

# Part 1: Principles of Debt Sustainability

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## Unit 1: Structure and Learning Objectives

### Video-Part 1 Introduction:

*Irina Yakadina:* Hello, and welcome.

In this part of our course, we will learn the key concepts of debt sustainability analysis.

What do we mean by debt sustainability?

What are the key concepts and definitions, and how we apply them?

We will go through a few derivations together, not only to help you brush up some basic calculus, but mostly guide you through the formal part underlying debt sustainability analysis. More specifically, we will derive debt dynamics for different types of debt, such as domestic and foreign currency denominated debt.

We will discuss the dangers of high debt and explore the role of fiscal adjustment and other macroeconomic policies.

Ready? Set. Here we go on this exciting quest to learn the principles of debt sustainability and get ready to apply them in the subsequent parts of the course.

### Video-Part 1 Structure and Learning Objectives:

*Irina Yakadina:* In this part, we will learn the key concepts of debt sustainability analysis, talk about the dangers of high debt, learn about debt dynamics for different types of debt, such as domestic and foreign currency-denominated ones. We'll also explore the role of fiscal adjustment and other macroeconomic policies.

Following this overview, we will start in Unit 2 by what we mean by debt sustainability. We will provide an academic definition and add several pragmatic considerations. We will also give the public policy perspective and explain what is meant by solvency, as well as what is meant by liquidity. We will then derive various debt indicators, which can be used to shed light on the concepts of solvency and liquidity. And we will provide a brief overview of the IMF-World Bank approach to DSA, Debt Sustainability Analysis.

In Unit 3 we will ask "why the concern with debt sustainability?" Well you probably already have a good idea why high debt may be problematic. After all, you have signaled an interest in this course by signing up for that. We will spell out the types of concerns one might have regarding high debt, including vulnerability to sudden stops, debt overhang, and debt restructuring.

In Unit 4 we begin our analysis of debt sustainability and focus on the simplest case where we can only have domestic currency-denominated debt. This unit is probably the most challenging one. We will start from the flow of budget constraint and derive the intertemporal budget constraint. We will then impose the transversality condition to obtain the definition of solvency.

In Unit 5 we will become more pragmatic. We'll derive an equation for the dynamics of the debt-to-GDP ratio and briefly discuss the main drivers of the debt ratios. We will then derive the so-called debt-stabilizing primary balance.

In Unit 6 we will generalize our analysis to include foreign currency-denominated debt. We will then derive expressions for the dynamic of debt and for the debt-stabilizing primary balance. We use the latter to examine the role of various macroeconomic variables for debt sustainability.

Units 7 and 8 are, in a nutshell, a course in economics for emerging markets. Why? Unit 7 adds some economic sophistication to the previous analysis by taking into account possible interactions between fiscal adjustment, the risk premium on government debt, etc. We will consider front and back-loaded adjustments and ask, "which economic circumstances favor each one?" We will compare their pros and cons. In this unit we also discuss various types of economic crises and provide an overview of what the typical debt crisis might look like.

Unit 8 considers the role of macroeconomic policies and supporting fiscal adjustment. We consider the role of expansionary monetary policy and discuss various policy tradeoffs: debt versus inflation; sustainable debt versus competitiveness; and sustainable debt versus fairness.

Finally, Units 9 and 10 focus on external debt. Unit 9 introduces the concepts which are necessary to conduct the analysis, notably the adjusted balance.

Unit 10 goes through the steps of the Unit 4 and 5, but in a sped up way deriving the solvency condition; deriving debt dynamics for debt-to-GDP and for debt-stabilizing adjusted balance; and analyzing the comparative statics.

## Unit 2: Defining Debt Sustainability

### Video-Unit 2 Objectives and Outline

*Irina Yakadina:* This unit defines debt sustainability.

The learning objectives for this unit are: to understand the concept of sustainability; to master the relevant terminology; and to identify the relevant indicators of debt burden.

We also want you to become familiar with various types of DSA, debt sustainability analysis, as conducted by the IMF and the World Bank.

We will study debt sustainability from different angles, then turn to debt burden indicators, and end up looking through the procedure and the structure of the IMF and World Bank debt sustainability analysis.

### Video-What is Sustainability?

*Irina Yakadina:* So what is sustainability?

Let us look at the academic definition and introduce some pragmatism into this definition.

Let's first turn to the concept of intertemporal solvency. This is an academic definition of debt sustainability. In essence, it says that initial debt, plus the future stream of primary expenditure, here, should equal the future stream of revenue. Future streams need to be discounted by calculating their present discounted value, using the interest rate paid on debt. When we talk about primary expenditures, we refer to expenditures which are net of interest. We will show how to derive this condition when we get to Unit 2.

To clarify what we mean by solvency, consider the following: when a government, or a business, or an individual is "solvent," it is able to repay its current debt out of future income or surpluses. So a person with a small debt and large future income is solvent. Consider an example of a business with debt of \$20,000, and the prospect of annual profits of \$10,000 per year. This business is clearly solvent, because this debt can be serviced. And from where? From future profits.

A counter example is a "Ponzi" scheme. In such schemes, initial debt is serviced by relying on new investors. So it's a pyramid, right? We are bringing in more and more investors, instead of servicing it out of future surpluses. Such schemes are clearly not solvent. Investors will not get paid back.

A problem with the concept of solvency is that it depends on how the future will evolve. A prominent economist Guillermo Calvo, who had worked extensively on sovereign debt and crisis, captures this idea as follows-- solvency is "very much like honesty: it can never be fully certified, and proofs are slow to materialize."

Formally, under the academic definition, debt is sustainable if the intertemporal solvency condition is satisfied. That is, if the expected present value of future primary balances covers the existing stock of debt.

What are the keywords here? It is sustainable if we fully cover what we have already, by the expected present value of something that may or may not happen in the future. And this future is talking about the primary balances. This basically talks about the ability to postpone generating primary surpluses to cover for the existing debt obligations into the future.

As some of you might suspect, that this is a rather weak requirement. Countries with high debt and deficits are solvent under this definition, as long as primary surpluses are sometimes expected, sometime in the future. So that's a weak requirement.

To recap, the academic perspective provides precise formula for debt sustainability, but involves unobservable information and forecasting the future.

What matters in practice is not the level of debt itself, but the ratio of debt relative to a measure of capacity to repay. To capture this idea, we use a pragmatic definition of debt sustainability. In this definition, debt is sustainable if projected debt ratios are stable, decline and are sufficiently low. Debt is unsustainable if the projected debt ratios either increase or remain high.

Pragmatism consists in recognizing that the ratio of debt-to-capacity to repay is what matters in order to avoid a debt crisis. To be sustainable, debt cannot grow faster than incomes and any other capacity to repay it. Projections may very well suggest that debt is rising. But the debt-to-GDP ratio is falling. That'll be the case when GDP growth is high relative to interest rates.

We will discuss this idea extensively further on. A declining path for debt ratios is not quite enough for sustainability. Debt ratios should be also sufficiently low. A debt ratio which is declining but high would still be unsustainable if it is associated with a high risk of default.

The economies are always subject to shocks, therefore, a debt ratio which is borderline can also be unsustainable.

This graph illustrates some of the issues. Is debt sustainable? The variable of interest here is debt as scaled by GDP. This is a common variable measuring the ability to repay because it's the value added of

all goods and services produced in the economy. It's part of the pragmatic approach, pretty much. As you can see on the graph, the ratio first rises and then falls. Under the pragmatic approach, this is fine, as debt only needs to fall at the end.

Debt appears to be sustainable, provided its ratio to GDP is sufficiently low. So we need to watch out for what are the levels of those debt-to-GDP ratios. A good question is: "what does a non-explosive debt ratio imply for solvency?"

It turns out that non-explosiveness guarantees solvency, as long as the interest rate exceeds the growth rate of GDP, which by the way, is not a favorable situation for the economy. When, on the other hand, the growth rate exceeds the interest rate, it is possible for the debt-to-GDP ratio to fall, even if an economy runs primary deficits. You will see that such primary deficits are not consistent with solvency, because solvency requires countries to run sufficiently large primary surpluses to pay back initial debt.

Strictly speaking, non-explosiveness does not always imply solvency. Since what really matters from a pragmatic point of view is that debt burden ratios should be falling, we will not be too concerned if this situation is not accompanied by the actual solvency.

In what follows, we will use solvency rather broadly, and we'll consider economies with non-explosive debt as solvent.

To sum up, if we are in the situation where the relationship between the interest rate and growth is unfavorable with the economy, and the ratio of debt-to-GDP is stable or declining in the long run, then we will use this broad solvency condition and go forward.

Let us now have a preview of the material that will be presented more in detail further down the road. Namely let's talk for a second, what are the risks to debt sustainability?

We have already mentioned that a debt ratio, which is declining but high, maybe unsustainable if it is associated with a high risk of default. When making assessments, it's really important that the projections be based on realistic assumptions about the underlying macroeconomic variables. Why? Because with optimistic assumptions, it's really not hard to project that the ratios start declining or end up sufficiently low.

Also, the resulting gross financing needs will have to be evaluated. It is possible for debt to be low, but gross financing needs to be high, for example, this is often the case when the short-term debt is high. And furthermore the market perception of the sovereign also has to be factored in. Such market perception is oftentimes based on debt maturity structure, its currency composition, its creditor base, and any other factors.

Liquidity is part of the assessment of sustainability, as we will discuss further below.

## Video-Define Debt Sustainability

*Irina Yakadina:* We are now ready for a formal definition of sustainability. In this lecture, we turn to the economic policy definition of sustainability. And here it is.

Debt is sustainable if the country, or its government, does not, in the future, need to default, or renegotiate, or restructure its debt, or recur to implausibly large policy adjustments.

This is the definition adopted by the IMF and World Bank.

Let's talk about some examples for a second. Argentina is an example of a country which did default. Greece had to restructure its debt. Note that the distinction between defaulting and restructuring is a

fine line. For example, involuntary restructuring is oftentimes considered as default. And we have seen cases of both default and restructuring. These are examples of countries that had to make large policy adjustments and if you follow the media, you know how painful it is to make such adjustments. This is why we need to ensure debt sustainability early on.

We are now ready to give the economic policy definition of debt sustainability. Sustainability rules out any of the following situations: first, a debt restructuring is already needed; second, when a borrower accumulates debt at a rate which is faster than the growth in its capacity to service debt, particularly so in the long run; and finally, if the borrower lives beyond its means by accumulating debt in the knowledge that a major retrenchment will be needed to service these debts.

Question: Can you spot the additional issue highlighted by this definition? Under the definition considered so far, debt could be sustainable if large fiscal adjustments were made sometimes, in the future. Here debt is only sustainable if it does not depend on an implausibly large, or major retrenchment, policy adjustment.

The bottom line as we move forward is that debt is sustainable if a country, or a government, is able to service its debt without the need for-- first, the implausibly large policy adjustments; second, renegotiating the terms of debt; or simply defaulting.

## Video-Debt Burden Indicators for Solvency and Liquidity

*Irina Yakadina:* In this lecture we discuss the concept of liquidity contrasting it to the concept of solvency.

We then turn to key solvency and liquidity indicators and their role. What do we mean by liquidity? We define an entity as liquid if, regardless of whether it satisfies the solvency condition, it's liquid assets and available financing are sufficient to meet or roll-over it's maturing liabilities.

The ability to roll-over maturing debt is the key. So let's go back a little bit to what we should be watching out for in our subsequent analysis, which we haven't just started yet. And we already mentioned that we should have some realistic assumptions for our underlying macroeconomic variables.

But we also have what we call the "risk factors." And those risk factors are first and foremost, driven by what we call the "market perception" of the sovereign. This can be measured with indices such as the EMBI, for example, global, the Emerging Market Bond Index. It's also very closely related to the debt maturity structure, which is measured by the debt repayment profile and also what is the share of the short-term debt.

It's important to watch out for the currency composition of debt, as well as for availability of liquid assets. Does the government own liquid assets, which it could sell to help meet the obligations? And the creditor base, notably the share of nonresident creditors, is also a very important factor. Investment by nonresident creditors tends to be less stable. Sometimes it can be really difficult to distinguish properly between insolvency and illiquidity situations.

Liquidity problems are often symptoms of underlying solvency problems, because creditors just refuse to roll-over maturing debt. In addition, liquidity problems may give rise to insolvency by fueling a rise in interest rates or pressuring the exchange rate.

Our final concept is that of vulnerability. Vulnerability is defined as a risk that the liquidity or solvency conditions are violated, and the borrower enters a crisis.

So how do we assess the debt burden? By examining the projected evolution of a set of debt burden indicators, and doing so over time.

Let's now see what those indicators are. We use two types of debt burden indicators. The first set involves ratios of the debt stock, or debt service, relative to what we define as measures of the ability to service debt or repayment capacity. These measures of the ability to repay include GDP, export proceeds, fiscal revenue, and maybe some others. The second set is the gross financing need, either in level or scaled by the above measures.

Let us see the various debt burden indicators and how we use them. For example, debt-to-GDP will be a misleading indicator when a country is a closed economy. In that case, debt-to-exports can be more useful.

What do all these indicators measure? When we take the ratios of debt stock, relative to the repayment capacity, we are talking about the indicators of solvency. When we take the ratios of debt service, we are talking more about the potential liquidity issues. And finally, gross financing needs is an indicator of potential liquidity problems.

How do we define gross financing needs? It's basically the amount of financing which is necessary to cover the forthcoming debt obligations, such as the deficit, plus amortization of debt.

We can redefine the gross financing need in terms of primary deficit and debt service. Why? Because debt service is basically equal to amortization, plus interest payments.

A final touch: the gross financing need can be positive or negative.

Just to repeat, debt service is interest plus principal payments on debt, including short-term debt repayments.

The principal payments are also called "amortization" of debt. We defined amortization as principal repayments coming due on medium and long-term debt, plus short-term debt, which is maturing within the year.

These two graphs illustrate the different results one might get from looking at debt-to-GDP and debt-to-exports ratios. Let's compare these two economies. Both of them have the same debt-to-GDP ratio, 60%, but as those graphs illustrate, the debt burden ratios for an open economy, where exports-to-GDP very high, 60%, versus a closed economy, in which exports-to-GDP are just 10%, are very different. This debt-to-exports ratio is about 160% for an open economy, which is already quite high, but it's closer to 700% for the closed economy.

To illustrate the importance of short-term debt, let us consider a couple of scenarios. In both scenarios the total debt-to-GDP ratio is 66%. Both countries have the same primary balance and interest rates. When we talk about the first scenario, short-term debt accounts for 10% of total debt, while in the second it's 60%. These scenarios show that with a higher share of short-term debt in the total, debt service and gross financing needs worsen dramatically. So does debt service: we are talking about worsening to 43% of GDP, versus 17% in the case of low short-term debt. And it's almost tripling the debt service-to-revenue ratio from 69 to 180% of GDP. Similarly, gross financing needs on both ratios to GDP and revenue, are dramatically worse in the case of short-term debt being six times bigger under the second scenario.

To recap, we have reviewed various debt burden indicators--whose function is to measure the repayment capacity, or the ability to service debt properly-- solvency, and liquidity.

## Video-Scope of Debt Sustainability Analysis

*Irina Yakadina:* Let us now define the scope of debt sustainability analysis for different types of countries and for different types of debt.

This lecture discusses the scope of the IMF/World Bank Debt Sustainability Analysis, or DSA for short, and DSAs for different types of debt. A DSA is produced for a particular country. And we also have a DSF-- or Debt Sustainability Framework-- which is a framework within which a DSA is produced for a particular country. We need this framework to ensure comparability across DSAs for different countries.

For public DSA for advanced and emerging economies with access to financial markets, we use Debt Sustainability Analysis for Market-Access Countries (MAC DSA). This is a new framework, operational since 2013. You may have come across the previous framework in the past. This new framework of MAC DSA will be studied in Part 2 of this course, focusing on public debt. And here is what Google would find on the IMF web page. This page summarizes the methodology and the key results for MAC DSA.

The Debt Sustainability Framework-- or DSF-- originated as a framework to assess Debt Sustainability in Low-Income Countries-- or LICs for short. This will be studied in Part 3 of the course, focusing on external debt. A similarly simple search will defer to the IMF web page with the history and the most recent updates of the LIC DSF.

This slide here shows the most comprehensive possible approach for external and public DSA. The public DSA, in general-- or the vertical dimension on the slide-- can cover public and publicly guaranteed domestic and external debt. The external DSA-- which is the horizontal dimension of this slide-- can cover public and private external debt. In practice, the external LIC DSA does not include

private debt, because it tends to be small. And the public MAC DSA does not include guaranteed debt, only public debt itself.

Let's see an example of public and external DSAs. In December 2013, the IMF report on Cyprus in the context of the second review under an IMF program is published on our website. You can see, by way of example, that my colleagues were kind enough to include both external and public DSA into the report.

Let's talk about the scope of each DSA type. To be explicit, the public DSA-- which we also call fiscal DSA-- covers total debt of the public sector to both external and domestic creditors. MAC DSA for public debt covers only public debt. It does not include publicly guaranteed debt. LIC DSA for public debt covers both Public and Publicly Guaranteed Debt-- or PPG. The external DSA covers external debt in the entire economy. MAC DSA for external debt covers debt owed by both the public sector and the private sector. So we do have private sector in the MAC DSA. LIC DSA for external debt covers public and publicly guaranteed external debt but no private debt.

As you will see in great detail later, the DSA for MACs and LICs combine the indicators of solvency and liquidity, the trajectory and the level of debt, and financing needs under a variety of scenarios, including a baseline scenario and the adverse scenarios, which recognize uncertainty and incorporate macro-fiscal risks, such as, for example, the economic cycle-- the boom-bust analysis-- or shocks to contingent liabilities, as well as the traditional shocks to growth, interest rate, exchange rate, and other variables.

MAC DSA is essentially risk-based. That means that it considers risks to the debt level and the debt profile and it also imposes a great deal of scrutiny on the underlying assumptions about how the economy will evolve over the projection period included into a DSA.

LIC DSA often takes a long-term perspective, primarily due to the longer average maturities of the LIC debt portfolio, which is often owed to the official external creditors, including donors. In addition, it takes into account the concessionality of debt to produce estimates of debt burdens below those based on the face value. It also produces an explicit assessment of the risk of external debt distress.

To sum up, we've talked about distinguishing between countries with and without market access-- so MACs versus LICs-- as well as between different types of DSAs-- public DSA or external DSA. The second part of this course will introduce you to the public MAC DSA. The third part of this course will cover external LIC DSA.

## Unit 3: Why is Debt Sustainability Important?

### Video-Unit 3 Objectives and Outline

*Irina Yakadina:* So let us now talk of why is debt sustainability analysis important. After completing Unit 3, you will understand the costs of high debt, such as debt overhang, know what we mean by "debt crisis," as well as currency and banking crisis, and understand the mechanism of a debt crisis.

### Video-Costs Associated with High Debt

*Irina Yakadina:* Let us talk now about costs which are associated with high debt. The negative consequences of high debt are many. We will consider in turn, vulnerability to sudden stops, crowding out of private investment, loss of policy flexibility, the so-called "debt overhang," and debt restructuring.

The main consequence of high debt, whether it is public debt or external debt, is that a country with high debt is really vulnerable to what we call a "sudden stop" in financing. If private investor sentiment turns, whether it's because global risk appetite has changed, or because of an adverse shock to the country, inflows could stop, and that can happen really fast. There could be even capital outflows by residents or nonresidents.

Countries with high debt are also more vulnerable to drying up of official flows. For example, when aid inflows stop or dry up. What's the impact of sudden stops? Sudden stops can be really devastating. On the side of external debt, a sudden stop could lead to current and capital account restrictions, basically in an effort to conserve foreign exchange reserves. Or it could lead to a currency crisis, or banking crisis, a recession, and ultimately, a default. On the side of public debt, it could force the government to implement drastic reduction in spending. Or it could lead to a currency crisis, banking crisis, recession, or default. The additional consequences for public debt include the possibility that interest rates become much higher, because the country risk premium would rise with high debt. This could in turn crowd out private investment. Also, there may be less flexibility to combat counter-cyclical policy, as the government can no longer afford to raise spending in a downturn, when debt is already high.

And finally, debt overhang. Let's see what we mean by that. The definition of debt overhang is a situation where the expected tax burden to finance debt is so high that it is basically a disincentive to the current investment and consumption. And therefore, it will cause a drag on economic activity. Why? Because when investors expect high tax rates, they basically reduce investment. This reduced investment, in turn, leads to lower growth and lower government revenues. As a consequence, we will have insufficient funds for primary expenditures. And you know what? If you have insufficient



money to spend, and you have lower revenues, your chance of default is higher.

This schematic representation illustrates a vicious cycle of debt overhang. If we start with worries about debt sustainability, it may lead to questions about how the sovereign will finance itself. And if this is the case, investors will become more and more reluctant to invest under those circumstances. Because higher taxes lead to a lower after-tax rate of return, there would be less incentive to invest, and that in turn would reduce economic growth. But concerns about growth then accelerate concerns about the deficit. Why? Because we have the scissors effect, right? If you have low economic growth, you really need more cyclical expenditure. Higher deficits trigger an increase in risk premium, and higher interest rate add to deficit directly, which accentuate concerns about sustainability. Take a few moments to study this cycle.

A final consequence of high debt is that the government may choose debt restructuring. While debt will be lower, restructuring has grave consequences, including political and economic penalties. In the past, the threat to restructure one's debt led to "gunboat diplomacy," where a government used military threat against another in order to secure debt payments.

One of the adverse consequences are spillovers across segments of the economy, especially if banks are major holders of government debt. But another one is contagion to other countries, as we've observed many times in the past, most notably during the Russian crisis of 1998, and more recently in the Greek crisis.

To recap, we've talked about the costs associated with high debt, such as increased vulnerability, the crowding out of private investment, loss of flexibility to run economic policies that help the cycle, or help offset the downturns, the risks from debt overhang, and the risks and dangers of potential debt restructuring.

## Video-Types of Economic Crises

*Irina Yakadina:* Let's talk about crises. As one of the most famous Russian writers Leo Tolstoy once stated, "Happy families are all alike, but every unhappy family is unhappy in its own way." Well, much like unhappy families, crises are hard to generalize. Yet, we will try hard to group them by type and study distinctive features of each type of crisis.

We will study these following different types of economic crisis-- the currency crisis, the banking crisis, the external debt crisis, and the sovereign debt crisis. Let's see how different they are.

So, what is a currency crisis? A currency crisis results from an attack on a country's currency. And it ends up with a combination of one, or maybe more than one, of the following-- a large devaluation, a sharp depreciation, a large increase in interest rates, or a large fall in reserves. These crises happen when there are concerns about the viability of the exchange rate regime or the level of the exchange rate. The examples of such crises are one, of the European Monetary Union in 1992 and the Argentine devaluation of 2001. There are many more. Such crises tend to occur quite frequently.

What are some underlying reasons for the loss of confidence? There are various types of scenarios, much like there are many types of unhappy families. Let's consider one in which inconsistent macroeconomic policies such as, for example, monetization of a large fiscal deficit, leads to a drain in foreign exchange reserves. As the reserves are about to run out, there is a speculative attack. The reserves of foreign exchange may be insufficient to cover short-term debt. That has been extensively modeled in the literature, starting by Krugman back in 1979 and all the way to Chang and Velasco and beyond.

In another scenario, the market expects the government to devalue in order to address a policy goal, for example, such as improved competitiveness. So we know the government is after solving a particular problem. And we expect that the means used to solve this problem will be a devaluation. The canonical model here is basically Obstfeld 1994, where investors are concerned that the government might devalue in order to support employment.

Let us now turn to banking crises. A banking crisis is defined by bank runs with depositors asking back their money, and just withdrawing their deposits, or a large scale government intervention needed to rescue banks. Banking crises occur when there are concerns about solvency and liquidity of the banks. The canonical model here is that of Diamond-Dybvig of 1983.

What are some underlying reasons for a banking crisis? Such crises are associated with a bursting bubble in equity, or real estate prices, or sequential or simultaneous adverse shocks to various macroeconomic variables, such as interest rates, exchange rates, or growth. It also can be that prior lending booms, which usually occur after financial liberalization or large capital inflows, burst and are followed by such a crisis.

The third type of crisis we are interested in are debt crises. Such crises can be associated with either sovereign, public debt, or commercial, private debt. Let us consider these in turn.

Let's start with sovereign debt crises, which consist of defaults, involuntary restructuring of sovereign debt, or the belief that one of these events is about to occur. They often happen simultaneously, or they may be immediately following a banking crisis. In fact, that was the case for over 60% of all sovereign debt crises between 1970 and 2010, as documented by Rogoff and Reinhart. The underlying reason for debt crises can be that the government is extending a helping

hand of financial rescue packages to the private sector, aka, the banks. It can be also an extended period of low growth, or it can be fiscal profligacy, including war financing. It can be the failed state-owned enterprises that have to be taken on the sovereign balance sheet. It can be a consequence of natural disasters, or any other reason.

Turning to external debt crises; they are characterized by payment arrears on a substantial fraction of external debt and they tend to occur when countries experience cash flow problems or difficulties in obtaining foreign exchange. They may be triggered by sudden stops following a capital inflow episode, or interest rate, or exchange rate, or growth shocks. Thus, shocks to the key macroeconomic variables can lead to corporate bankruptcies. It can also make it difficult for corporates, banks, and the government to service their debts.

To recap, we have reviewed the definitions and origins of various types of crises.

And I now invite you to watch a video on Latvia, which is covering a debt and balance of payment crisis with internal devaluation and fiscal adjustment.

## **Video-How Latvia Recovered from Crisis (IMF public video)**

### **Video-Mechanism of Sovereign Debt Crises**

*Irina Yakadina:* Let us now talk about the sample mechanism of sovereign debt crisis. We start the lecture by illustrating what the typical debt crisis might look like. And then we discuss bank-sovereign interdependence: the two-way links between bank-holds and the sovereign debt.

A typical sequence of events would go like this: A banking crisis requires rescue by the government. The financial sector rescue package, in turn, weighs on public debt, and adds to the deficit, even though not entirely, but in part. Together with the banking crisis, this causes economic activity to nosedive. Because of that, fiscal revenues collapse, while expenditures skyrocket. But the story, actually, doesn't end there. The spike in deficits and debt, causes concerns about the fiscal balance, and ultimately about debt sustainability. As a consequence, a cost of borrowing for the sovereign increase, and the fiscal position worsens further. Thus a story which started with trouble in the banking sector ends up as sovereign debt difficulties.

We will see that there is interdependence between the banking sector and the sovereign. Bank-sovereign interdependence can, in general, go both ways. There is a mechanism we just described-- damage to bank balance sheet leads to bailout costs, and an increase in sovereign debt.

But there is also the reverse, an effect of the sovereign onto bank balance sheet. How does the latter work? Well, an increase in sovereign debt would lead to high probability, or possibility, of sovereign default. So that will lower the ratings. And, as you may recall, the rating of the company cannot exceed the rating of the sovereign. So this, in turn, spills over and damages the balance sheet of the banks that own sovereign debt, and maybe own in the company debt. In fact, we often find vicious circles, where banks adversely affect the sovereign. That then exacerbates bank difficulties, and the vicious circle continues.

This slide summarizes the interactions we have discussed so far. Let us start from the bottom bubble. The financial system fragilities lead to bank bailout costs, and may cause a recession. A recession would, in turn, lead to increase in debt and deficit. So it will further highlight country's fiscal vulnerabilities, and worsen its fiscal fundamentals.

This situation of increased vulnerabilities will have two types of effects. First, the spillovers from the sovereign to the banks, through the mechanism we described earlier-- an increase in debt tends to lead to a decline in prices of sovereign bonds, and hurts banks that are holding sovereign bonds. But there is also another effect-- going back to our discussion of the consequences of high government debt. Funding strains lead to crowding out and debt overhang. An ensuing recession affects banks through non-performing loans.

To recap, we've talked about sample mechanism of debt crisis. And we also covered the bank-sovereign interdependence.

## Unit 4: Public Debt Sustainability in a Closed Economy, Part 1

### Video-Unit 4 Objectives and Outline

*Irina Yakadina:* Let us now turn to the analysis of public debt sustainability. In Units 4 and 5 we will cover public debt sustainability in the closed economy. In such a stylized economy, we have in mind that there's only one type of debt expressed in local currency. Then we'll turn to Unit 6 and study an open economy, where we'll introduce a second type of public debt: debt expressed in foreign currency.

Our objectives for this unit are two-fold. First we'll set up the expression for the law of motion for public debt, also called public debt dynamics, which is the key equation on which the analysis is based. Then we will learn how to derive the formal solvency condition for public debt.

## Video-Law of Motion for Public Debt, Part 1 of 3

*Irina Yakadina:* Let us start with the law of motion for public debt. After deriving the expression for the law of motion of public debt, we will introduce the concept of present value, which we will need to derive the solvency condition.

In what follows, we will explore the relationship between debt and deficit. We will rely on the concept of primary balance, also known as a non-interest balance, the difference between non-interest revenues and spending. And then we will show the government budget constraint and how it can be expressed as a law of motion for debt.

Intuitively, a country's debt reflects the past deficits it has incurred. How, precisely, are debt and deficits related? The current stock of debt equals the past stock of debt, plus deficit, plus other flows, plus the exchange rate valuation. We will discuss "other flows" shortly. Exchange rate valuation is a term, which is only relevant when some of the debt is expressed in foreign currency, and we will study this term when we study the open economy in Unit 6.

Deficits add to debt, but the relationship is more complex than that. Why? Because debt pays interest. Consider the following graph. A deficit needs to be financed so it leads to borrowing unless it is financed by another way. Borrowing adds to debt, on which interest needs to be paid. This, in turn, adds to deficit. This is a vicious circle of debt and deficits.

To recap, a budget deficit can be financed by borrowing or other means, such as printing money or selling assets. New borrowing necessary to finance a budget deficit adds to the current stock of debt.

## Video-Law of Motion for Public Debt, Part 2 of 3

*Irina Yakadina:* We will now turn to the analysis of public debt dynamics. We will show that the flow budget constraint and the law of motion for public debt are, in fact, the two sides of the same coin.

The notation we will use is as follows:

Let us define the stock of debt as  $D(t)$ , and note that it always equals past debt, plus the change in debt.

We define interest spending as  $I(t)$ .

We will assume that the interest spending can be derived as the interest rate, times past debt. For that, we can always define the nominal interest rate as a weighted average of interest rates on each of the loans with different maturities that usually compose the portfolio of public debt.

Now,  $i(t)$  is nominal interest rate, and the  $R(t)$  are government revenues, net of interest earned on public sector assets, if any.

Let  $G(t)$  be government primary spending.

We are ready to define  $PB(t)$ , which is the primary surplus equal to  $R$  minus  $G$ . This will be our key fiscal measure, extensively used in debt sustainability analysis.

You may wonder what we mean by debt. There are two concepts of debt, gross debt and net debt, the latter being debt, net of financial assets. The analysis in this section can proceed at either level.

Now a technical point: When debt refers to gross debt, the earnings on assets, notably interest, should be included in revenues. When debt refers to net debt, these earnings should be excluded.

Continuing the notation, we use  $r(t)$  for real interest rate and  $\pi(t)$  for inflation rate. The nominal interest rate is roughly equal to the real interest rate, plus inflation. The precise formula here is (1 plus the nominal interest rate) is equal to (1 plus inflation) multiplied by (1 plus the real interest rate).

Finally,  $g(t)$  is the real growth rate of GDP, and we denote nominal GDP by  $P(t)$  times  $Y(t)$ . Nominal GDP at time  $t$  can be related to its past,  $P(t-1)$  times  $Y(t-1)$ , by multiplying (1 plus the inflation rate) and (1 plus the real GDP growth rate).

Using this terminology, we can write the flow budget constraint as follows: On the left hand side of the equation, we have the deficit, primary spending plus interest spending, minus revenues. We also have what we call "other flows."

These other flows are a residual category, which includes two items. First, spending items not captured in  $G$  and second, non-debt sources of financing. Examples of items included into the first category, asset purchases and expenditure items are bank recapitalization or assumption of guaranteed state enterprise debt. For the second category, non-debt sources of financing, we can have asset sales, such as privatization revenues, and we can also have the so-called seigniorage or revenues from money creation.

Please note that the second category items enter other flows with a negative sign as they really belong to the right hand side of the budget constraint, but have been grouped in other flows for simplicity.

So back to our formula. On the right hand side, we have debt financing.

Please note that there is an assumption hidden in this equation, which may not be immediately obvious. We assume that the change in debt provides so equivalent financing to the government. The implicit assumption here is that there is no debt relief, neither debt restructuring.

### Video-Law of Motion for Public Debt, Part 3 of 3

*Irina Yakadina:* Let's turn to other flows. Other flows include asset purchases and expenditure items which are not included in  $G$ . What are the examples? Well, bank recapitalization or assumption of guaranteed state enterprise debt, or it can be non-debt sources of financing such as asset sales, say, privatization revenues, or seigniorage.

Non-debt sources of financing enter as a negative sign in other flows as they really belong on the right hand side of the government budget constraint, but have been grouped in other flows for simplicity.

Next, we will use a definition of primary balance. The primary balance is equal to revenues minus primary - or non-interest expenditures; assume that other flows are 0 for simplicity.

In Unit 6, which presents a more general case, we will reintroduce other flows. Solving for  $D(t)$ , we find this simple, yet very powerful, expression relating debt to past debt and the primary balance.

We find that  $D(t)$  equals (1 plus the nominal interest rate), multiplied by past debt, minus primary balance. To recap, we have started from the budget constraint and through simple manipulations, found an expression for the change in debt over time.

Because we capture what happens to debt over time, we call this equation the Debt Law of Motion, or Debt Dynamics. In essence, the

law of motion for debt is reorganized budget constraint. We will use these terms interchangeably.

How can this equation be used to project debt over time? Suppose that  $D(t-1)$  is equal to 100, that the nominal interest rate is 2%, and the primary balance is minus 2% of GDP. So there is a primary deficit of 2%. Simply plug-in these numbers to find  $D(t)$ . To find debt in years further out, plug in the result for  $D(t)$  in the same equation and find  $D(t+1)$ . The results are shown in the graph.

## Video-Deriving the Solvency Condition, Part 1 of 2

*Irina Yakadina:* In this lecture, we next turn to solvency condition. To give you a preview, we start from the flow budget constraint to derive the intertemporal budget constraint. We then impose the so-called transversality condition, and find a condition for solvency.

Let's pause for a moment and see what we have done so far. We've talked about the debt-deficit relationship. We've derived the government budget constraint. And we've derived what we have: the law of motion for public debt, and so far we've derived it for just one period.

So we started in period  $t-1$  when we had our stock of debt outstanding, and we obtained the outcome for the period  $t$ .

What's next? We will start with the flow budget constraint again, and we will use this forward substitution to derive the intertemporal budget constraint, which we need to obtain the solvency condition, and we will do so in present value terms.

## Video-Deriving the Solvency Condition, Part 2 of 2

*Irina Yakadina:* We are now going to do the following: start from the flow budget constraint to derive the intertemporal budget constraint. We then define and impose the so-called transversality condition and obtain the condition for solvency.

Now please recall the flow budget constraint at time  $T$ . Let us apply this formula to  $T=1$  and  $T=2$  to obtain the flow budget constraints for  $T=1$  and  $T=2$ . These are the first two expressions.

What we do next: we substitute the flow budget constraint for  $T=1$  into the flow budget constraint for  $T=2$ . We expand the expression. The resulting expression relates  $D(2)$  to  $D(0)$ , as well as the primary balances at times 1 and 2, with all terms being multiplied by the appropriate interest factors. We call this expression the intertemporal budget constraint for  $T=2$ .

What do we do next? We obtain the intertemporal budget constraint for  $T=3$  by simply plugging in the intertemporal budget constraint for  $T=2$  from the previous slide into the flow budget constraint for  $T=3$ , then, going forward, make the substitution for each subsequent period to obtain the intertemporal budget constraint for terminal year  $T=N$ .

The resulting expression relates  $D(T)$  to  $D(0)$  and the primary balances between time 1 and time  $N$  with all the terms again multiplied by appropriate interest factors.

That is, we have  $D(N)$  is equal to  $(1+i)^N$  times  $D(0)$ , minus the sum between period 1 and period  $N$  of primary balances scaled by  $(1+i)^{N-t}$ .

We are now just two steps away from the solvency condition. To obtain the solvency condition from the intertemporal budget constraint, we will first divide the left hand side and the right hand

side of the constraint by  $(1 + i)$  to the power of  $N$  and solve for the initial debt.

We find that initial debt is related to the intervening primary balances and terminal period debt. To be precise, initial debt equals the sum of the present discounted value of future primary surpluses plus the present discounted value of terminal period debt,  $D(N)$ . Each term is discounted by an interest factor involving the number of years into the future when these primary balance or debt occur.

Does this condition impose any real constraint on the government? Not really. Why? Actually, any path for the primary balance is consistent with the intertemporal budget constraint. Higher deficits will simply translate into higher debt, which means that to get a meaningful condition which serves a constraint on the policies, we need to impose a condition on terminal debt,  $D(N)$ .

The condition we impose on terminal debt is the so-called transversality condition. This condition is sometimes referred to as a "No Ponzi Scheme" condition. The name Ponzi Scheme refers to a scheme used by Charles Ponzi back in 1920 to defraud investors. I'm sure many others used it even earlier but didn't manage to get it named after them. Under the scheme, Ponzi paid investors not out of any profits, but out of the capital obtained from new investors.

So the formal condition is as follows: The limit of  $D(N)$ , the terminal debt, scaled by the interest rate,  $(1 + i)$  to power minus  $N$ , is equal to zero. This condition states that, in the limit as time goes to infinity, the present discounted value of terminal debt is 0.

The condition does not rule out the existence of debt in the terminal period and does not even rule out growing debt. However, debt has to grow at a rate less than the nominal interest rate.

How does this condition rule out a Ponzi scheme? Well, in a Ponzi scheme, when all debt is rolled over and new debt is issued to cover interest payments, this debt would grow at the speed of the rate of interest and not less than at the speed of the rate of interest. So transversality rules out a Ponzi scheme.

Imposing the transversality condition, we find the condition that states that the outstanding initial debt should be covered by the present value of future primary surpluses. This is our solvency condition.

This condition does impose a constraint on government. If initial debt is positive, the government needs to run primary surpluses in the future.

To recap, there is an intimate relationship between debt and deficits. Deficits add to debt and then lead to high interest rates. What we did was we introduced the flow budget constraint on the government and showed its close relationship with the law of motion for debt.

## Unit 5: Public Debt Sustainability in a Closed Economy, Part 2

### Video-Unit 5 Objectives and Outline

*Irina Yakadina:* Let us continue with the case of a closed economy. Our objectives are to get to debt dynamics of the ratios and analyze how key macroeconomic variables affect such dynamics. And what will it take to arrest rising debt ratios?

## Video-Law of Motion for Public Debt-to-GDP in a Closed Economy, Part 1 of 2

*Irina Yakadina:* This lecture does the derivations. And we start with the law of motion, and then we analyze the comparative statics, so the impact of the key macroeconomic variables, such as primary balance, initial level of debt, growth, and real interest rate.

First and foremost, let's check: where are we? We are still in the case of a closed economy, just to avoid worrying about the exchange rate and the variations in debt that these variations in exchange rate are inducing.

What do we do next? We are able to get really pragmatic and do derivations in terms of ratios. And these are ratios to the economy's capacity to repay, as measured by GDP. And once we get the law of motion for debt-to-GDP, we can really look at the impact of the variables.

## Video-Law of Motion for Public Debt-to-GDP in a Closed Economy, Part 2 of 2

*Irina Yakadina:* Let us derive the law of motion for the ratio of public debt-to-GDP. Once we have the condition, we'll examine how debt is affected by macroeconomic variables, such as growth and interest rate.

We start from the now familiar expression for the dynamics for the level of debt, equation 1, and divide both sides of the equation by nominal GDP. For the first term on the right-hand side of the equation, we express GDP as: past GDP, multiplied by an interest and growth factor.

Next, use lowercase letters for variables expressed as ratios to GDP. So  $d(t)$ ,  $d(t-1)$ , and  $pb$ . Also, notice that we can simplify the expression involving interest and inflation into an expression involving only the real interest rate, based on the formula relating nominal and real interest rates given earlier.

With this new notation, we find the following expression:  $d(t)$  is equal to the ratio of  $(1 + r(t))$  over  $(1 + g(t))$  times  $d(t-1)$ , minus  $pb(t)$ .

We will use this expression many times.

Sometimes, we will find it easier to denote the coefficient in front of the past debt-to-GDP ratio as "phi." Phi is defined differently in the closed and open economy cases.

In the closed economy case, which we are considering now, it is equal to: the ratio of  $(1 + r(t))$  over  $(1 + g(t))$ .

Using the equation, we can easily infer the direction at which various parameters affect debt.

Can you see how each variable affects the debt-to-GDP ratio?

Let's examine them one by one.

A high primary balance leads to lower debt-to-GDP ratio.

A high initial debt leads to a higher debt-to-GDP ratio.

A higher growth rate leads to a lower debt-to-GDP ratio.

And finally, last but not the least, a high real interest rate leads to a higher debt-to-GDP ratio.

These results are all as one would intuitively expect. A higher primary balance or higher interest payments add to the deficit, and therefore to debt. What about growth? Why intuitively does higher growth lead to a lower debt-to-GDP ratio? It is because of its effect on GDP. High



GDP means a low debt-to-GDP ratio. Later on, we will consider another way in which growth affects debt-to-GDP.

## Video-Stabilizing Debt in a Closed Economy, Part 1 of 4

*Irina Yakadina:* OK. We saw the debt dynamics. Let's now go to the next step and see how we can stabilize debt.

We will start with the debt dynamics equation, derive the debt-stabilizing primary balance, talk about debt stability conditions, and flag the dangers of debt momentum.

## Video-Stabilizing Debt in a Closed Economy, Part 2 of 4

*Irina Yakadina:* To get further insight, deduct past debt from both sides:

$d(t) - d(t-1)$  is equal to our usual fellow,  $\phi$ , times  $d(t-1)$ , minus the primary balance at time  $t$ .

The change in debt depends on two terms. The first term is called the "automatic debt dynamics." The second term is the familiar "primary balance."

Why is this first term called the automatic debt dynamics? It tells us what will happen automatically to debt as a result of interest rates and growth when the primary balance is 0.

Take a good look at the first term. Automatic debt dynamics can be favorable or unfavorable. When are they favorable? When  $r$  is less than  $g$ . In that case, the automatic debt dynamics, or the interaction of the  $(r - g)$  term and the initial debt, will have a negative sign, provided initial debt stock is positive, which is usually the case.

In the case of  $r$  bigger than  $g$ , the automatic debt dynamics are unfavorable. If initial debt is positive, the automatic debt dynamics have a positive sign.

Note that favorable automatic debt dynamics does not mean debt will fall. This depends on primary balance. With a high deficit, debt can increase even when automatic debt dynamics are favorable.

To take it one step further, favorable dynamics implies stability. If debt is perturbed from its equilibrium, it will return to that equilibrium. Unstable debt dynamics implies instability. Debt moves away from its equilibrium after perturbation. We shall see that on the diagrams.

## Video-Stabilizing Debt in a Closed Economy, Part 3 of 4

*Irina Yakadina:* Let's talk about stability of debt. We can distinguish stable from unstable debt dynamics.

Assume that  $d(t-1)$  is bigger than  $d^*$ , where  $d^*$  is the level of debt which can be sustained by the primary balance at time  $t$ ,  $pb(t)$ .

$r$  minus  $g$  implies that debt converges to  $d^*$ .

$r$  bigger than  $g$  implies that debt explodes.

Stable debt dynamics can be easily illustrated on the so-called cobweb diagram.

If the slope of the debt dynamics equation, which is equal to  $\phi$ , is smaller than 1, the original debt ratio,  $d(0)$ , will, with time, converge to its unique steady state level,  $d^*$ .

This is exactly the point where the debt dynamic equation crosses the 45-degree line, along which debt is stable. Why? Well, because simply along this line,  $d(t)$  is always equal to  $d(t-1)$ . If  $d(0)$  is higher

than  $d^*$ , the debt ratio will fall over time. That is,  $d(t)$  will be below  $d(t-1)$ . This happens because the economy is growing faster than the cost of borrowing does.

However, if  $\phi$  is bigger than one, debt will diverge from its steady state level,  $d^*$ . This is because the economy is not growing fast enough to cover the real cost of extra borrowing.

## Video-Stabilizing Debt in a Closed Economy, Part 4 of 4

*Irina Yakadina:* OK, so we now have this formula for the debt-stabilizing primary balance,  $pb^*$ , which is equal to the automatic debt dynamics, times the past stock of debt, at which we want to stabilize.

The automatic debt dynamics are equal to the difference between  $r$  and  $g$ , scaled by the growth rate of economic growth,  $(1 + g(t))$ . And the primary balance which is needed to stabilize debt would be a surplus if the initial stock of debt is high. And it is proportional to the gap between the real interest rate and the real growth rate, so the speed at which our resources increase in the economy and the cost of additional debt.

The primary balance which is needed to keep the debt-to-GDP constant, will also rise directly with the size of the initial debt-to-GDP, particularly if the cost of debt,  $r$ , is bigger than the growth rate,  $g$ . We can also interpret the equation as telling us the level of debt which can be sustained for a given primary balance.

As we just saw, the primary surplus needed to keep the debt-to-GDP ratio constant will rise directly with the size of the initial debt-to-GDP. It means that the higher is the initial debt stock, the more difficult it is to stabilize debt-to-GDP ratio. It is difficult politically. It is difficult administratively to implement all those measures that we call "fiscal

adjustment," which are necessary to reach the required high primary surplus. If the necessary primary surplus is not reached, debt will continue rising.

High debt brings with it the likelihood that it will be rising ever further. We can say that debt has momentum. More debt leads to greater vulnerability. Hence, there is this reference to the "danger of debt momentum."

Let us close with a numerical illustration. Assume that initial debt is 100% of GDP, and that the interest rate is 2% of GDP, so that the real growth is 5%. Well, that's good news, because we have the situation where economic growth is faster and bigger than the cost of borrowing. And that the primary balance is minus 2% of GDP, or 2% deficit. Do you think debt-to-GDP will be a rising or falling? On the one hand we have the automatic debt dynamics that are favorable. On the other hand, there is a primary deficit. The advantage of the formula is that it tells us exactly what will happen, as you can see in the graph.

To recap, the most important things to remember from Unit 5 are: how to derive the law of motion for debt-to-GDP ratio, the meaning of automatic debt dynamics, and how to calculate the debt-stabilizing primary balance.

## Unit 6: Public Debt Sustainability in a Closed Economy

### Video-Unit 6 Objectives and Outline

*Irina Yakadina:* Now we are ready for an upgrade. We now turn to public debt sustainability in an open economy.

In Unit 6, we study debt sustainability in the open economy and start with reviewing methods and concepts which we introduced in the closed economy case, but allowing for foreign currency debt. This will include the law of motion for debt, the debt-stabilizing primary balance, and comparative statics.

## Video-Law of Motion for Public Debt in an Open Economy, Part 1 of 4

*Irina Yakadina:* So let's start with the law of motion.

In studying the law of motion, we will start from the flow budget constraint with external financing. Then we will proceed to the law of motion for the debt-to-GDP ratio, and the automatic debt dynamics, much as you've seen in the case of the closed economy.

## Video-Law of Motion for Public Debt in an Open Economy, Part 2 of 4

*Irina Yakadina:* We now turn to public debt sustainability in the open economy, allowing for foreign currency debt. In studying the law of motion, we will start from the flow budget constraint, much like we did, but we'll add external financing. And then we'll proceed with the law of motion for the debt-to-GDP ratio and the automatic debt dynamics.

We will need some additional notation. Here the superscript  $D$  denotes domestic currency-related variables, and the superscript  $F$  stands for foreign currency-related variables.

The nominal exchange rate,  $e(t)$  is defined as domestic currency per dollar. So an increase in  $e(t)$  over  $e(t-1)$ , which is by the way equal to  $\epsilon(t)$ , means depreciation of domestic currency.

If the government can borrow abroad in foreign currency or even domestically in foreign currency, debt dynamics will be more complex, because we need to take into account the exchange rate and its impact.

The expression for debt in local currency equals domestic currency debt, plus value of foreign currency denominated debt, evaluated at the end-of-period exchange rate.

Why do we use the end-of-period exchange rate? This is very simple. We are talking about stocks, and stocks are pictures taken on the 31st of December at one minute to midnight, if we are talking about annual variables.

The flow budget constraint with foreign currency debt is similar to the one we studied for the closed economy, but more general. This left-hand side of the equation shows the value of  $D(t)$ , expressed in local currency. The right-hand side equals previous period debt, plus interest, minus the primary balance, plus other flows. The interest on domestic debt is captured as  $i$  times  $D(t-1)$ , and, as before, grouped with the past debt.

The foreign component is similar to the domestic component except that it is evaluated at the new exchange rate,  $e(t)$ . Interest on foreign debt is paid at the new exchange rate, since it is paid at time  $t$ . For that reason,  $i(t)^f$  times  $D(t-1)^f$  is multiplied by  $e(t)$ . Past debt, naturally, is also valued at the current exchange rate.

We can rewrite the expression as an equation that introduces the share of domestic and foreign debt and the rate of depreciation. That is, the percentage change in the exchange rate from the time  $(t - 1)$  to  $t$ . In the equation, we have replaced domestic debt by its share, which is  $(1 - \alpha)$ , times total debt and have replaced foreign debt by its share, which is  $\alpha$  times the total debt. In making the

substitutions, we need to use the definition of alpha, noting that is related to the exchange rate.

Alpha(t-1) is related to e(t-1). When making the substitution, the ratio of current e(t) to the past exchange rate, e(t-1), will appear and can be replaced by (1 + epsilon).

Actually, do me a favor. You can convince yourself by plugging in the expression for alpha(t-1) into the equation above and find the equation.

To sum up, equation 1 that we just obtained is the basis for what follows.

### Video-Law of Motion for Public Debt in an Open Economy, Part 3 of 4

*Irina Yakadina:* Using a technique which is similar to that used for the case of a closed economy, we divide our equation by GDP. And for the first term on the right-hand side, express GDP as the past GDP multiplied by terms for real GDP growth rate and inflation.

You've seen that before. After introducing the concept of the weighted average of the domestic and foreign interest rates,  $i^w$ , where weights are the previous years' shares in total debt, and doing a bit of simple math, we find equation 2.

Note that the structure of equation 2 is the same as we found for the closed economy. The debt-to-GDP ratio equals the past debt-to-GDP ratio, multiplied by the coefficient  $\phi^*$ , minus the primary balance, combined with other flows, all expressed as share of GDP. The star on  $\phi$  differentiates this general case from the closed economy case.

Equation 2 is the key equation for debt dynamics in the open economy, that we will use for the subsequent analysis.

As an aside, I would like to show you a different way of expressing the equation, 2', as directly obtained from equation 1, by dividing by GDP and using the definition of the real interest rate on domestic debt.

Returning to our formula for the evolution of debt-to-GDP and subtracting  $d(t-1)$  on both sides, we find, as before, an expression for the change in debt, which consists of two main components. The first is called the automatic debt dynamics. And the second consists of the effect of the primary balance, combined with any other flows.

What do the automatic debt dynamics look like in the case at hand? Namely, when we have an economy with foreign currency-denominated debt, we obtain the automatic debt dynamics by deducting 1 from the expression for  $\phi^*$  we found in equation 2. This expression is quite complicated, compared to the case of closed economy.  $(\phi^* - 1)$  equates to the weighted nominal interest rate, minus a term involving inflation, minus the growth rate, plus a term involving the share of foreign currency debt and the nominal exchange rate depreciation.

Substituting this expression into the expression for the change in debt, equation 3, an expanded equation, we find that the automatic debt dynamics can be broken into first, contribution of the effective real interest rate, second, contribution of real GDP growth, and last but not the least, contribution of the nominal exchange rate.

### Video-Law of Motion for Public Debt in an Open Economy, Part 4 of 4

*Irina Yakadina:* Let's now go back to our formula for the evolution of debt-to-GDP, and subtract  $d(t-1)$  on both sides, much like we did in

Unit 5 to find an expression for change in debt, which consists of two main components. The first one is called-- you guess? Correct. The automatic debt dynamics. And the second one consists of the effect of the primary balance, combined with any other flows.

How would the automatic debt dynamics look in the case at hand, namely of that of an economy with foreign currency-denominated debt? We can obtain the automatic debt dynamics by deducting 1 from the expression for the  $\phi^*$  that we found in equation 2. This expression is quite complicated, compared to the case with the closed economy.  $(\phi^* - 1)$  is equal to the weighted nominal interest rate,  $i(t)^w$ , minus a term involving inflation, minus the growth rate, plus a term involving the share of foreign currency debt and the nominal exchange rate depreciation. Substituting this expression into the expression for the change in debt and expanding the equation here's what we find.

The automatic debt dynamics can be broken into, in the first place, the contribution of the effective real interest rate, the contribution of real GDP growth, and the contribution of the nominal exchange rate. This exact breakdown will be used in Part 2, when you study debt sustainability analysis for the market-access countries.

## Video-Stabilizing Debt in an Open Economy, Part 1 of 3

*Irina Yakadina:* Next we turn to the derivation of the debt-stabilizing balance. The aim of this unit is to derive the debt-stabilizing primary balance and study how it is related to key macroeconomic variables, such as initial level of debt, growth, interest rate, exchange rate, and other flows.

## Video-Stabilizing Debt in an Open Economy, Part 2 of 3

*Irina Yakadina:* We next turn to the derivation of the debt-stabilizing balance. The aim for this derivation is to show a primary balance that stabilizes debt and study how it is related to key macroeconomic variables: initial level of debt, growth, interest rate, exchange rate, and other flows. We start once more from the general expression for the change in debt, or equation 3, which is replicated here.

How can we obtain the debt-stabilizing balance? Well, as before, we simply set debt equal to the previous period's debt and solve for the primary balance. When we substitute for  $\phi^*$ , we find that  $pb^*$ , the debt-stabilizing primary balance, is equal to the previous period's debt,  $d(t-1)$ , pre-multiplied by this big and scary expression, which is nothing else but the automatic debt dynamics. And it composes of the weighted average of the nominal interest rate, the inflation term, the growth term, and the term which is related to the foreign interest rate, times the rate of depreciation of the currency, and the share of the foreign currency-denominated debt.

The debt-stabilizing balance equals the automatic debt dynamics, times  $d(t-1)$ , plus other flows.

## Video-Stabilizing Debt in an Open Economy, Part 3 of 3

*Irina Yakadina:* Based on this equation, we can do comparative statics. The required primary balance is higher when the real interest rate growth differential is large. That echoes what we have already seen in the case of a closed economy. Other flows contribute to an increase in debt. For example, financial sector support measures or nationalization of private pensions. There is also the real exchange

rate depreciation,  $E$ , which in countries with large foreign exchange-denominated debt, where  $\alpha$  is large, plays a very important role. Note, that the last two are examples of the so-called "stock-flow adjustments" because they help reconcile the changes in debt with the deficit.

Let us now use the expression for the debt-stabilizing primary balance to give a numerical example. Consider four countries,  $X$ ,  $Y$ ,  $Z$ , and  $W$ .  $W$  and  $X$ -- these two-- only have local currency debt, which is, in both cases, 100% of GDP. The difference between  $W$  and  $X$  is in the patterns of  $(r \text{ minus } g)$ . So they're actually having opposite patterns of  $(r \text{ minus } g)$ .

At the bottom of the table, we find the debt-stabilizing primary balance, which corresponds to each country. Primary balance, which stabilizes debt is positive and requires about one percentage point surplus for the case of the country  $W$ , while it can be up to 1% deficit for country  $X$ . Why? Well, this is simply reflecting the fact that they have opposite automatic debt dynamics.

Country  $Y$  has a small amount of debt, so it's just 20% of GDP. It's real interest rate and growth are as for country  $W$ . So in this sense,  $Y$  relates to the pattern of  $W$ . But with debt of about 1/5 of the country  $W$ , this primary balance, which is required to stabilize debt, also falls to about 1/5 of what's needed in country  $W$ .

Finally, country  $Z$  adds foreign currency debt and real exchange rate depreciation. Domestic currency debt and foreign currency debt are each 50% of GDP so the real interest rate on foreign debt is 5%, and the rate of real depreciation is also 5%. The real interest rate on domestic debt and growth are the same as in country  $W$ , so we're back to the familiar scheme. The only difference with  $W$ , therefore, is that there is this real depreciation, which is affecting the value of foreign currency debt-to-GDP ratio. As a consequence, you can see how striking the increase in the debt-stabilizing primary balance is.

This is now almost 3 and 1/2 percentage points of GDP required in surplus.

How exactly did we obtain these numbers? We actually applied the formula for the primary balance while being careful to divide interest rates and growth rates by 100, so that we get everything in percentage points.

Using similar methods as for the closed economy case, we derived the debt law of motion and the debt-stabilizing primary balance, and studied the impact of the key macroeconomic variables on this balance.

## Unit 7: Chipping Away at Public Debt

### Video-Unit 7 Objectives and Outline

*Irina Yakadina*: OK, It is time to turn a little bit to policies and talk about chipping away at public debt.

In this unit, we will explore various aspects of fiscal adjustment. We will explore different adjustment paths and their implications, and we will examine how fiscal adjustment may affect GDP and the risk premium on government debt. This unit also includes a video on the history of past fiscal adjustments.

### Video-Adjustment Paths and their Implications, Part 1 of 3

*Irina Yakadina:* So let's start with different paths of fiscal adjustment. We actually have two possibilities. One path involves an upfront increase in the primary balance, and the other involves adjustment, which is delayed.

It's easy to see it on the graph that front-loaded fiscal adjustment quickly raises the primary balance to the targeted level. The back-loaded adjustment, to the contrary, phases in this adjustment, so there is only gradual passage from a deficit range to the surplus. Front-loaded fiscal adjustment quickly raises the primary balance to the targeted level, while the back-loaded adjustment phases in the measures.

What are the circumstances that affect the timing of fiscal adjustment? Front loading may be necessary when facing severe financing constraints-- or if you are already in crisis-- or to build credibility, or seize the opportunity of political support. Back-loading may be preferable if the objective is to support economic activity and take time but ensure that measures are quality measures. It takes time to design, agree, and implement high quality measures. An issue with back-loading is that the adjustment amounts to a promise and may not be fully credible.

Credibility is crucially important in the context of high debt because of its effect on the risk premium and therefore debt dynamics. To enhance credibility, institutional mechanisms, such as balanced budgets rules and other fiscal rules and procedural rules, may be very, very useful.

Fiscal adjustment may have a negative impact on growth, which would undermine debt sustainability. Fiscal adjustments which are undertaken in the upswing of the business cycle or when the global environment is supportive or when the monetary policy can be accommodating or helpful to implement the adjustments, are definitely most favorable to growth.

We consider some of the issues in the next lecture, and we'll come back to that in Unit 8.

In what follows, we will provide a formula for the fiscal adjustment which is necessary to reduce debt over a given number of periods. This formula allows us to distinguish front-loaded adjustment from back-loaded adjustment.

## Video-Adjustment Paths and their Implications, Part 2 of 3

*Irina Yakadina:* Sometimes stabilizing the debt ratio is not enough. We will need to reduce it.

The next few slides show how to derive a more pragmatic estimate of the primary surplus, the one which is necessary to reduce debt to only  $\gamma^*$  % of its original stock over a finite number of periods, which is equal to  $k$ . For that, we need simplifying assumptions that the real interest rates, real GDP growth rate, and the necessary primary surplus are constant over time; that is, we are talking about, what they will be, on average.

Using the solvency condition for debt-to-GDP ratio, and substituting in the target for  $d(t+k)$ , we obtain the following formula, a formula for the sum of geometric series, and apply it to the previous slide.

As a result, we obtain  $pb^*$ , the primary balance as percent of GDP, necessary to reduce the debt ratio to the target over  $k$  periods. As before, the primary balance is proportional to the initial debt ratio.

Let us see how the level of ambition in debt reduction, as measured by these two parameters  $\gamma^*$  and  $k$ , affect the  $pb^*$ . Lower  $\gamma^*$ , which is the higher the chunk of debt we aim at chipping

away, the larger primary balance is required to achieve the target. Similarly, trying to reduce debt faster or over a small number of  $k$  periods, requires a more significant fiscal adjustment or a better primary surplus.

## Video-Adjustment Paths and their Implications, Part 3 of 3

*Irina Yakadina:* Let's recap.

The trade-off between front-loading or back-loading fiscal adjustment is very simple. You need front-loading to ease financing constraints and to gain credibility. And if you can afford, you would prefer back-loading in order to support growth and work out quality measures.

## Video-Fiscal Adjustment and the Business Cycle

*Irina Yakadina:* Let's now turn to some links between fiscal adjustment and the business cycle.

We will consider how the budget balance affects growth and GDP and how the budget balance affects the risk premium. So far we have assumed that the fiscal balance does not affect growth rates or interest rates in the debt dynamics equation. However, fiscal adjustment could lead to lower growth or lower interest rates. This chart here shows the many directions between fiscal adjustment and the economy and how they ultimately affect debt-to-GDP.

Both of these issues are widely recognized as important, though no consensus exists in the literature on the magnitude of the effect.

Let's see it on a chart. On the left-hand side we can see the effect of the fiscal consolidation and how it affects debt-to-GDP. On the right-

hand side we see the impact of the growth and inflation on the ratio of debt-to-GDP.

Our main interest in this unit is the impact of fiscal consolidation altogether. However, we will discuss the impact on growth, on debt-to-GDP, because it is necessary background for what's coming in Part 2 of the course.

Let's focus first on the effect of fiscal consolidation on the left-hand side. The outer upper arrow-- this one-- shows how consolidation has a direct impact on debt through reducing the deficit. This is the so-called "direct effect." The lower arrow shows how the consolidation affects output, with a magnitude which depends on the size of the so-called multiplier. A large positive multiplier means that fiscal consolidation will reduce the output substantially. So now let's turn to the inner arrows that shows how consolidation helps improve debt dynamics by reducing the risk premium.

We are going from consolidation to lower interest rates because the risk premium is now lower. And then the inner lower arrow shows how low interest rates might support GDP. So we have this positive impact on growth, mainly through investment.

On the right-hand side we can see the effect of growth on the ratio. Growth has direct effect on  $Y$ , obviously, better growth improving  $Y$ . And in addition, growth affects the primary balance, depending on the size of automatic stabilizers. Revenues tend to move proportionally with output, falling in recession. Various spending may increase in recession, especially in countries with automatic stabilizers, such as unemployment benefits.

As a result, the primary balance tends to deteriorate in a recession. In Part 2, we will take another look at the effect of the business cycle on the fiscal balance.



To review - there are 3 main channels through which the primary balance affects debt-to-GDP: first of all, directly, via the primary balance in the debt dynamics equation; second of all, via demand, which in part comes through lower government spending and higher taxes that have a negative impact on demand; and also via interest rate; first through credibility (the risk premium) and then through the demand. If interest rates go up, we have this crowding out phenomenon. These elements will be further discussed in Part 2, where links between the primary balance, demand, and interest rate are built into several scenarios.

What are the circumstances favoring each of the channels? Fiscal consolidation will have a large impact on demand when multipliers are large. This is the case for a closed economy, when unemployment is high, so that the economy behaves in a very Keynesian way, and when foreign partners are also consolidating, so that external demand does not substitute for domestic demand. Higher multiplier or coordinated consolidations will slow growth by a lot. In this case, one sometimes refers to a "doom loop," from fiscal consolidation to lower growth and to a higher deficit. Instead of the intended lower deficit, one gets a higher deficit.

On the other hand, fiscal consolidation may lead to reduction in the risk premium and lower interest rates, helping debt sustainability. This tends to be the case for high debt countries, since such countries start out with credibility problems.

To recap, the main take away from this lecture is that fiscal adjustment can be front or back-loaded. Front-loading tends to be more credible; hence, will likely benefit from a reduced risk premium, compared to back-loading. Front-loading, depending on the context, may be associated with a slower growth. The speed at which debt can be reduced depends on how fiscal adjustment affects GDP and the interest rate.

## Video-Chipping Away at Public Debt (IMF public video)

## Unit 8: The Role of Macroeconomic Policies

### Video-Unit 8 Objectives and Outlines

*Irina Yakadina:* We have now reached the final point in our analysis. So, we are now ready to elevate our discussion to the role of macroeconomic policies. And also, we are ready to complete the derivation of the key analytical results for external sustainability.

In this unit, we will study the role of macroeconomic policies. And this unit has two objectives: to teach you how monetary policy interacts with fiscal policy, and what trade-offs you may encounter.

In this lecture, first we will discuss the effect of monetary policy stance on debt. Then, we'll turn to the policy trade-offs.

### Video-Monetary Policy Stance and Debt

*Irina Yakadina:* In general, there are four variables that are affected by monetary policy, and they are: the interest rate, inflation, exchange rate, and last, but not the least, growth. So, how these variables will be affected?

We have a textbook model that suggests that expansionary monetary policy will first of all and utmost lead to lower interest rates, starting with lower nominal interest rates, as well as lower real interest rates, which are the inflation-adjusted ones. If nominal interest rates go

down and inflation is unchanged or even higher, then obviously real interest rates will fall. We have this impact of the expansionary monetary policy, which lead to our interest rates going down.

What happens then? Lower interest rates stimulate demand and this may end up either being inflationary, because the economic capacity is insufficient; or alternatively, it could lead to higher growth because the economy is in excess capacity.

On the flexible exchange rate system, there is another channel through which interest rates affect inflation and growth. It's the exchange rate. If (lower) interest rates lead to a more depreciated exchange rate. Why? Because investors will basically be shifting funds out of domestic assets, because they now bear lower returns, to foreign assets. The depreciation of the exchange rate, in turn, may stimulate growth through higher net exports. So we can have higher growth, or we have higher inflation, much like before.

Why would inflation be higher? Inflation may rise because of higher demand from net exports, or it can be from the so-called "imported inflation."

How does this all affect debt dynamics? Let us first consider a simple case without foreign currency-denominated debt. As we have seen, higher growth will lead to improved primary balance. First of all, through the operation of automatic stabilizers. As you recall, we will have more cyclical revenues and we'll have less of the primary expenditures, which are cyclical. So here on this slide we have replicated the formula of the key equation, and we now add arrows for the direction of the change in interest rates, the primary balance, growth, and see how it all adds up.

Well, the great result is in the case of no foreign currency debt, the result is unambiguous. The debt dynamics are sure to go down, so future debt stock, as a percent of GDP, will decline if our real interest

rates go down, which stimulates growth. And therefore stimulated growth improves the primary balance. All the three variables act in the same direction.

A small recap, in the case without foreign currency-denominated debt, expansionary monetary policy will lead to improved debt dynamics.

Now let's consider the case with foreign currency-denominated debt. The impact on debt dynamics will no longer be so clear cut. Why? We now need to add the effect of the exchange rate depreciation. And basically this effect of the exchange rate depreciation-- we know for a fact that depreciation worsens debt dynamics. So the only way it will not affect debt dynamics adversely would be if this depreciation will be fully offset by what we call a "pass-through." The pass-through is this transmission from the exchange rate to inflation. So if there is a one-for-one relationship between the depreciation and inflation, then these two terms will cancel out, and then we will have the part of the domestic currency-denominated debt contributing positively to debt dynamics, meaning that debt dynamics here would be contributing to decreased debt-to-GDP ratio.

But unless we have this pass-through offsetting effect, we see that the foreign component may be contributing negatively. So the total is ambiguous. In this particular case, the increase in inflation would offset the impact of high depreciation, and we can see that the two terms could cancel out, depending on how much the inflation changes.

Let's illustrate this idea with some examples. Let us first place the equation from the previous slide in the approximate form, which is-- I'm replacing the ratios with differences, and the products with sums of the net rates, rather than gross rates. And this is an approximation, which is valid if the variables are small and relatively close to 0.

Going from left to right, the first column illustrates a base case with no monetary easing. And the second one is a simple case of monetary easing when there is no change in the exchange rate, while the third and the fourth have depreciation of the currency, each by 10%, and differs in the level of their pass-through, which is assumed. In the no-easing case, with a primary deficit of 2%, the automatic debt dynamics, adverse as we can see, because in the domestic economy, the real interest rate is 6% while the real growth is only 3%.

In total, applying the formula above, we can see that debt increases to 105% of GDP, from 100%. In the next column, easing of monetary policy actually lowers the real interest rate. So we're getting from 6 to 4% real interest rate. And that leads to high investment, and therefore we should expect that growth picks up somewhat. So it's passing from 3% to 4%. So we have the automatic stabilizers impact on the primary balance, which improves the primary balance to the deficit of 1.5% rather than 2, and as a consequence we have the resulting debt dynamics leading to an increase to debt-to-GDP ratio to only 103%.

In the third column, we assume that there is this 10% depreciation, and let's start by assuming that it only has a minor impact on inflation, it leads to 2 percentage point additional inflation from 0, just because of the exchange rate depreciation. Because of the real depreciation, growth will increase further, so we have a 5% growth up from 4, but we still have worse debt dynamics, which mainly follow from the increase in this ratio of 1, the difference between  $\epsilon$  and  $\pi$ . We end up at 106.1% of GDP, debt-to-GDP ratio.

If you contrast this result to the fourth column, where the pass-through to inflation is 100%, there is no impact on the real exchange rate. Growth is exactly as in column two then, because we don't have any additional boost to growth, and so is the primary balance.

As a bottom line, we are back to the case of 103% of GDP, exactly in the case of no change in exchange rate. Why? Because we had the offsetting impact from inflation.

Before moving to the next topic, we need to make one more point. So far, we have not distinguished between short-term interest rate and long-term interest rate. While the expansionary monetary policy tends to reduce the short-term interest rate, because oftentimes they are the instrument of the monetary policy, it may very well lead to higher long-term interest rates. Why? Because the long-term interest rates are built around the expectations of future inflation.

How does that work? The Fisher equation, which is now familiar to you, is relating the nominal interest rates to a required real return, and the expected inflation. The underlying assumption here is that people have in mind a required real return. So when expectations of inflation go up, it translates into higher required nominal interest rates.

The bottom line is: if expansionary monetary policy leads to higher expected inflation, then long-term nominal interest rates may go up.

A final note: While we have mentioned that people have a required real return, that does not mean that they will be automatically obtaining this exact return ex-post. If inflation is higher than anticipated, then the ex-post return will be less than the required real return.

## Video-Policy Tradeoffs

*Irina Yakadina:* Our next topic is policy tradeoffs. We will discuss three tradeoffs: the tradeoff between sustainable debt and inflation; the tradeoff between sustainable debt and competitiveness; and the tradeoff between sustainable debt and fairness.

During our discussion we will introduce the concepts of fiscal dominance, fear of floating, and fairness, or income distribution. Let us first turn to sustainable debt versus inflation and define what we call fiscal dominance.

Fiscal dominance is the inability to conduct contractionary monetary policy because it would jeopardize government debt dynamics. Thus, the government has a choice between fighting inflation, which would obviously result in contractionary monetary policy, and this would also result in higher real interest rates. Therefore, as we know, that may lead to lower growth. And ultimately, from our debt dynamics, both of which would definitely imply a higher debt-to-GDP ratio.

Next consider sustainable debt versus competitiveness. Calvo and Rogoff coined the term "fear of floating." What does that mean? It's the reluctance to allow a floating exchange rate to adjust freely. Why? Calvo and Rogoff has suggested various reasons, one of them being that large devaluations tend to cause recessions because of large foreign currency debt. Depreciation would lead to debt-servicing difficulties and even default. That means that the government may be really reluctant to allow depreciation, even when this is necessary to raise competitiveness. So loose monetary policy is helpful for competitiveness and growth, but will raise the value of foreign currency debt expressed in local currency. And that may cause bankruptcies.

This fear of floating is pervasive among emerging economies, but not necessarily advanced economies. Why? Emerging markets often have difficulties in issuing external debt in their own currency. Eichengreen and Hausmann refer to this condition as "original sin," basically invoking the idea that this structural feature of debt is the source of endless suffering.

Our final tradeoff is the one between sustainable debt and fairness. It's a thin line, right? In a way inflation, if it is unexpected, is some form of default. It basically erodes away the value of debt. Put differently, real interest rates that are negative, ex-post, once people had expected them to be positive, are some sort of tax or implicit default. In particular, in addition, if there is a, what we call, "financial repression," which we define as capping nominal interest rates, then real interest rates will actually be negative ex-ante. That means that inflation creates a redistribution of wealth from creditors to debtors. This may be helpful to reduce government debt, but obviously at the cost of fairness.

## Unit 9: External Debt Sustainability

### Video-Unit 9 Objectives and Outline

*Irina Yakadina:* OK, now we are in the final units, 9 and 10, of this part of the Principles of Debt Sustainability, and we will explore how to make external debt sustainable.

The objectives of Unit 9 are threefold. We want you to understand similarities between external and fiscal sustainability. We want you to learn what are external debt creating flows, and to comprehend the solvency condition, but this time around, for external debt.

### Video-External Debt-Creating Flows

*Irina Yakadina:* In this lecture we'll start by comparing and contrasting the external debt sustainability analysis with the public debt sustainability, explain the different types of debt-creating flows, and

last but not least, we want to introduce the concept of the "adjusted balance."

External and fiscal sustainability require very similar methodologies in their analysis, so we'll just simply replace the focus from being on public debt to being on external debt of a country, which means that this debt would include the private sector debt. Why's that? Well, recall that in a country you may have firms or corporations that are issuing debt to non-residents, so this would be the private external debt.

The current account balance, or the balance of payment, will be now our workhorse, and that will take place of the overall budget balance in our earlier analysis.

Now what are the differences between the public debt analysis? There are several key differences, and the first and the utmost is the government actually has very little, if any, control over the current account balance. In addition to that, we may have some trends which are underlying both exports and current account balance. For example, if we take an example of a healthy cycle, then both exports and current account balance may be improving over time, which would be automatically allowing for some repayments of debt. And last but not least, obviously the exchange rate plays a much larger role in the analysis of external sustainability.

One of our main goals in the lectures of this unit is to derive a law of motion for external debt which would link debt to its past stock and the current account balance. We will derive the following formula: Current debt is equal to its past stock, multiplied by (1 plus the interest rate), from which we subtract the adjusted balance. Just hold your breath, we'll define the adjusted balance very soon but you can already use your intuition. What would it be? It would be that, you need to follow the logic of the fiscal sustainability analysis, and add back the interest payments-- OK, once. But just hold on for a second--

there will be, maybe, some other adjustments that we should include, once defining this balance. Let's see.

OK, let's first introduce some notation. All variables are expressed in US dollars, for easy correspondence to the balance of payment, because the balance of payment records all variables in US dollars by convention. So we define variables as current account balance, we define  $AB(t)$  as adjusted balance, interest payments.

We distinguish between the current account balance, the capital account, and the financial account balance.

We also have external assets that we define as  $A(t)$ , as well as external liabilities which are  $L(t)$ . The liabilities basically consist of two types of external liabilities-- external debt liabilities and external equity liabilities.

And last but not the least, we now have our nominal GDP in US dollars, which just implies that we have to divide by the exchange rate.

It's now time to define the adjusted balance. It will be the current account, to which we add back interest payments. And we need to introduce or add the change in external equity liabilities, and subtract the change in external assets.

Let's consider the first term for a moment. We have the operation with interest, which is quite intuitive, because interest is basically part of the income item on the current account, which itself is the sum of net exports, income, and current transfers. So we have to add back interest to improve-- or have our adjusted-- current account balance. OK, fine.

What about the second term? The first term would be very similar to the primary balance in our fiscal DSA. Now let's make the adjustment

for the non-debt financing, so that the current account is financed by items which are other than debt, and so those items will not add to debt. It's equivalent to our "other" flows in the fiscal analysis.

We need this adjustment for the non-debt financing. And this is the financing through equity. So if you increase equity (financing), or you decrease your external assets, you draw on them. So that's why you have the negative term-- negative sign--- in front of the external assets.

To recap, drawing down our assets in the form of financing, and therefore our adjusted balance, equals the non-interest current account balance, plus non-debt financing.

## Video-External Financing Constraint & Debt Law of Motion

*Irina Yakadina:* In this lecture, we derive the Debt Law-of-Motion for External Debt. In order to do this, we need to first discuss the external financing constraint, and then derive the debt law-of-motion.

Let's start with the external financing constraint. We have the equation such as, the current account plus the capital account is equal to the financial account, which is just to say, that we have the balance of payment presented as the accounting identity.

Intuitively, a positive balance on the current account will allow the country to accumulate external assets. Hey, that's great news, right? If you're saving, then it's natural that you are accumulating assets. If you run a deficit on the current account, then you need financing, and this financing will either come in the form of capital account or from the financial account.

Why don't we for now abstract from financing via the capital account. The capital account mostly is composed of capital transfers, and let's for now rule out this possibility. Assuming that the capital account is zero that means the current account should be financed by the financial account.

What about the financial account? The financial account of the BOP actually records the acquisition of assets and the incurrence of liabilities - something which is the result of external borrowing. We call such flows transactions. The incurrence of liabilities could be on both the equity liabilities and debt liabilities. Whichever way, these are still transactions.

The value of those liabilities and assets can change over time for a couple of reasons. First of all, the transactions we just mentioned, or we may have what we call valuation effects.

What are the valuation effects? Well, if your domestic currency or other currencies fluctuate against the US dollar--which is the unit of recording in the balance of payment-- then it's natural that the value of your assets, denominated in dollars, will change over time, and so will the one of liabilities. If this is the case, we call these changes, which are just due to fluctuations of exchange rate, valuation effects.

So let's for now assume out, or assume away, the valuation effects. OK? So we say they're 0, and if so, then what we have here is that the change in value of assets and liabilities, so this difference in the change in the value of assets and liabilities should be exactly equal to the value of these balance of payment transactions. Well, that's just by assumption, right? With these couple of simplifying assumptions, we have obtained a very useful result, which is, that the current account is equal to the change in assets minus the change in liabilities.

Our next step will be to separate out debt liabilities from equity liabilities, given that we are now interested in debt. And in particular, this debt is, per se, a composite thing. It includes securities. It includes loans. It includes currency. It may include bank deposits. And equity is also not a monolith. It can be shares, or it can be foreign direct investment.

We have this expression, which is now relating the current account to the change in debt liabilities and the change in equity liabilities. Next, we can rewrite the previous equation by bringing current period debt over to the left hand side, and the current account balance over to the right hand side, and adding and subtracting the interest payment. A simple trick we've done before.

Now the question is, do you recognize the adjusted balance? Let's see here. With a negative sign, we have here current account, to which we add back the interest, and we adjust it by what we call the non-debt financing, which is this term. So all we need to do now is to make a simplifying assumption as before, which is that the interest payments are now proportional to the past stock of debt, and for that we can always use the effective interest rate. After replacing the expression for the adjusted balance with AB, and using this formula for interest payments, we can now group terms, which involve lagged debt, and we are home because what happens is we are back to our debt law of motion that we were set out to find.

Current debt dynamics, current debt stock, will equal past debt, which is multiplied by (1 plus the interest rate), and we have to subtract the adjusted balance. That should be very familiar by now, right? This is exactly the debt law-of-motion in which we substituted the primary balance and replaced it with adjusted balance.

## Video-Solvency Condition for External Debt

*Irina Yakadina:* Now we are ready to derive solvency condition for external debt. The exposition in what follows exactly mirrors that for solvency of public debt. It can go a bit faster this time around. And we can derive the debt law-of-motion, which is a one-period budget constraint, from which we can obtain the intertemporal budget constraint through repeated substitutions, exactly as we did for public debt. And from there, we can obtain the solvency condition for external debt by imposing your favorite, the transversality condition. Remember the one which is No-Ponzi? No-Ponzi scheme condition? OK, that one. In particular, we require that the present discounted value of external debt at time infinity has to be zero. That's my transversality.

Here we're using the same method as for public debt. And the intertemporal budget condition was just extended to N periods. Does this look familiar? Remember that one? It's the exact same condition that we found for public debt, where external debt is taking the place, or taking the role, of the public debt. And the adjusted balance is taking the role of the primary balance.

As before, we extended this formula to time infinity, and we need to impose the transversality condition, which brings us to the condition for solvency. The present value of all surpluses of the adjusted balance of the current account has to equal the initial debt.

Lets recap Unit 9. External debt sustainability analysis follows a very similar analysis as public DSA. We have discussed external debt-creating flows and the adjusted current account balance. Finally, we derived the debt law-of-motion and the solvency condition for external debt.

## Unit 10: Deriving External Debt Law of Motion

### Video-Unit 10 Objectives and Outline

*Irina Yakadina:* Let us now proceed with deriving debt dynamics for external debt. The key objectives for this unit are to derive the external debt law of motion, do comparative statics, namely to check how the results will change with different parameters, and finally to understand the automatic debt dynamics for the case of external debt.

### Video-Deriving External Debt Law of Motion

*Irina Yakadina:* Now we are ready to derive the External Debt Law-of-Motion. And in this lecture we'll first derive this Law-of-Motion-- which is very easy, we will just use our previous result and divide it by GDP-- and then get to the automatic debt dynamics. We will start with the evolution of external debt at time  $t$ , which we obtained previously. And all we need to do is divide it by GDP, which is now expressed in US dollars,  $(P(t)Y(t))^*$ . So that's easy, we are doing the same trick of collecting terms and decomposing the  $(P(t)Y(t))$ -- the GDP expressed in local currency-- as its previous period value, times growth rate of growth, times growth rate of inflation,  $(1 + \pi(t))$ .

All I need to do now is to take care of what to do with the exchange rate, because my goal is to have all the variables here expressed in period  $(t - 1)$  in terms of past values and their rates of changes, such as the rate of inflation, rate of interest, rate of depreciation. Well that's easy. All I need to do is to use what I know is the gross rate of growth of the exchange rate. What is it? Well, it's just my rate of depreciation,  $(1 + \epsilon(t))$ . Now what I can do is to add this growth rate of depreciation to my otherwise familiar term, and this way I have my ratios to GDP, as expressed in foreign currency, as  $D(t)^f$  is

equal to its past value,  $D(t-1)^f$  pre-multiplied by something which looks very, very familiar,  $(1 + \text{nominal interest rate})$ , divided by  $(1 + \text{plus growth rate})$ ,  $(1 + \text{inflation rate})$ . Then I have to pre-multiply by a new term, which involves or is related to my local currency depreciation,  $(1 + \epsilon(t))$ .

With that in mind, what I need to do now is to convert those expressions involving the nominal rates, such as the inflation rate and nominal interest rate, and derive the Debt Law-of-Motion for the debt-to-GDP ratio in more familiar terms of real interest rate and real exchange rate. How do I do that? Well, by using a couple of definitions here, which is the real interest rate is defined, by Fisher's law, as a ratio of the gross nominal interest rate and gross inflation.

But careful there, the inflation I need to use now is the inflation abroad, or foreign inflation rate. Similarly, what I need to do here is to define real exchange rate depreciation as a product of my nominal exchange rate depreciation and the ratio of the gross inflation abroad, adjusted by the ratio of gross inflation at home, or in my country.

We denote this real exchange rate as  $\epsilon^*$ , and that's real exchange rate, again defined as positive  $\epsilon^*$ , means depreciation. And you can check that the expression that I need to transform, which was the expression from the previous slide, can now very easily be written in a very familiar form of  $(1 + r(t))$ -- in terms of real interest rate-- times  $(1 + \text{real depreciation rate})$ .

With that, we have the expression here that shows our final result for the Debt Law-of-Motion. Debt-to-GDP equals previous year debt-to-GDP multiplied by a coefficient which incorporates the following: It has the real interest rate, the real exchange rate, and the real growth, and it is affected negatively by the adjusted balance. We denote this coefficient as  $\phi(t)$ , as we did in the past.



For what follows, higher  $d(t)^f$  would result from either higher real interest rates or greater real exchange rate depreciation or lower real growth, and all of those will make debt dynamics worse.

A few things are really interesting about this formula and worth noting. Let's first assume that the adjusted balance, this one, is equal to 0 and consider only cases where the starting point of debt is positive, just to make life interesting. In that case, if my  $\phi$  is less than 1, what I have is this beautiful debt dynamics where debt basically converges to 0. Why? Because I don't have any primary deficits-- or adjusted deficits, in this case-- which are fueling debt buildup. Conversely, if my  $\phi(t)$  is bigger than 1, then I have adverse debt dynamics. What does that mean? Well, it basically leads to debt exploding.

To summarize, as in the previous analysis with public debt, the size of  $\phi$ , the coefficient on debt dynamics, is key to determining whether debt explodes or converges. From Debt Law-of-Motion, what we can do now is to derive an expression which will relate debt, or changes in debt, to its past value, the one of the adjusted balance, and the automatic debt dynamics.

How do we do that? We just simply subtract debt from both sides and simplify, exactly like we did in the past. So as a consequence, I have my automatic debt dynamics, which is now a function of  $(r \text{ minus } g)$ -- so my expression of  $(r - g)$ -- which I'm still dividing by  $(1 + g(t))$ , but I have a new term here, which is related to the real exchange rate depreciation, times the gross real rate of interest.

If I am to re-evaluate this whole expression there, then I have my change in debt ratios, which is captured by essentially two terms. The first one has this complicated automatic debt dynamics, pre-multiplying the past stock of debt, and the second one is, as in the past, related to the adjusted balance.

That means that our next step should be familiar to you. You can anticipate it now. We know that we can now solve for the automatic debt dynamics that stabilize debt. What we are after is to find the level of the adjusted balance that will keep debt ratios constant. And to find this answer, we simply need to set  $d(t)$  equal to  $d(t-1)$ , on the previous slide, and introduce some notation.

So we introduce the stars to denote the constant level of debt by  $d^*$ , and the debt-stabilizing adjusted balance by  $ab^*$ , and here we are. We have the debt-stabilizing adjusted surplus,  $ab^*$ , which is equal to our expression of  $(r - g)$ -- so the familiar part from the past-- plus the part which is related to the real depreciation and the interest rate, and we are dividing, as in the past, by  $(1 + g)$ .

Now, a simple question: What if I have  $ab$  in excess of  $ab^*$ ? Well, congratulations then, right? You have a better adjusted balance than is required to stabilize debt. What does it mean? That means that my debt will be falling, so I will have improving debt dynamics. Conversely, if I don't have a sufficient adjusted balance, then it's clear that my debt will rise.

Just the final touch: The debt-stabilizing balance, what is the sign of it? So we talked about the level, we defined-- we found the level as a function of our key macroeconomic rates, such as growth rate, the interest rate, rate of real depreciation, past stock of debt, or the targeted stock of debt. Depending on the constellation of those variables, the debt-stabilizing balance can be positive or negative.

We had found that debt dynamics will be affected by the real interest rate, the growth rate, the current level of indebtedness, the net exports-- because ultimately this is what will be the large driving force behind our current account balance, and, as a consequence, adjusted current account balance. It's also affected by long term level of other flows-- remember the foreign direct investment-- and it also will be affected by the real exchange rate changes.

Let's see what is the direction of those effects? What if my real interest rates increase, or my growth falls, or there is an increase in my current level of indebtedness, or I have net exports which are underperforming, or, worse than that, I have a reduction in my foreign direct investment or other non-debt financing, or if my real exchange rate depreciates. What do you think happens? Obviously in this case I will have adverse debt dynamics. So my debt-to-GDP ratios of external debt will go up.

Now, the final point: We were talking about the impact on the ratios without talking about the impact on the level of debt. Well, there is a good reason for that. This is because the effect of the level of debt is ambiguous. If the coefficient on past debt in the automatic debt dynamics is greater than 0, which is the equivalent of saying that  $\phi$  is bigger than 1, then higher debt leads to higher debt-stabilizing balance, and therefore worse debt dynamics. On the other hand, if the coefficient on past debt is less than 0, which is the same as saying  $\phi$  is smaller than 1, then higher debt leads to a larger debt-stabilizing deficit, or better debt dynamics.

Let us close with an illustration. So let's consider three countries, X, Y, and Z. What we are after is deriving the debt-stabilizing adjusted balance for each of the countries. So country X has 5% real interest rate, 4% real growth, no change in the exchange rate, but a foreign debt of 100% of GDP. Country Y has the same real interest rate and growth rate, but foreign debt which is only 20% of GDP. And country Z is same as country X, but there is something that's happening in country Z which is not happening in countries X and Y, namely something is happening to the exchange rate. So let me quickly quiz you. -2 in the line for  $\epsilon$ , which is my exchange rate depreciation, means that my currency is... appreciating.

What does that mean? My goal is to figure out which of the countries will have the most favorable debt dynamics. We can guess that a

positive adjusted balance is required for debt stability for countries X and Y. Why? Well, because we have this little  $(r - g)$  component here, and we can see that this part of the dynamics are adverse for debt ratios. We can also get that debt dynamics of Y will be definitely better than X. Why? Well, for the simple fact that the debt-to-GDP ratio is much lower in Y than in X. It is the case-- I'll let you do the calculations-- but we can easily show that country X needs an adjusted surplus of just 1.1% of GDP, while country Y needs an adjusted surplus of only 1/5 of that amount, and country Z is best ever, because it can actually afford an adjusted deficit.

As I said, please take a moment to do the calculations, but let me illustrate of how we do them. All you need to remember is to make calculations in percent. So, for example, for country Z, your adjusted balance will be equal to my 5% real interest rate, minus 4% percent of growth, minus-- minus because I have this real appreciation-- times gross rate of real interest, and divided by  $(1 + g)$ , times 1, which is 100% of GDP.

That gives us that 1.06%, or approximately 1.1% percent.

## Video-Summary: Key Takeaways for External Debt

*Irina Yakadina:* Let us summarize the key takeaways for external debt.

The debt law-of-motion is: current debt is equal to past debt multiplied by  $(1 + r)$ , minus the adjusted balance.

The adjusted balance equals the non-interest current account balance, plus non-debt financing.

The debt law-of-motion, expressed as a change in the ratio of debt-to-GDP, relates the change in the debt-to-GDP ratio to the automatic debt dynamics and the adjusted balance.

And finally, debt dynamics are affected by the real interest rate, growth rate of the economy, current level of indebtedness, net exports, long-term level of other flows, and real exchange rate changes.

And we are done!

## Video-Final Remarks

*Irina Yakadina*: Congratulations for making it thus far. I'm really impressed with you. This part of the course is now coming to an end.

Let me try and review the key messages of what we have learned.

This part has reviewed the basic principles of debt sustainability analysis. We covered the key concepts and derived a few equations describing debt dynamics and the effort that will be needed to stabilize or reduce debt.

By now I'm convinced I can call you up at 3:00 AM in the morning, ask about the formula for debt- stabilizing primary balance in the case of open economy and, most importantly, get the correct answer!

We also talked about the dangers of high debt and different types of crisis and why it is important to have the proper macroeconomic policy mix to prevent from a costly fiscal adjustment.

I hope you have found our quest for the meaning of debt sustainability exciting and useful.

Please keep up with your good work and continue with the subsequent parts of the course.

Best of luck with earning your course certificate!