Financial Market Analysis (FMAx)
Module 3

“Bond Price Sensitivity”

This training material is the property of the International Monetary Fund (IMF) and is intended for use in IMF Institute for Capacity Development (ICD) courses.
Any reuse requires the permission of the ICD.

Main Question of This Module

If interest rates change (due to monetary policy, etc.), the YTM that financial investors apply for pricing will change.

The Main Question:

How sensitive is the bond price to change in the YTM?

$$P = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{M}{(1+y)^T}.$$

P = price
C = coupon
M = face value (principal)
y = YTM
T = maturity.

This sensitivity can be an approximation of the price risks that bond holders face.

The Relevance to You

You might be…

- A financial **investor** for yourself or for your country (managing reserves or sovereign wealth fund).
- An **economist** (policymaker, bank supervisor, etc.) interested in the sensitivity of asset values to interest rate changes.
  - Involved in bank stress testing, for example.

The Relationship between Bond Price and YTM

Formal Pricing Formula:

To understand the relationship between bond price and YTM.

For quick intuition, use the following:

- **Analytical Example** – Consider the simplest possible bond.
- **Numerical Example** – Make up an example of a hypothetical bond and experiment using Excel.

Price is Decreasing and Convex in YTM – 1

Analytical Example:

- One-period bond, delivering $1.
- Price: $P = 1/(1+y)$.

Price is Decreasing and Convex in YTM – 2

Typical "**price-yield curve**" (relating bond price and YTM):

- **Decreasing** – As the YTM increases, the price decreases.
  Intuition: The higher the YTM, the more future cash flows are discounted.

- **Convex** – The price is more sensitive to the YTM change when the YTM is low.
  An increase in the YTM results in smaller price changes than a decrease.

Price is Decreasing and Convex in YTM – 3

[In Excel]

Numerical Example:

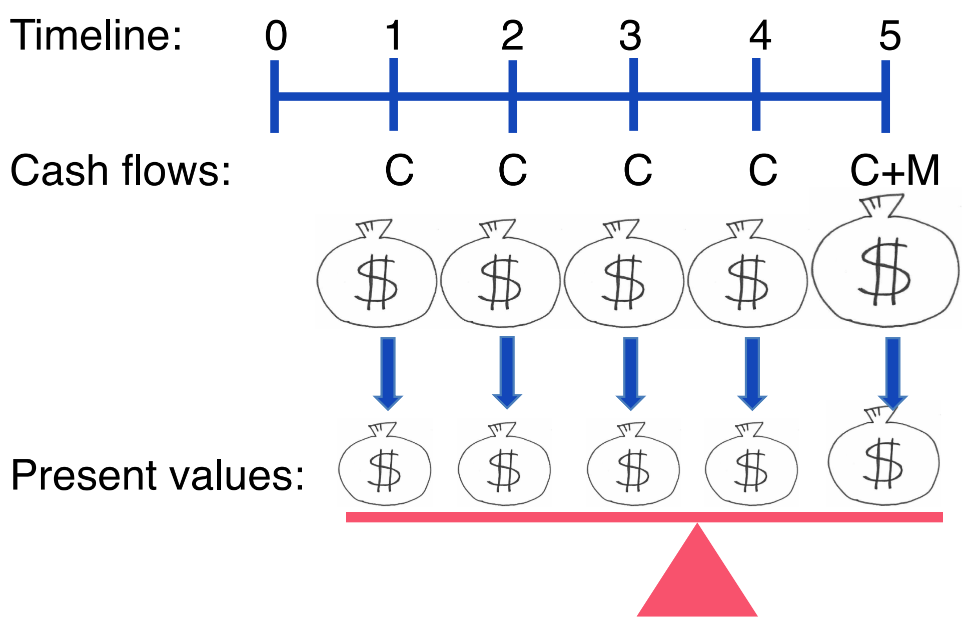
- Consider a 30-year bond with 5% coupon rate and semiannual coupon payment.
- How is the price affected by a change in the YTM?

Price Sensitivity and (Macaulay) Duration

Price sensitivity of bonds is closely related to one key characteristic of bonds: "(Macaulay) Duration."

- Refers to **the average time for which an investor must wait** to receive the cash flows from the bond.
- For **zero-coupon bonds**, "**Duration = Maturity**," as there is only one cash flow, at maturity.
- For **coupon bonds**, "**Duration < Maturity**," as some cash flows exist before maturity.

(Macaulay) Duration – 1



(Macaulay) Duration – 2

Defined: Is the average of the time to the bond's promised cash flows.

Duration (*D*) is

$$D = \sum_{t=1}^{T} t W_t$$

where

$$W_t = \frac{CF_t}{(1+y)^t} / P.$$

P = bond price
CF_t = cash flow at t
y = YTM
n = maturity

(The weight is the cash flow's present value as share of today's value.)

Duration as a Measure of Price Sensitivity

Recall:

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}.$$

Then,

$$\frac{\partial P}{\partial y} = -\sum_{t=1}^{T} t \frac{CF_t}{\left(1+y\right)^{t+1}},$$

But a more informative measure might be the **percentage change in P**:

$$\frac{\partial P / P}{\partial y} = -\sum_{t=1}^{T} t \frac{CF_t}{\left(1+y\right)^{t+1}} / P = -\frac{1}{1+y} \sum_{t=1}^{T} t \frac{CF_t}{\left(1+y\right)^{t}} / P = -\frac{D}{1+y}$$

Hence, ***D* can be understood as a measure of price sensitivity**. Geometrically, *D* is related to the **slope of the price-yield curve** ($\partial P / \partial y$).

Duration of a Portfolio

The duration of a portfolio of bonds is the weighted average of the durations of the bonds in the portfolio.

The weights are the ratios of the values of each bond over the total value of the portfolio.

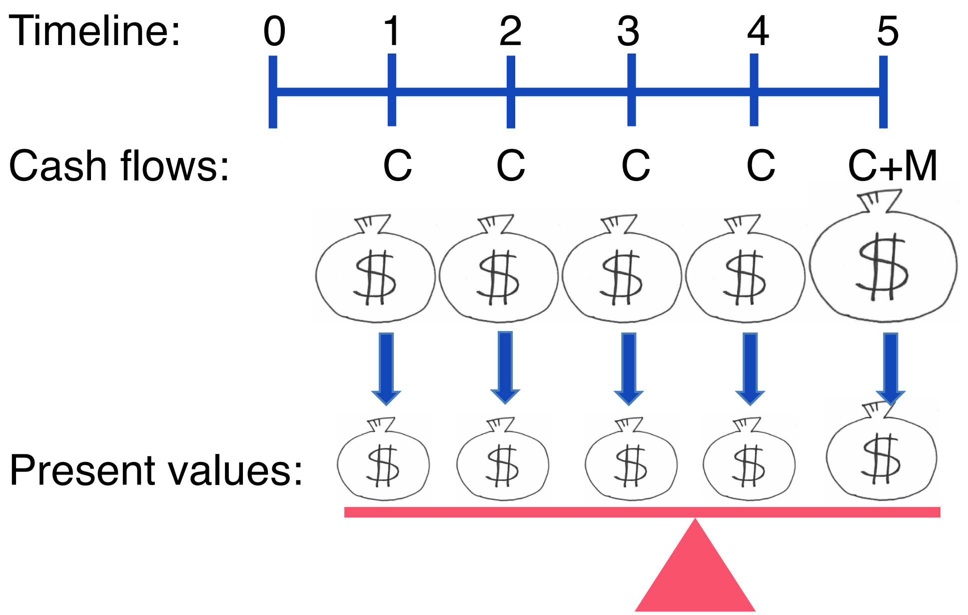
Example: 1/3 of your portfolio value… Duration: 5

2/3 of your portfolio value… Duration: 8

Then, your portfolio has a duration of ___________.

Property 1: A bond's duration is higher when the coupon rate is lower – 1

Intuition: A larger proportion of the cash flows will be at maturity.



Since duration is a measure of bond price sensitivity, Property 1 implies: **Bond price is more sensitive to YTM changes when coupon rate is lower**.

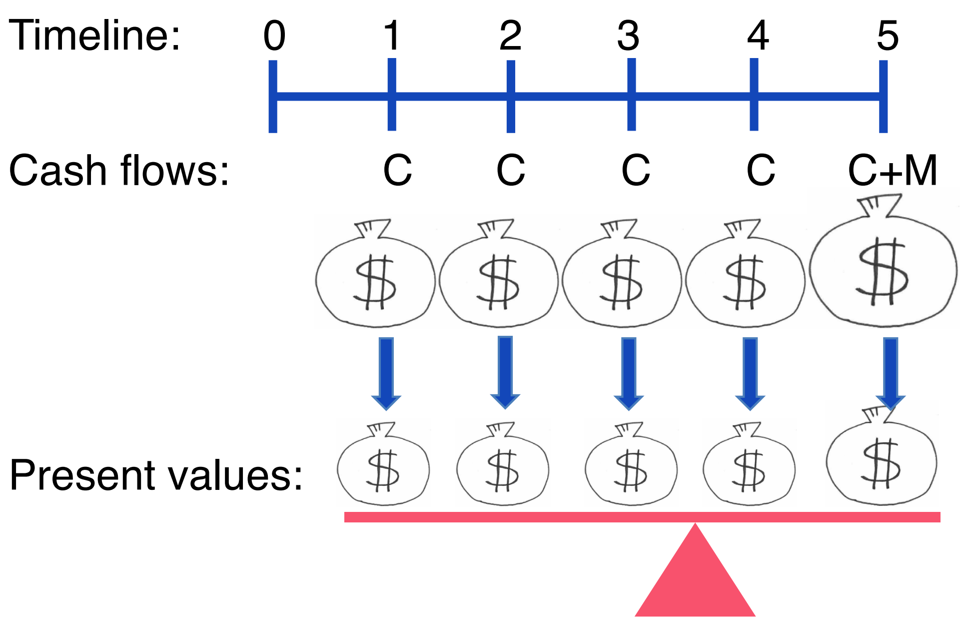
Property 1: A bond's duration is higher when the coupon rate is lower – 2

Special Case: For a zero-coupon bond, duration is the same as maturity.

- When the coupon rate is zero, the duration will be at its maximum.
- The maximum possible duration is the maturity itself.

Property 2:
A bond's duration is higher when the maturity is higher

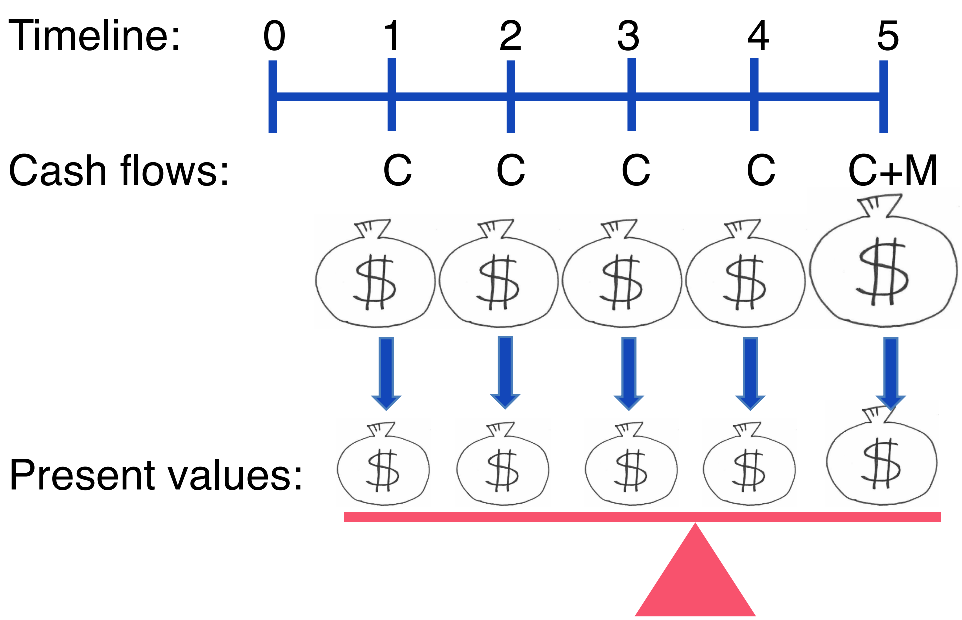
Intuition: There will be more cash flows far in the future.



Implication of Property 2:
Bond price is more sensitive to YTM changes when the maturity is higher.

Property 3:
A bond's duration is higher when YTM is lower

Intuition: If the YTM is lower, the future is more highly valued today.



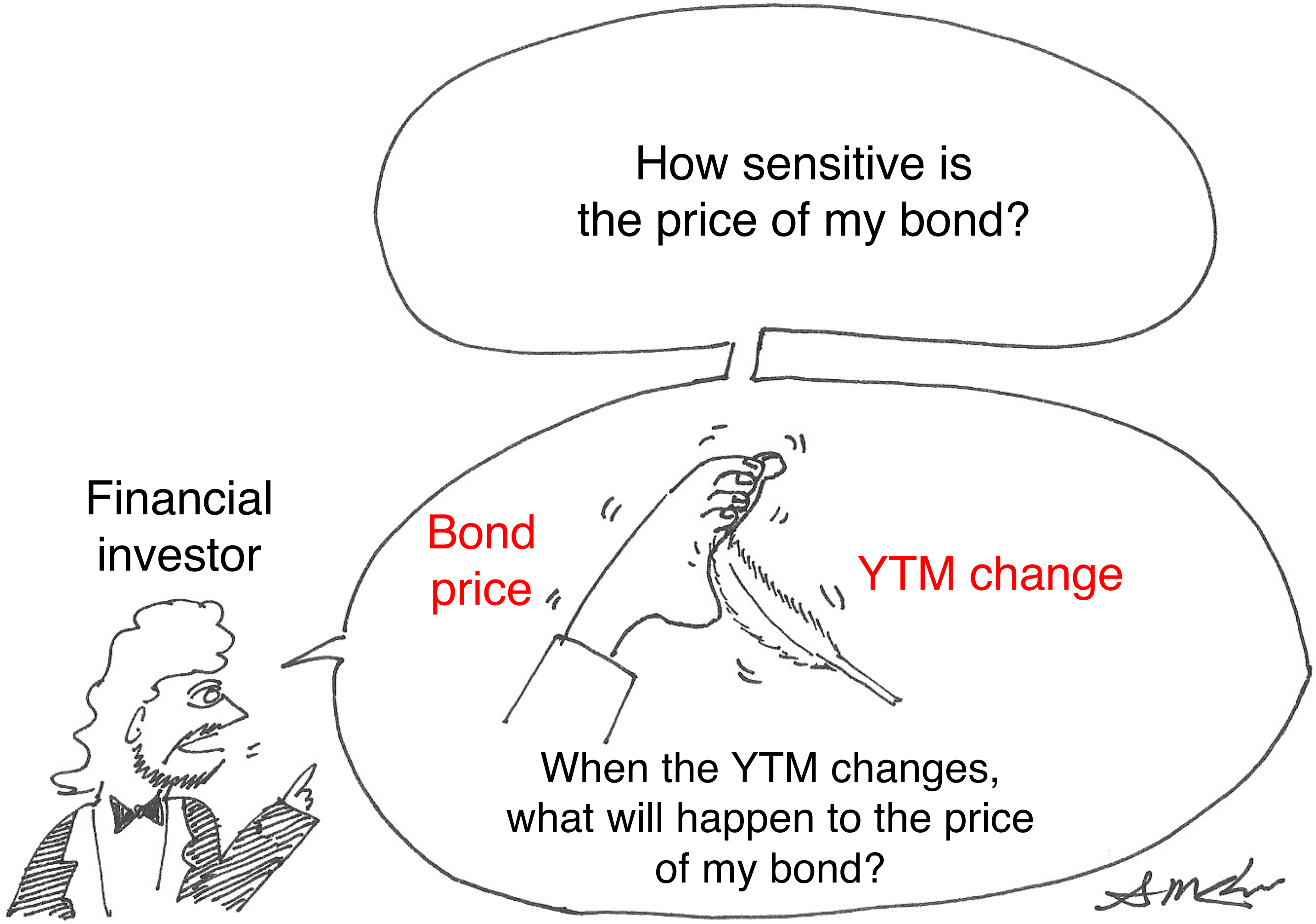
Implication of Property 3:
Bond price is more sensitive to YTM changes when YTM is lower. (The bond price is convex in YTM.)

Properties of Duration

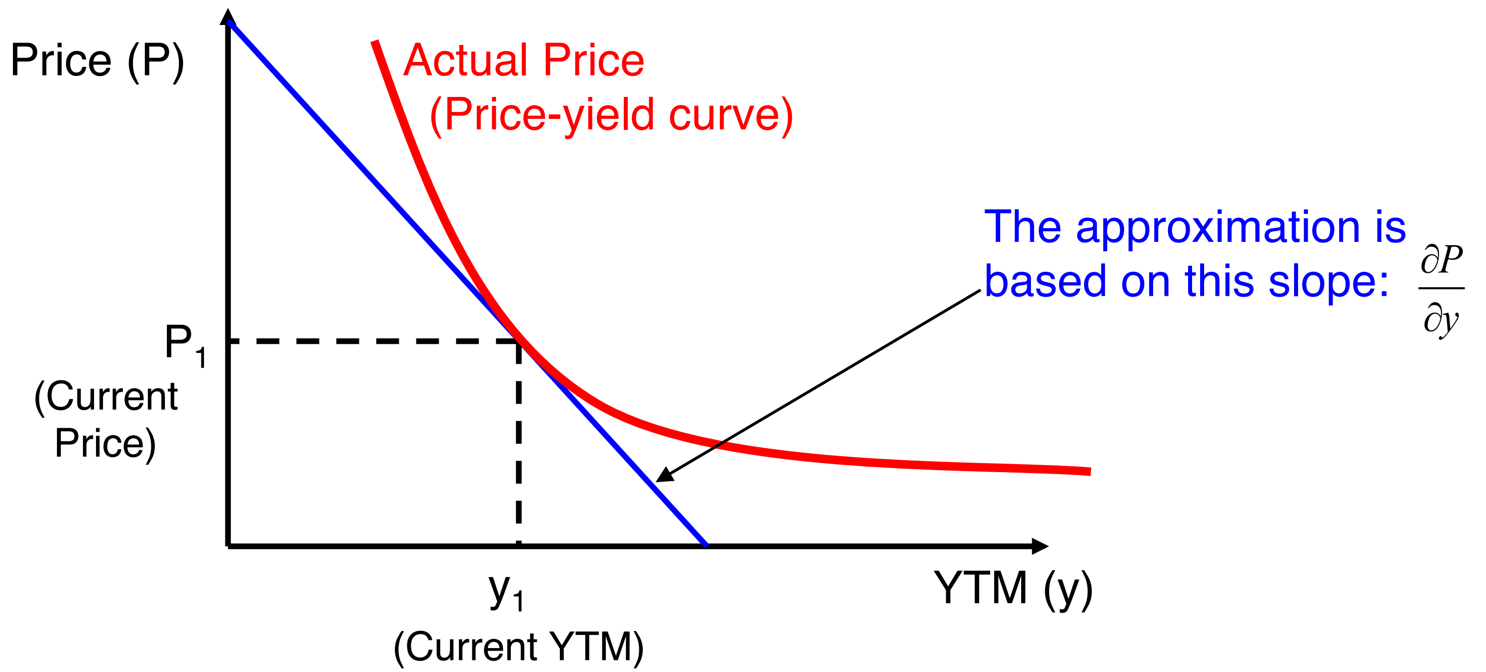
In Review:

- Prices of the bonds with lower coupon rates are ___________ sensitive to YTM changes than prices of the bonds with higher coupon rates.
- Prices of longer-term bonds are ___________ sensitive to YTM changes than prices of short-term bonds.
- Price is ________________ and ________________ in YTM.

Recall: The concept of “duration” is closely linked to the sensitivity of the bond price to YTM changes.


How sensitive is
the price of my bond?
Financial
investor
Bond
price
YTM change
When the YTM changes,
what will happen to the price
of my bond?

Price Approximation Using Duration: Geometric Analysis



The approximation using duration can be understood as a tangent line approximation.

Price Approximation Using Duration – 1

So how to obtain this slope? Recall

$$\frac{\partial P / P}{\partial y} = -\frac{D}{1+y}$$

Hence,

$$\frac{\partial P}{P} = -\frac{D}{1+y}\partial y.$$

This equation holds generally for very small changes in y, but can be used to approximate the (relative) price change in response to discrete changes in *y* (or *Δy*):

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y}\Delta y.$$

Price Approximation Using Duration – 2

Again, **duration (*D*)** satisfies the following approximation:

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y}\Delta y.$$

- That is, percentage change in price is approximated by *–D/(1+y)* multiplied by the change in YTM.
- Hence, *–D/(1+y)* measures the price sensitivity (semi-elasticity) to the change in YTM.
- *D/(1+y)* is called "modified duration" (*MD*). Hence,

$$\frac{\Delta P}{P} \approx -MD \times \Delta y.$$

Approximation Using Duration: An Example

If D is 5 (years), y is 3% per year, then

$$-\frac{D}{1+y} = -\frac{5}{1.03} = -4.85$$

Hence, 100-bps change in y (say from 3% to 4%) will make the following percentage change in price:

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y}\Delta y = -4.85 \times 0.01 = -4.85\%.$$

That is, the price will decrease by 4.85% (say from \$100 to \$95.15).

Review of Duration – 1

Question: What is the duration of a bond?

Answer:

#1 – The average of the time to the bond's promised cash flows.

- Higher when the coupon rate is lower.
- Higher when the maturity is higher.
- Higher when the YTM is lower.

Review of Duration – 2

Question: What is the duration of a bond?

Answer:

#2 – A measure of the price sensitivity (semi-elasticity) to a change in YTM.

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y}\Delta y.$$

Application of Duration – 1

Many financial institutions hold short-term liabilities (checking and savings accounts, certificates of deposit, etc.) and long-term assets (car loans, home mortgages, etc.).

Assets		Liabilities/Equity	
Assets	300 million (Duration: 5 years)	**Liabilities**	285 million (Duration: 3 years)
		Equity	15 million (Duration: 43 years)

Duration of a portfolio is the average of the durations of the portfolio's components weighted by the values. That is…

$D_{Portfolio} = D_1 \times P_1/(P_1+P_2) + D_2 \times P_2/(P_1+P_2)$.

$5 \times 300/(300-285) + 3 \times (-285)/(300-285) = 43$.

Note a minus sign in $P_2 = -285$ (because this is the amount *owed* by the bank)

Application of Duration – 2

Duration of 43 for a financial institution's equity?

- The Fed often changes the target federal funds rate by 25 bps.
- Using our approximation (assuming YTM is small), a **100-bps increase** in YTM will **reduce the bank's equity by 43%**.
  - 25-bps increase: By 11%.
  - With a just over 200-bps increase, the financial institution's equity could be completely wiped out.
- Regulators might take an action against the financial institutions operating at this level of duration mismatch (i.e., high duration for equity).

Immunization

When the durations of a firm's assets and liabilities are significantly different, the firm has a **duration mismatch**.

- The firm may attempt to eliminate the duration mismatch as much as possible.

Portfolio managers can "**immunize**" their portfolio from changes in YTM by setting duration to zero.

Immunization Exercise – 1

Assets		Liabilities/Equity	
Assets	300 million (Duration: 5 years)	**Liabilities**	285 million (Duration: 3 years)
		Equity	15 million (Duration: 43 years)

Bank XYZ would like to **reduce the duration** of its equity from 43 to **0**. That is, it wants to **immunize** its portfolio.

Immunization Exercise – 2

Summary:

Duration:

	Assets	Liabilities	Equity
Before selling	5.0	3.0	43.0
After selling	**2.9**	3.0	**0.0**

Bank XYZ reduced its risks. (Why?)

- However, Bank XYZ gave up the returns from mortgages.
- “Zero risk” is not (usually) optimal – Bank XYZ’s business is to achieve returns by taking some risks.

Immunization Exercise – 3

How to obtain the value "$80.63 million"? (That is, how can one know how much mortgages or any other assets to sell for immunization?)

Before selling:

43	=	$8 \times X/15$	+	$Y \times (15-X)/15$
(portfolio duration)		(mortgages to be sold)		(other assets and liabilities)

After selling:

0	=	0	+	$Y \times (15-X)/15$
(portfolio duration)		(cash)		(other assets and liabilities)

X = 80.625.

Limitations of Immunization

As the YTM changes, the duration of the portfolio changes.

- Maintaining an immunized portfolio requires continuous adjusting as the YTM changes.

A duration-neutral portfolio is protected when the YTMs for all maturities change by the same percentage point (i.e., when there is a parallel shift in the yield curve).

Immunization is costly.

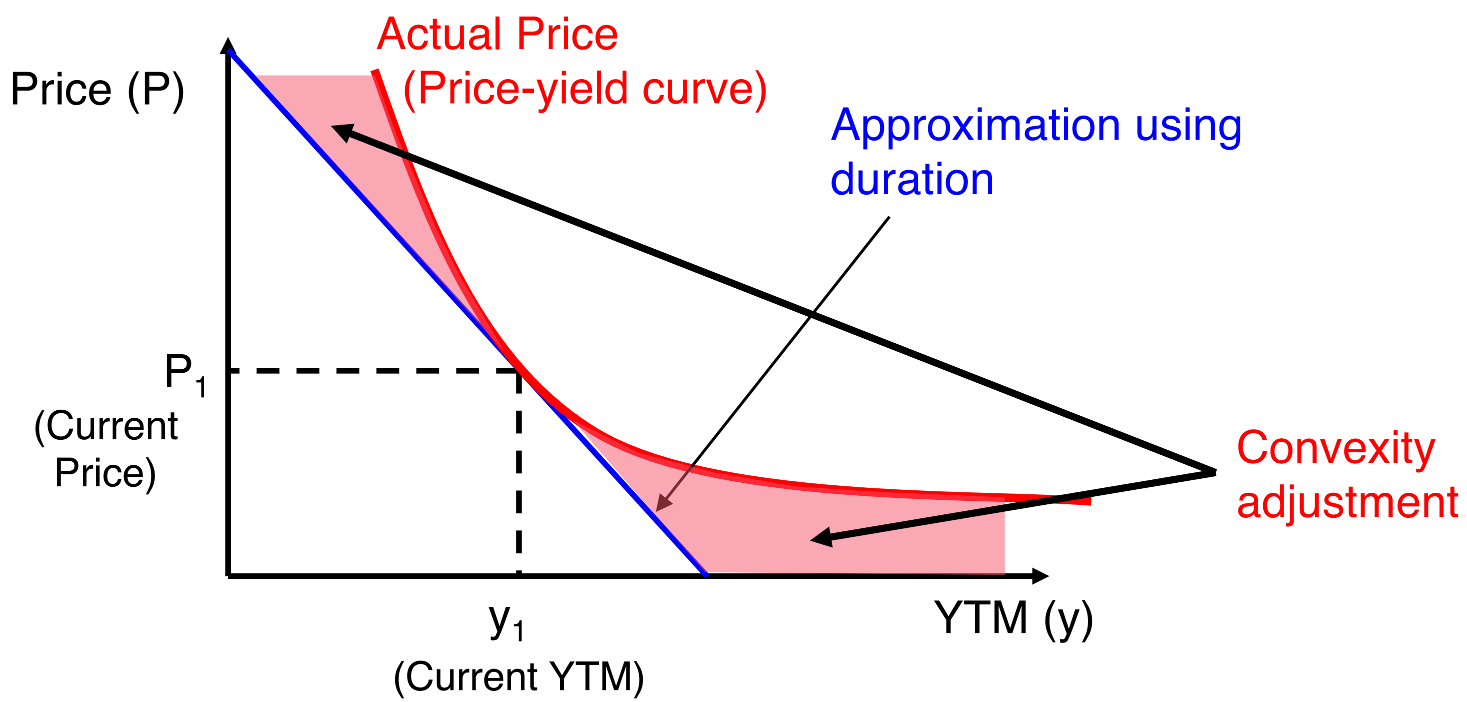
- In our example, exchanging mortgages for cash entails giving up future revenue.

Convexity:
Motivation – 1

The approximation using duration can be improved.

- This is a linear approximation.
- The actual price is higher than the price approximate with duration.

Convexity: Motivation – 2



Approximation using duration fails to give accurate account of the impact of large YTM changes on price.

Convexity – 1

A measure of **convexity (*C*)** defined:

$$C = \frac{1}{P(1+y)^2} \sum_{t=1}^{T} \left[\frac{CF_t}{(1+y)^t} t(1+t) \right].$$

P = bond price
CF_t = cash flow at t
y = YTM
T = maturity

Then, one can show

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y} \Delta y + \underbrace{\frac{1}{2} C (\Delta y)^2}_{\text{"Convexity Adjustment"}}.$$

Convexity – 2

Improves the approximation for the price change by considering the fact that the price-yield relationship is convex.

The adjustment is always positive. This reflects that the actual price-yield curve is above the straight line.

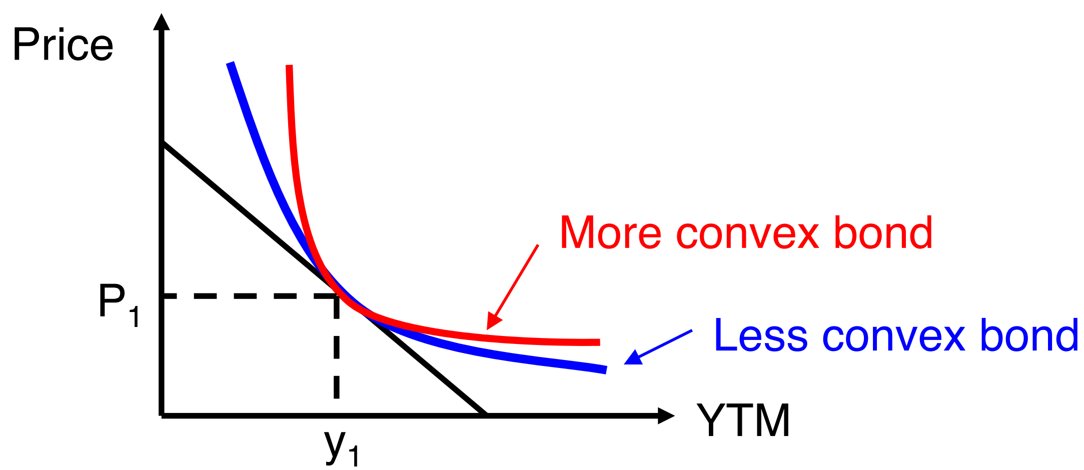
Convexity of a Portfolio

The convexity of a portfolio of bonds is the weighted average of the convexities of the bonds in the portfolio.

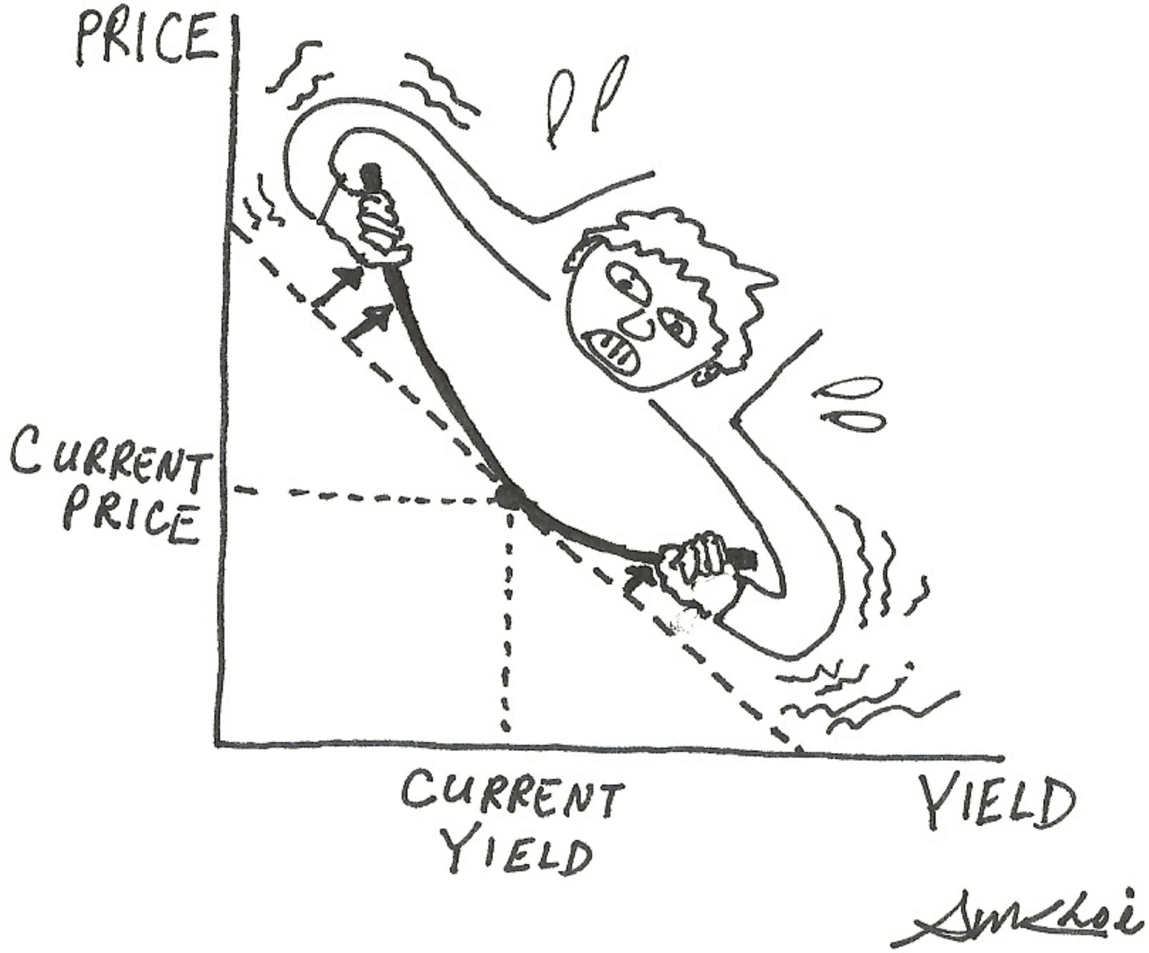
- The weights are the values of the individual bonds over the total value of the portfolio.

Comparing Bonds with Different Convexity

A bond with higher convexity will have a higher price than otherwise equal bonds when the YTM changes, regardless of whether the YTM rises or falls.



A **Barbell strategy** takes advantage of **increased convexity** by duplicating the duration of an existing bond using a portfolio with one shorter-term bond and one longer-term bond.


PRICE
CURRENT PRICE
CURRENT YIELD
YIELD

Barbell Strategy: An Example – 1

Issue	YTM (%)	Duration	Convexity
2-year	0.55	1.91	0.05
5-year	1.40	4.73	0.25
10-year	2.00	8.82	0.87

April 29, 2015, U.S. Treasuries. Source: Bloomberg.

Suppose an investor holds $1-million value of the 5-year Treasury note.

- This could be sold and used to purchase a portfolio of \$*X*-million value of the 2-year note and \$(1-*X*)-million value of the 10-year note.

Barbell Strategy: An Example – 2

Issue	YTM (%)	Duration	Convexity
2-year	0.55	1.91	0.05
5-year	1.40	4.73	0.25
10-year	2.00	8.82	0.87

Now we find X that (approximately) equalizes the durations between 5-year note and this portfolio:

$$1.91 \times X + 8.82 \times (1 - X) = 4.73$$

Then, $\boldsymbol{X=0.592}$.

The convexity of the portfolio is

$$0.05 \times 0.592 + 0.87 \times (1 - 0.592) = \mathbf{0.38}$$

Barbell Strategy:
An Example – 3

Issue	YTM	Duration	Convexity
2-year	0.55	1.91	0.05
5-year	1.40	4.73	0.25
2-year (59%), 10-year (41%)		4.73	0.38
10-year	2.00	8.82	0.87

In a previous module, we discussed how to obtain the YTM of a portfolio.

- This is not just an average of the YTMs of the bonds in the portfolio. You will need detailed information on coupons to construct the cash flows.
- A more complete comparison can be made by considering the YTM in addition to duration and convexity.

Barbell Strategy:
An Example – 4

Situation: You sold a 5-year bond to construct a Barbell portfolio.

Convexity has increased while duration remains the same. Hence, this Barbell portfolio is preferred.

In response, you are likely to sacrifice something else for this Barbell strategy. (For example, you might have to accept a lower YTM.)

Module Wrap-Up – 1

YTM change adds uncertainty to bond prices.

- Price is decreasing and convex in YTM.
- The sensitivity of price to YTM changes can be understood using the price-yield curve.

But the concepts of duration and convexity help to understand the bond price sensitivity to YTM changes.

Module Wrap-Up – 2

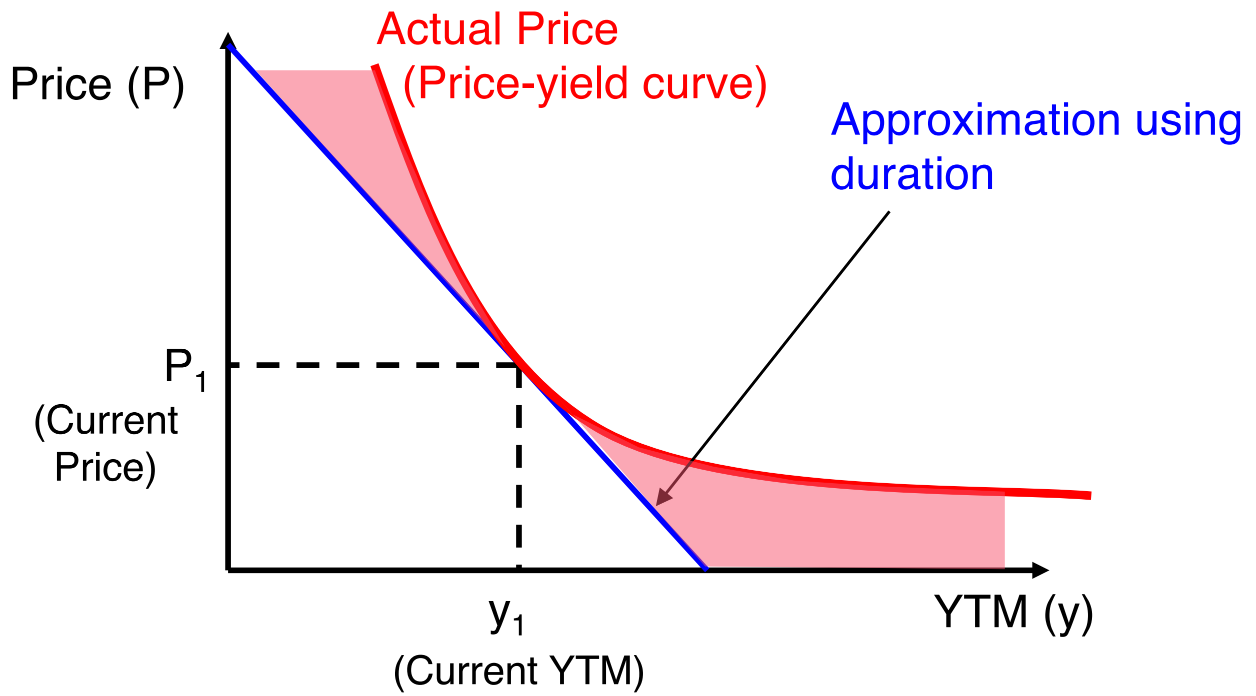
(1.) **Duration:**

$$D = \sum_{t=1}^{T} t W_t, \qquad W_t = \frac{CF_t}{(1+y)^t} / P.$$

Duration approximates the price sensitivity to YTM changes.

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y} \Delta y.$$

Module Wrap-Up – 3



Module Wrap-Up – 4

(2.) **Convexity:**

Convexity adjustment improves the approximation of the price sensitivity to YTM changes.

$$\frac{\Delta P}{P} \approx -\frac{D}{1+y}\Delta y + \underbrace{\frac{1}{2}C(\Delta y)^2}_{\text{"Convexity Adjustment"}}.$$

Module Wrap-Up – 5

Immunization and the **Barbell Strategy** can be understood as applications of duration and the convexity measure.

The main idea of… **Immunization**	"High duration is risky because it implies a high price sensitivity to YTM changes."
The main idea of… **Barbell strategy**	"High convexity is good because the price for bonds with higher convexity is higher when the YTM changes (than for bonds with lower convexity, other things being equal)."