

Capacitors in Series and Parallel

Capacitors in Series

The capacitors on the top left share the same current, i_s , which is related to their individual voltages by

$$i_s = C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt} = C_3 \frac{dv_3}{dt}$$

Note also,

$$v_s = v_1 + v_2 + v_3$$

Now for the circuit on the bottom, it must be true that

$$\begin{aligned} i_s &= C_{\text{eq}} \frac{dv_s}{dt} \\ &= C_{\text{eq}} \left(\frac{dv_1}{dt} + \frac{dv_2}{dt} + \frac{dv_3}{dt} \right) \\ &= C_{\text{eq}} \left(\frac{i_s}{C_1} + \frac{i_s}{C_2} + \frac{i_s}{C_3} \right) \end{aligned}$$

which leads to

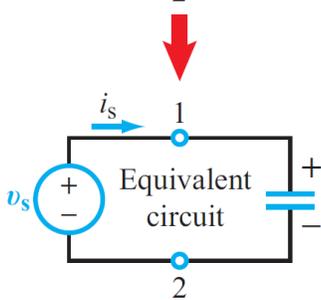
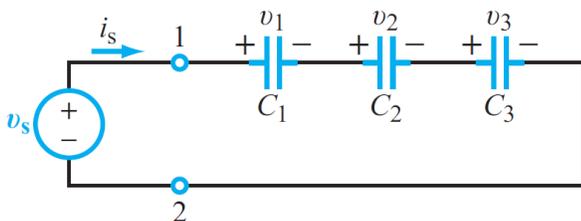
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For the general case of N capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \sum_{i=1}^N \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

(capacitors in series).

Combining In-Series Capacitors



$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$$

Capacitors in Parallel

The three capacitors on the top left are connected in parallel. Hence, they share the same voltage v_s , and the source current i_s is equal to the sum of their currents,

$$\begin{aligned} i_s &= i_1 + i_2 + i_3 \\ &= C_1 \frac{dv_s}{dt} + C_2 \frac{dv_s}{dt} + C_3 \frac{dv_s}{dt}. \end{aligned}$$

Now for the circuit on the bottom, it must be true that

$$i_s = C_{\text{eq}} \frac{dv_s}{dt}$$

Equating the expressions leads to

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

For the general case of N capacitors in parallel:

$$C_{\text{eq}} = \sum_{i=1}^N C_i \quad (\text{capacitors in parallel}).$$

