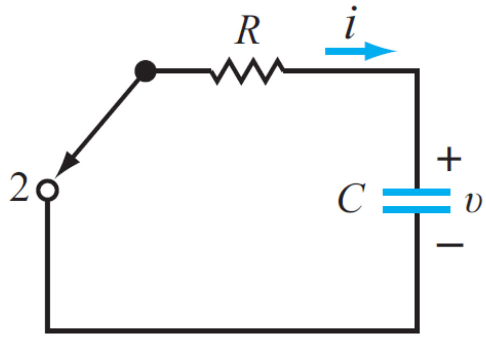
Natural Response of an RC Circuit

Consider the circuit below. Assume we know that the capacitor, C, has an initial voltage v(0) across it. What is the voltage, v, across C, for t ≥ 0?



Applying KVL, we can write:

$$Ri + \upsilon = 0$$

Substituting the i-v relation for a capacitor, we obtain:

$$RC\,\frac{d\upsilon}{dt} + \upsilon = 0$$

We can clean this up a bit by dividing by RC:

$$\frac{d\upsilon}{dt} + a\upsilon = 0$$

Where:

$$a = \frac{1}{RC}.$$

This is a differential equation. It turns out the solution to the differential equation in the blue box above is:

$$\upsilon(t) = \upsilon(0)\, e^{-t/\tau}$$

where:

$$\tau = RC \qquad \text{(s)}$$

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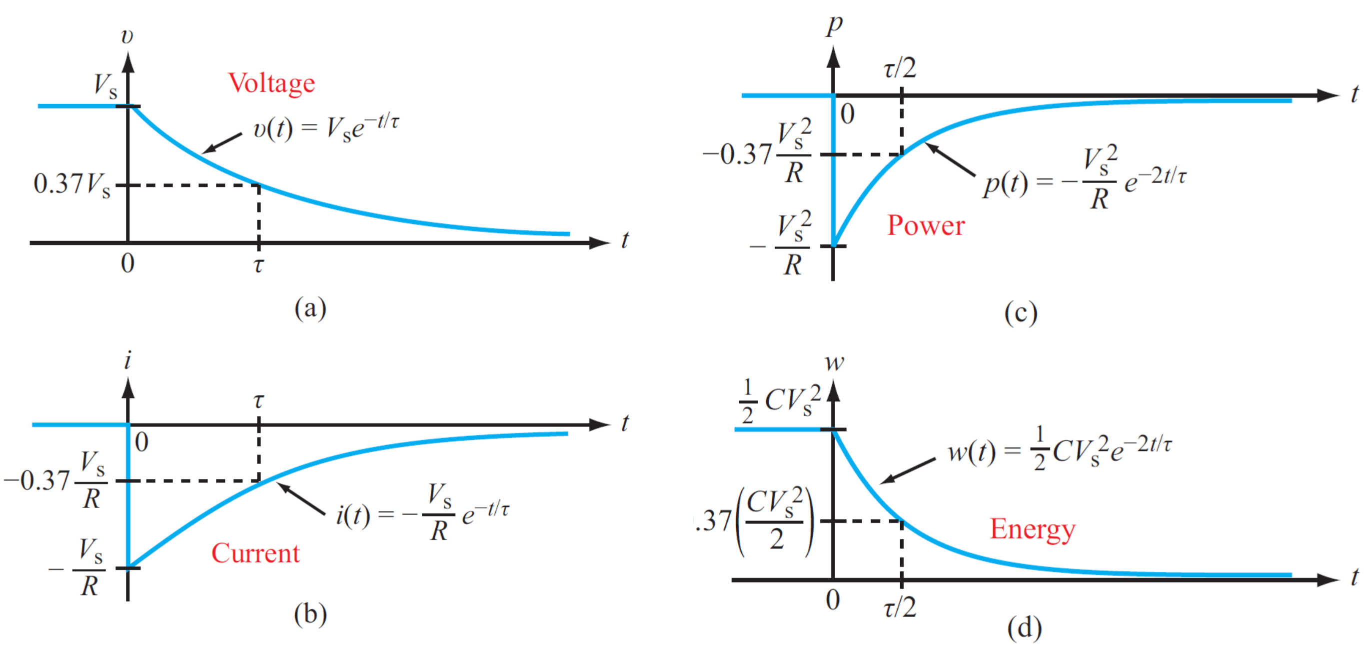
Once we know the voltage, v, we can also determine:

- the current, i (since i = C*dv/dt for a capacitor)
- the power being absorbed or injected by the capacitor (since P = i*v)
- the energy stored in the capacitor at any time (since U = ∫P or ½*Cv^2)

All four variables are plotted below for this circuit.



What does the *time constant*, τ, tell us?

The magnitude of the time constant τ is a measure of how fast or how slowly a circuit responds to a sudden change.

- Notice that the units of τ are seconds (that is ohms * farads = seconds).
- Notice in figure (a) above, that after 1 τ, the capacitor has discharged to 0.37 of the initial value.
- After about 5τ, v(t) has dropped to <1% of its original value. Engineers assume 5τ is long enough for particular RC circuit to charge or discharge to its final value.