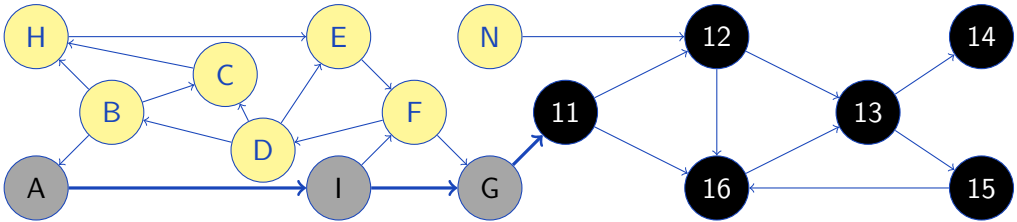
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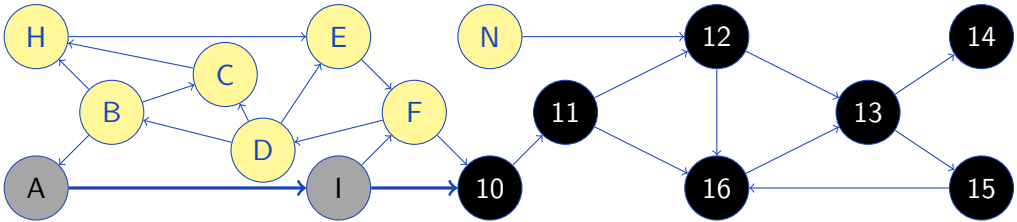

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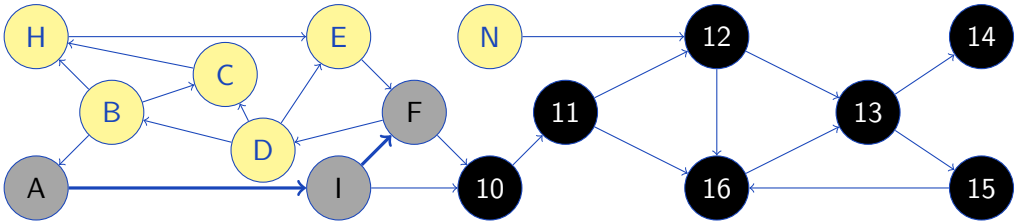

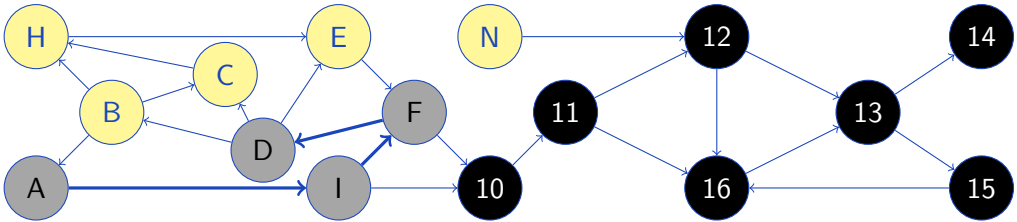
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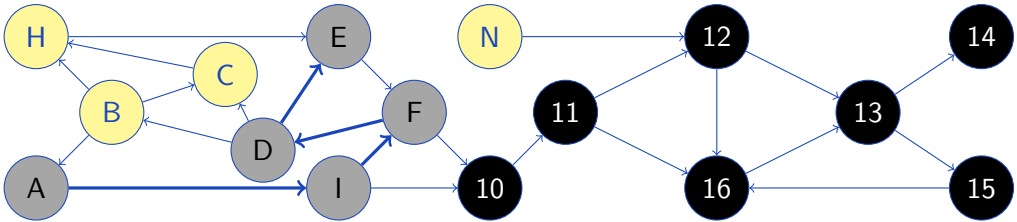
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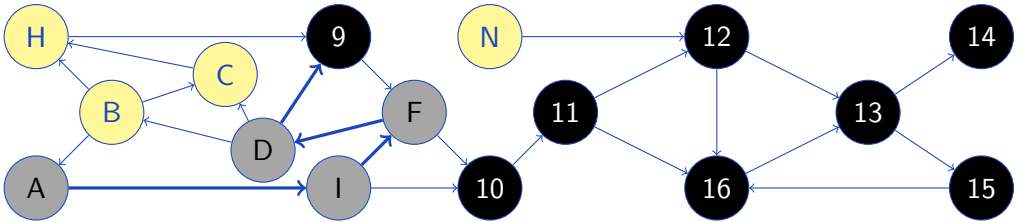
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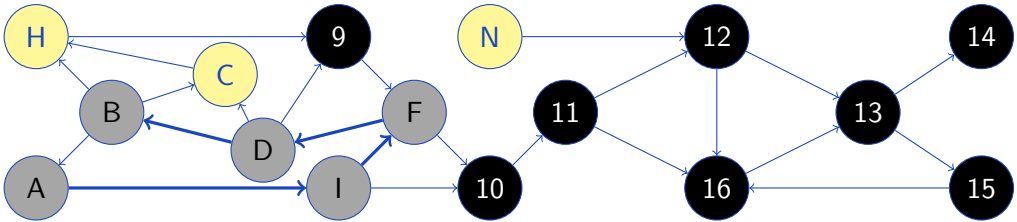
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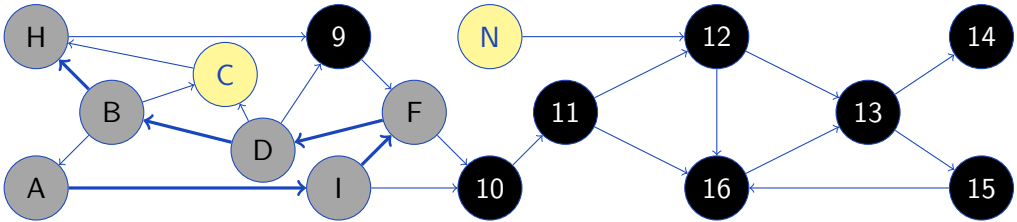
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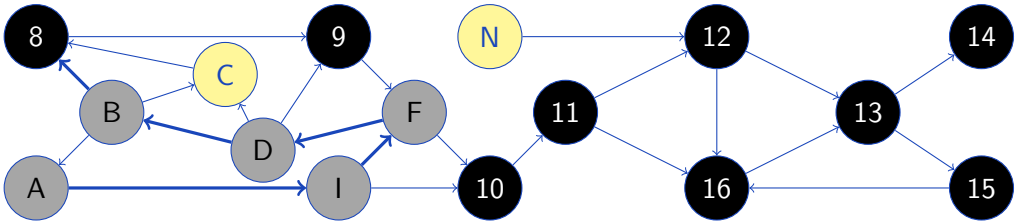
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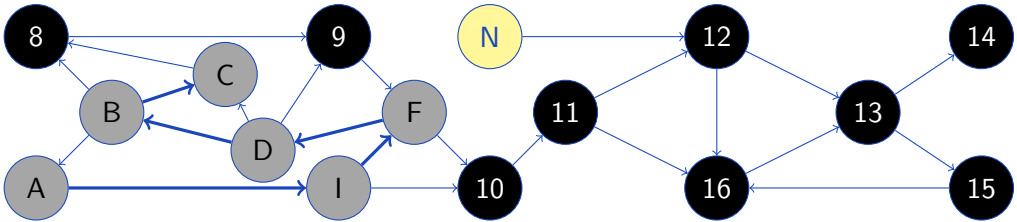
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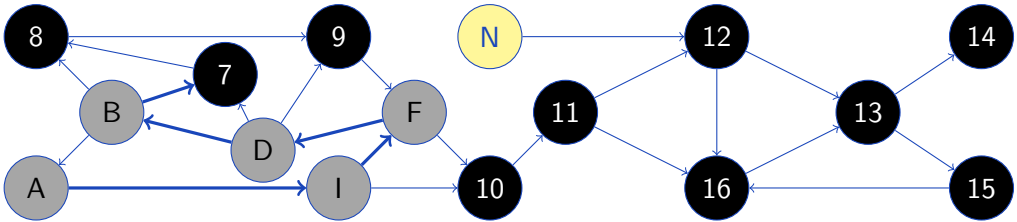
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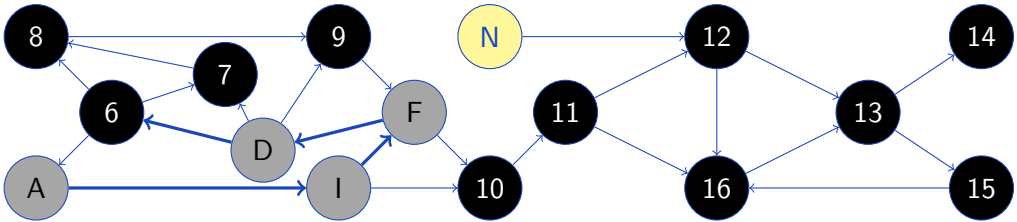
Example of condensation



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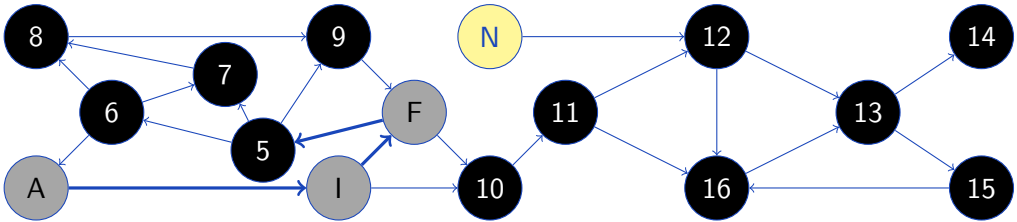

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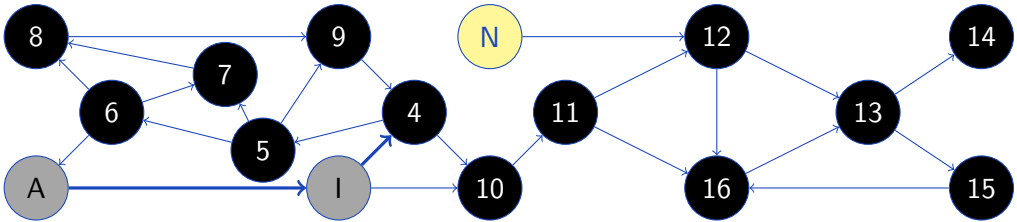

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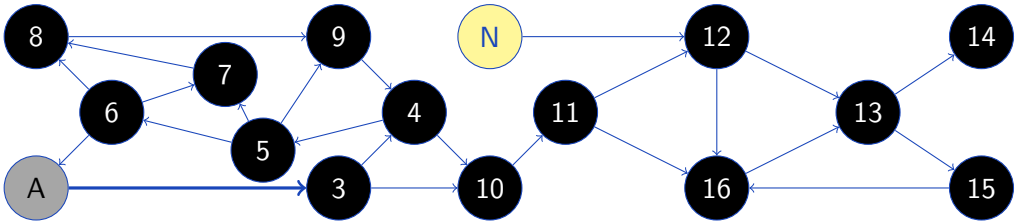

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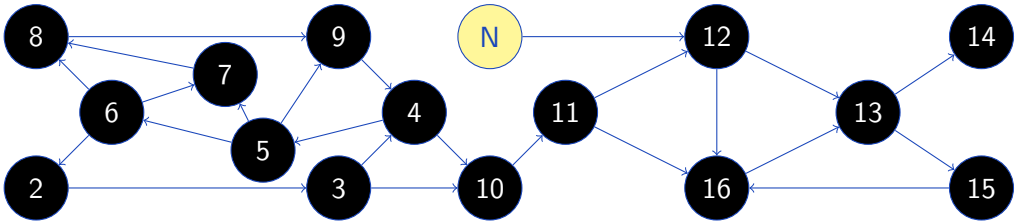

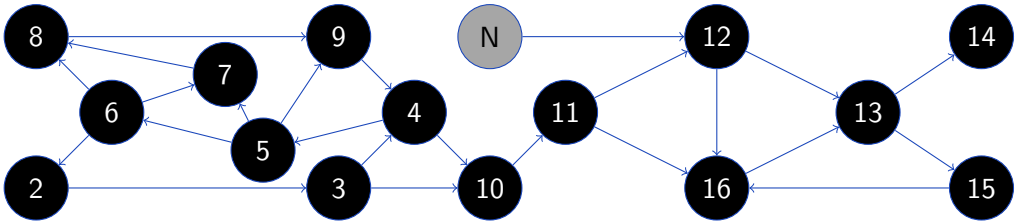
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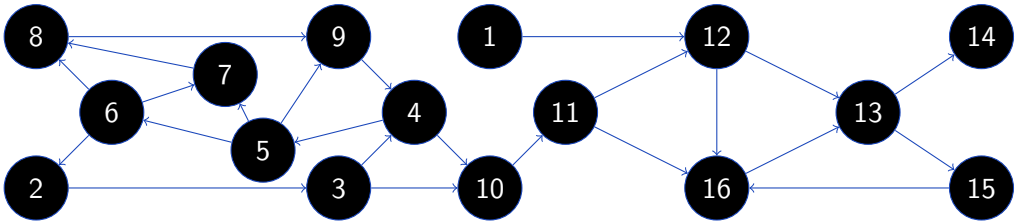

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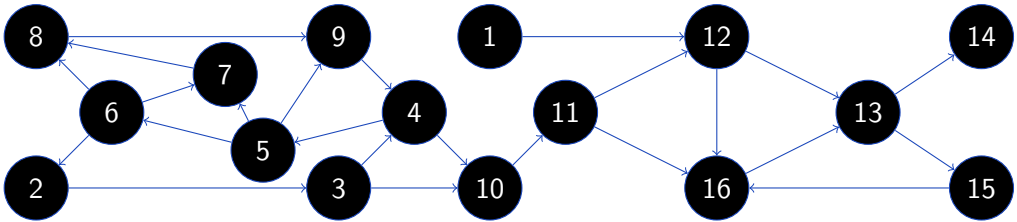

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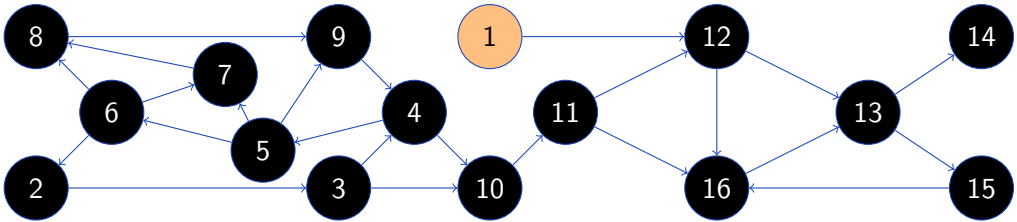

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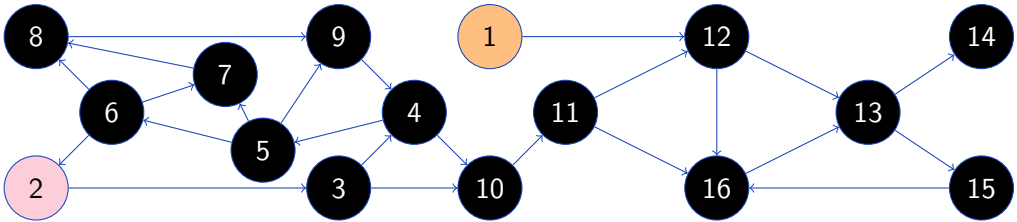

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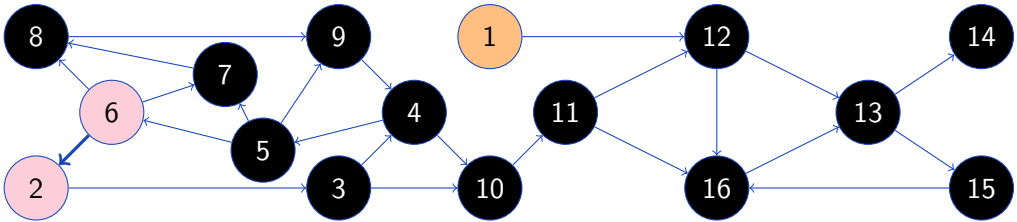

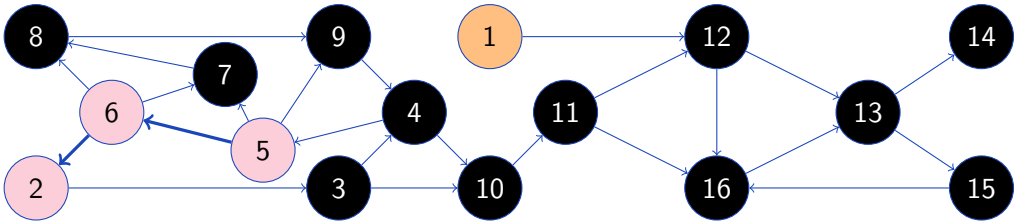
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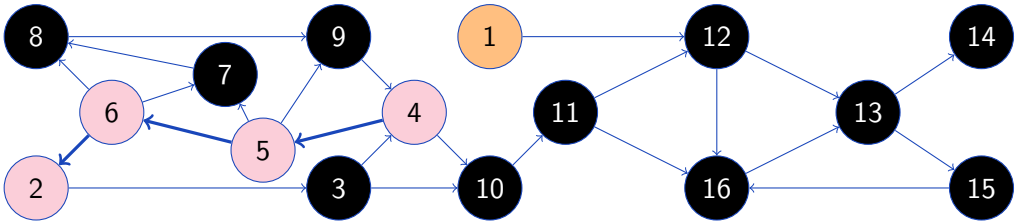

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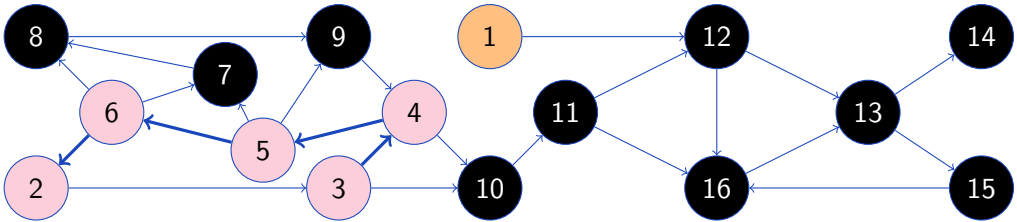

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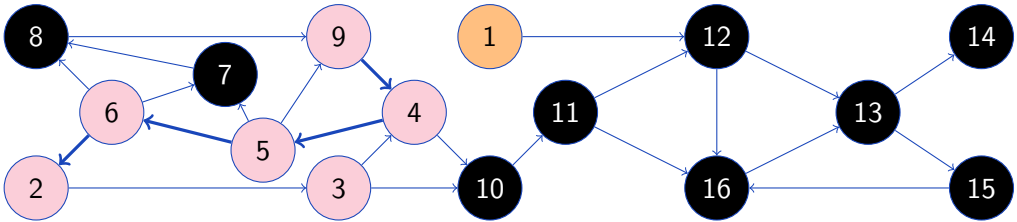

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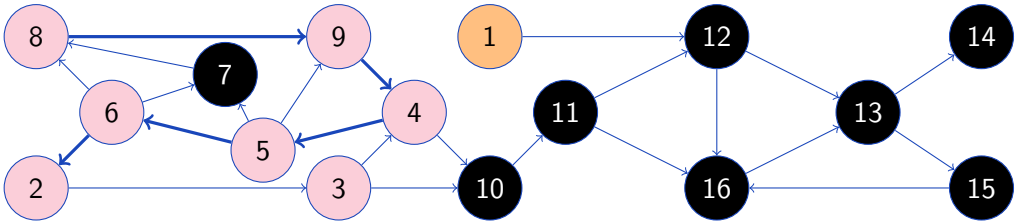

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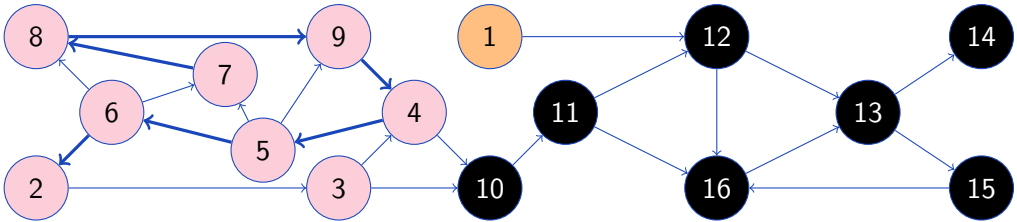

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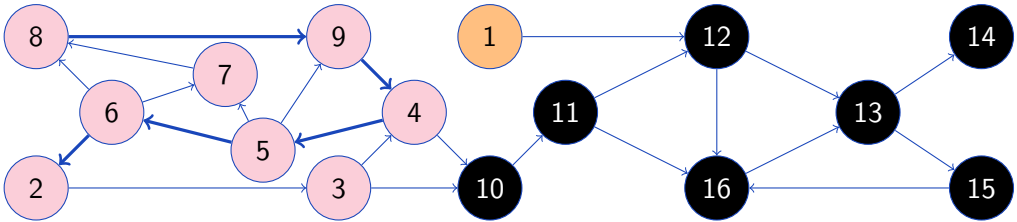

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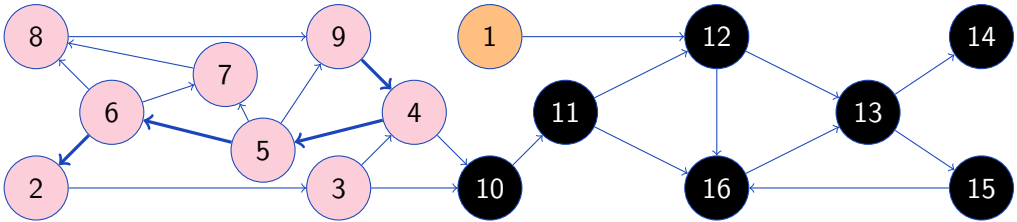

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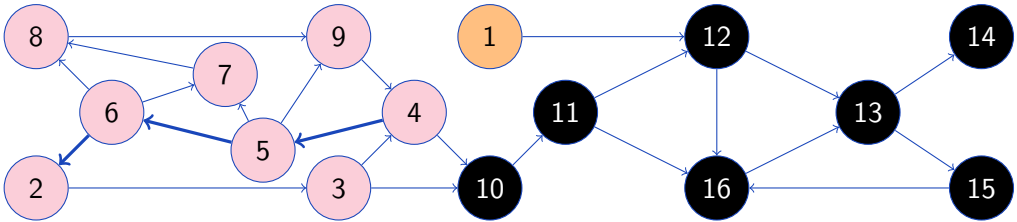

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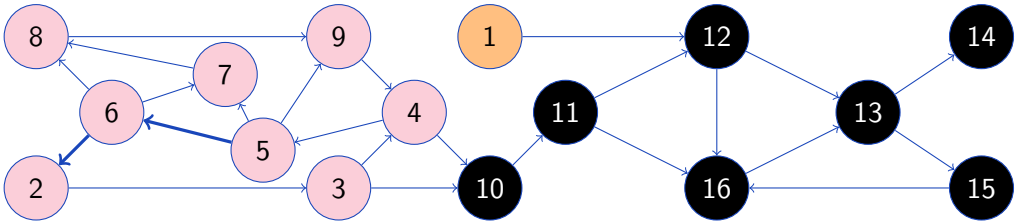

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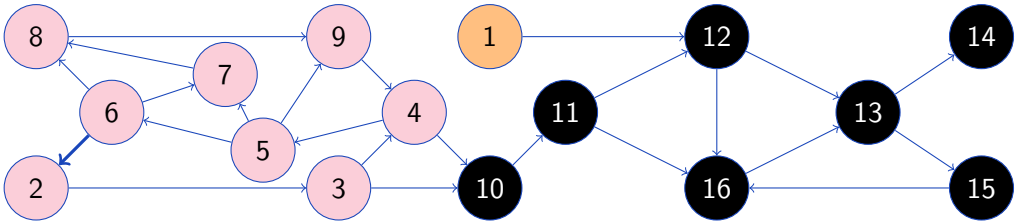

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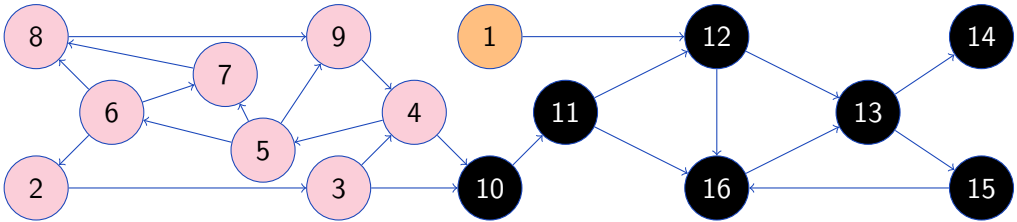

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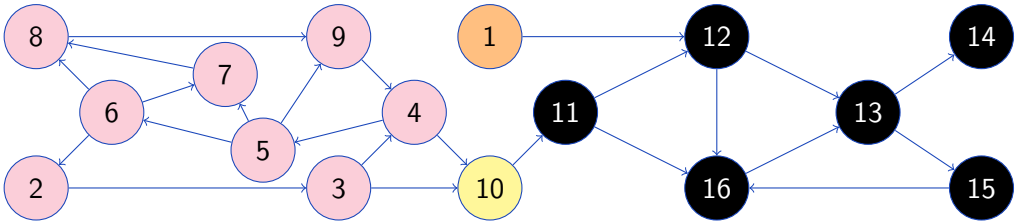

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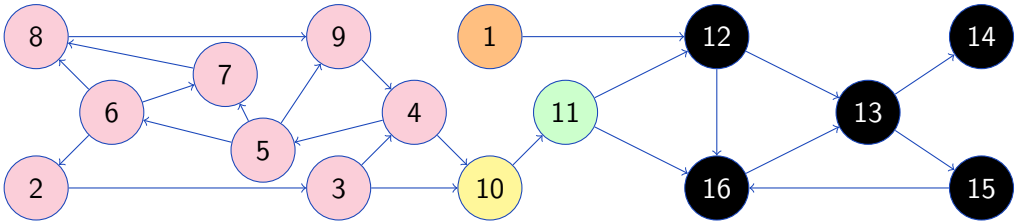

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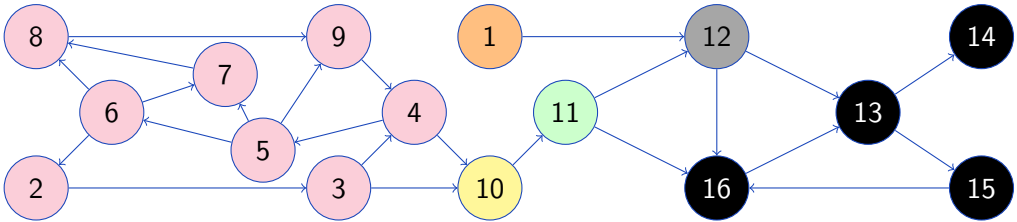

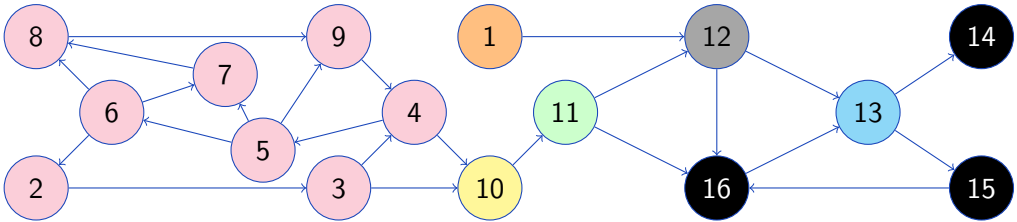
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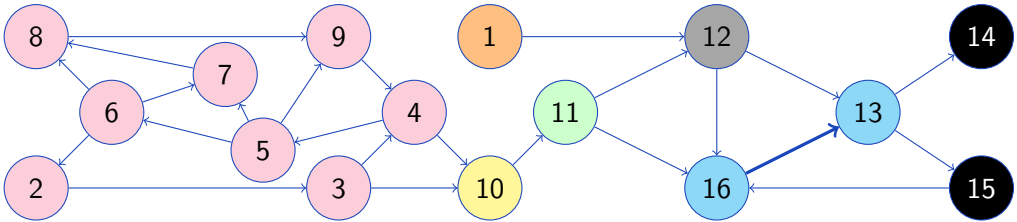

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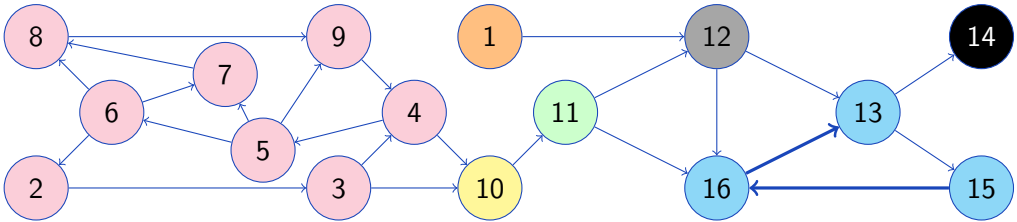

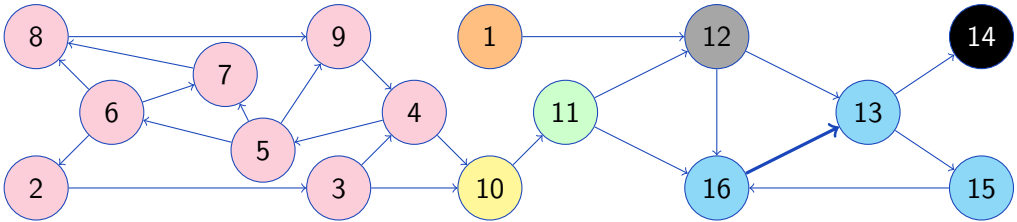
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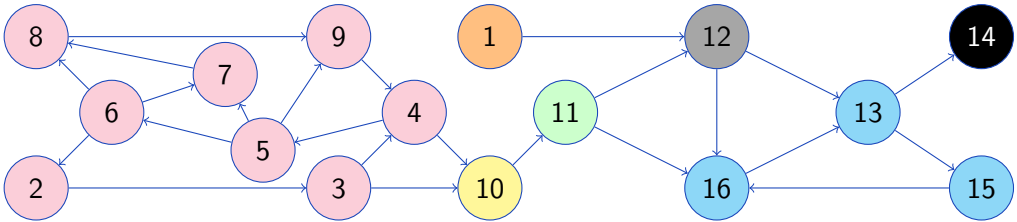

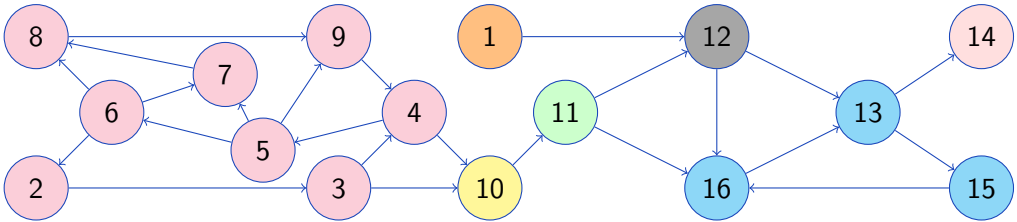
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