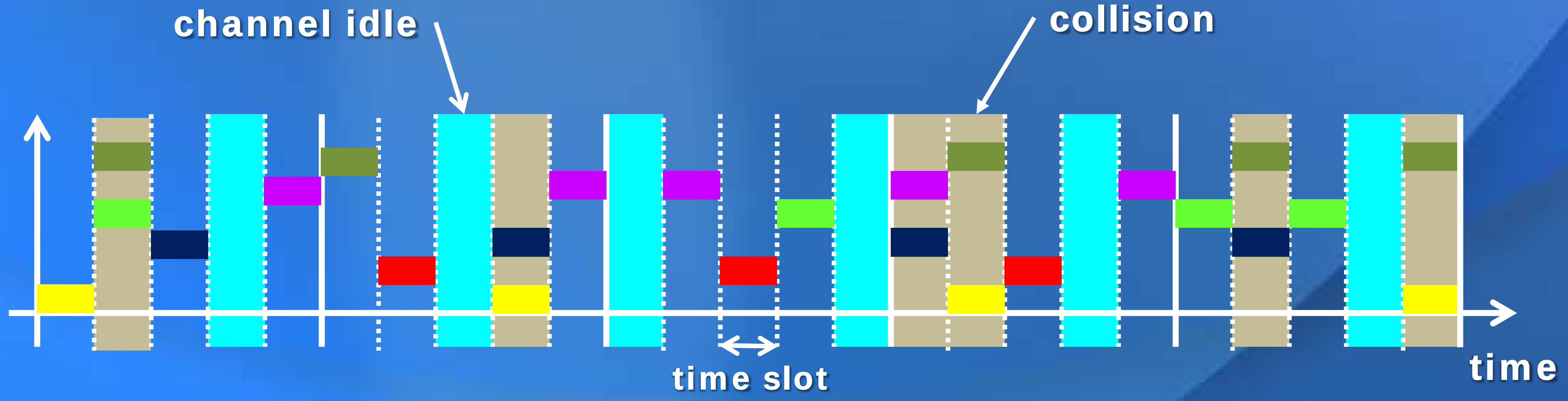
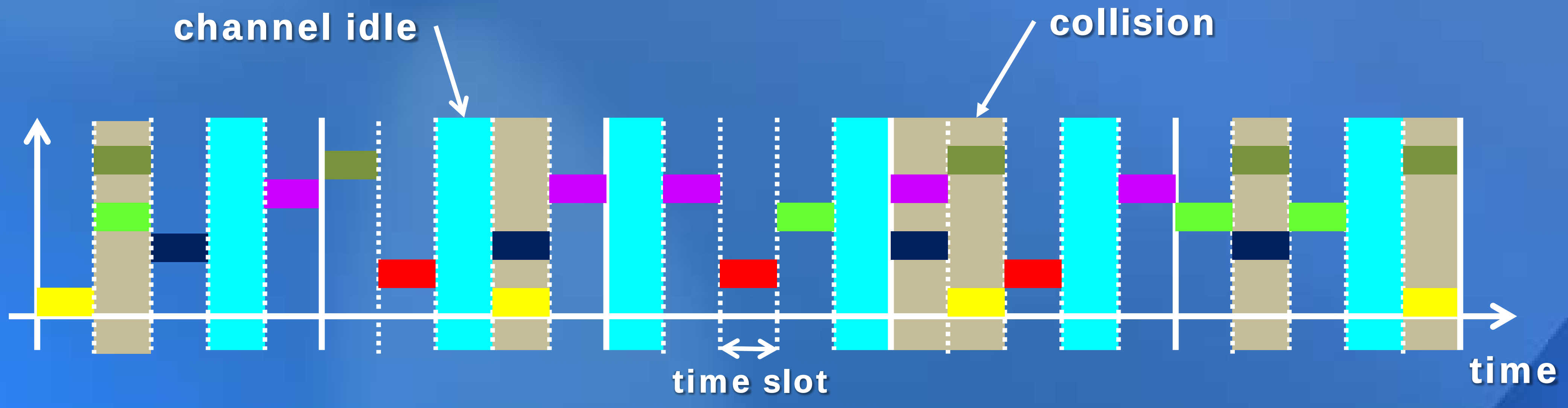
Efficiency of Slotted Aloha

Assumptions

Time divided into slots equal to the length of one packet.
Packets transmitted starting only at the start of each slot.
If collision, both packets are lost.



Efficiency



Efficiency: The fraction of time slots with a successful transmission (13/25 above).

Throughput: The number of packets successfully transmitted per time interval (depends upon slot length).

Slotted Aloha Assumptions

- Nodes operate independently.
- Each node has a queue to hold packets for transmission.
- If the queue is non-empty, the node is backlogged.
- Backlogged nodes send packets in each time slot with probability p.
- N nodes are backlogged

Slotted Aloha Efficiency

If N nodes are backlogged, and each transmits packets with probability p in each time slot:

probability a node transmits

probability a node does not transmit

Efficiency: $$E = N \cdot p \cdot \underbrace{(1-p)^{N-1}}_{\text{probability (N-1) nodes do not transmit}}$$

number of backlogged nodes

Maximum Efficiency

Differentiating with respect to p,

$$E = N \cdot p \cdot (1-p)^{N-1}$$

$$\frac{dE}{dp} = N(1-p)^{N-1} - N(N-1)p(1-p)^{N-2}$$

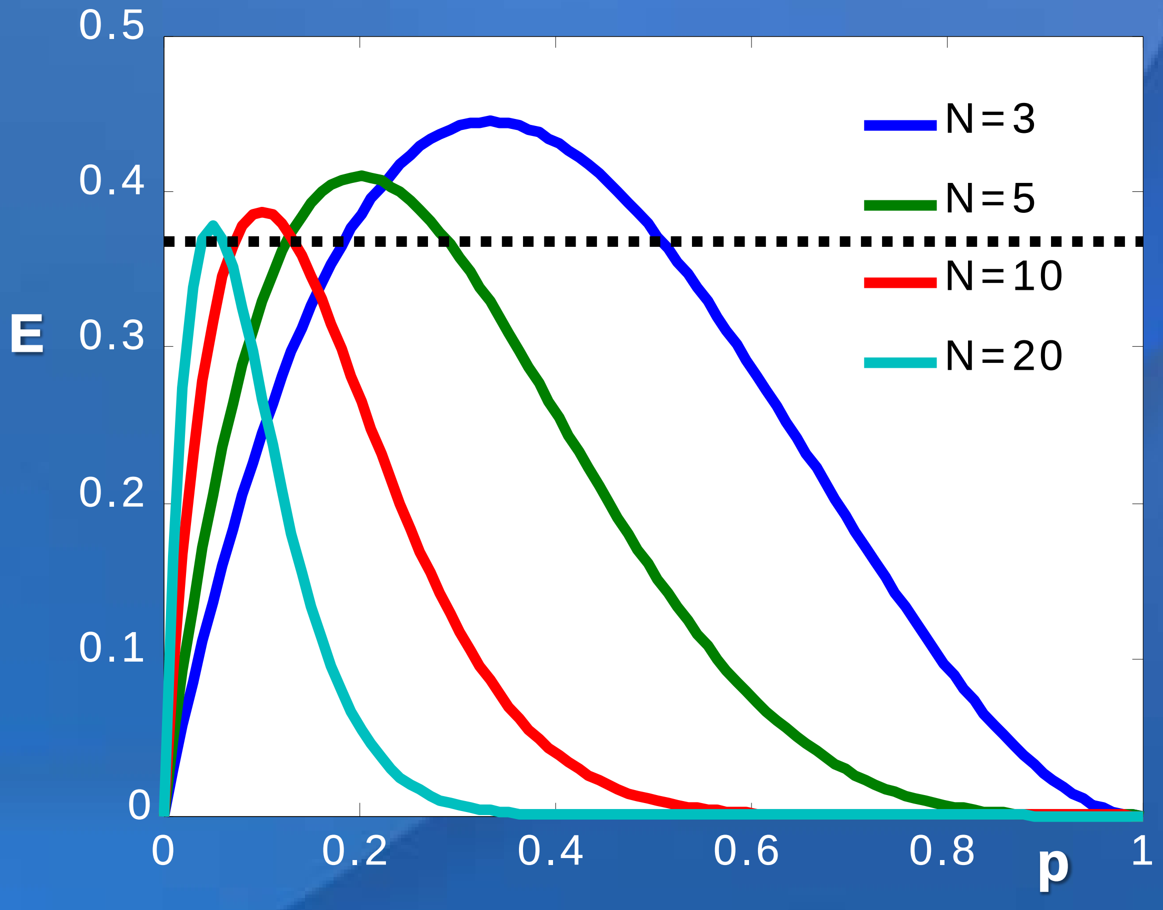
$$0 = (1-p) - (N-1)p$$

$$0 = 1 - Np$$

$$p = \frac{1}{N}$$

$$E_{max} = \left(1 - \tfrac{1}{N}\right)^{N-1} \xrightarrow[N\to\infty]{} e^{-1} \approx 37\%$$

Channel transmitting successfully only 37% of the time!



Stabilizing Aloha

To achieve maximum efficiency, each node must set $p=1/N$, where N is the number of backlogged nodes.

How can it set p without centralized information?

By adjusting its value of p up and down, depending upon the number of collisions.

- Collision $\rightarrow$ too much traffic $\rightarrow$ adjust p downwards.
- Successful transmission $\rightarrow$ adjust p upwards.

Binary Exponential Backoff

Given initial guess of p.

If no transmit,	no change in p
If collision,	$p = \max(p_{min}, p/2)$
If transmit OK,	$p = \min(2p, p_{max})$

These updates ensure that $0 < p_{min} \leq p \leq p_{max} < 1$.