Welcome to MATH226.3x: Nonlinear Differential Equations: Order and Chaos. This syllabus provides a general description of the course content, the schedule, the assessments and grading, and general guidelines. Please check the syllabus if you have any questions regarding the operation of this course.

Nonlinear Differential Equations

Phenomena as diverse as an automobile's suspension system, the swaying of a bridge, and the damping of a skyscraper are governed by differential equations. MATH226x is an introduction to the mathematical theory of ordinary differential equations. This course adopts a modern dynamical systems approach to the subject. That is, equations are analyzed using qualitative, numerical, and if possible, symbolic techniques.

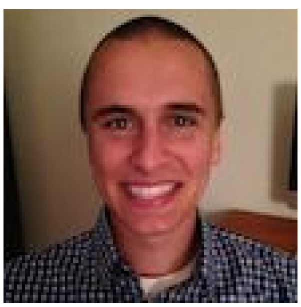
MATH226.3x is the capstone course in the three-MOOC sequence of courses that together are MATH226x. We will apply the theory and relevant techniques from MATH226.1x and MATH226.2x to the study of nonlinear systems of ordinary differential equations. There is no systematic approach that applies to all nonlinear systems of differential equations, but in this course we present a handful of techniques that address a wide range of systems.

We do not hesitate to sacrifice rigor for intuition. We are able to discuss advanced topics such as linearization and Hamiltonian systems by ignoring certain technical details and by restricting our discussion to autonomous systems with two dependent variables. In the final module of the course, we discuss a three-dimensional system that was first studied by Edward Lorenz at MIT in the early 1960s. In that discussion, we introduce some of the terms and concepts related to the study of "chaotic" dynamical systems. Overall our treatment in 226.3 will be less systematic than it was in 226.1 and 226.2, but we hope that it will persuade you to continue your study of nonlinear differential equations after completing this course.

About the Team

Paul Blanchard is professor of mathematics at Boston University. He grew up in Sutton, Massachusetts, USA, and spent three undergraduate years at Brown University. During his senior year, he decided to have an adventure and learn a new language, so he was an occasional student at the University of Warwick in England. He received his Ph.D. from Yale University. He has taught mathematics for more than thirty-five years, most at Boston University. His main area of mathematical research is complex analytic dynamical systems and the related point sets---Julia sets and the Mandelbrot set. He is a Fellow of the American Mathematical Society.

For many of the last twenty years, his efforts have focused on modernizing the traditional sophomore-level differential equations course. That effort has resulted in numerous workshops and minicourses. He has also authored five editions of *Differential Equations* with Robert L. Devaney and Glen R. Hall. When he becomes exhausted fixing the errors made by his two coauthors, he heads for the golf course to enjoy a different type of frustration.



Patrick Cummings is a Ph.D. candidate in the Department of Mathematics and Statistics at Boston University. His research involves extending the theory of finite dimensional dynamical systems to infinite dimensional dynamical systems defined by partial differential equations. Patrick received his Bachelor of Arts degree in Mathematics from Marist College in 2012. While at Boston University, he has been a teaching assistant for MA226, the residential equivalent of MATH226x.

Course Outline

Module	Content
Module 1: Nonlinear Systems Released on Thursday, July 30 at 1:00PM EDT	We introduce four important examples of nonlinear systems. These examples illustrate many important concepts and will reappear throughout the course.
Module 2: Equilibrium Point Analysis Released on Thursday, July 30 at 1:00PM EDT	In MATH226.2x, we were able to understand the solutions of linear systems both qualitatively and analytically. Unfortunately, nonlinear systems are in general much less amenable to the analytic and algebraic techniques that we have developed, but we can use the mathematics of linear systems to

	understand the behavior of solutions of nonlinear systems near their equilibrium points.
Module 3: Qualitative Analysis Released on Thursday, August 6 at 1:00PM EDT	The process of linearization discussed in Module 2 gives us a powerful technique for understanding the behavior of solutions of a nonlinear system near an equilibrium point. Unfortunately it provides "local" information only—information that can be used only near equilibrium points. So far our only general techniques for the study of the behavior of nonlinear systems away from equilibrium points are numerical. In this module we develop qualitative techniques that can be used in combination with linearization and numerics.
Module 4: Hamiltonian Systems Released on Thursday, August 6 at 1:00PM EDT	Nonlinear systems of differential equations are almost impossible to solve explicitly. The solution curves of systems behave in many different ways, and there are no qualitative techniques that are guaranteed to work in all cases. Fortunately there are certain special types of nonlinear systems that arise often in physical systems and for which there are special techniques that enable us to gain some understanding of the phase portrait. In this module we discuss one of the most important of these special types.
Module 5: Dissipative and Gradient Systems Released on Thursday, August 13 at 1:00PM EDT	The Hamiltonian systems discussed in Module 4 are idealized systems. In this module we discuss systems for which there is a quantity that dissipates over time.
Module 6: Chaos and the Lorenz Attractor Released on Thursday, August 20 at 1:00PM EDT	In 1963 Edward Lorenz published a paper that would eventually have a profound effect on the mathematical analysis of nonlinear equations. We end MATH226x with a discussion of the system that now carries his name. We will also attempt to convey its place in the development of the mathematics that underlies chaos theory.
Final Exam Released on Thursday, August 27 at 1:00PM EDT Due on Thursday, September 3 at 1:00PM EDT	This exam will test all topics presented in this course and will be worth 60% of your overall grade.
End of Course Monday, September 3 at 1:00PM EDT	The course officially ends at this time. The content will still be available after the course closes, but those seeking a certificate must achieve an overall grade of 50% by this date.

Assessments and Grading

Each module consists of a series of videos interspaced with brief exercises designed to help you assess your understanding of the material discussed in the video. These "content check" exercises will be worth 10% of your overall grade.

At the end of each module there will be an exercise set that will provide more detailed practice with the concepts presented in the module. These exercise sets will be worth 30% of your overall grade.

The final exam for the course will be released on Thursday, August 27 at 1:00 PM (EDT). It will cover all of the material discussed in all six modules. To receive credit, you must submit your answers by September 3 at 1 PM (EDT). The final exam will be worth 60% of your overall grade.

The deadline for all assessments will be the end of the course, that is, September 3 at 1 PM (EDT). You may delay completion of the content check exercises and exercise sets until the end of the course while still getting credit. However, we strongly recommend that you complete all exercises as you go.

Discussion Forum Guidelines

We hope that you find the discussion forums to be a useful component of this course. They are meant to be an area where the students can interact with each other, ask questions, or talk to the course staff. We greatly encourage you to use these forums on a regular basis.

We support and encourage the use of the forum to discuss or ask questions about exercises and consequently their solutions. We will not delete questions or discussions that contain solutions; however, we do ask that you do not abuse the forums as a way to share answers to exercises.

We ask that you do not post comments that are derogatory, defamatory, or in any way attack other students. Be courteous and show the same respect you hope to receive. Discussion forum moderators will delete posts that are rude, inappropriate, or off-topic. Commenters who repeatedly abuse this public forum will be removed from the course.

There is a feature in the discussion forums that allows you to select from two post types, Question and Discussion. The Question type is meant for specific issues with the platform or with content, and the Discussion type is meant to share ideas and start conversation. Please keep this distinction in mind when posting to the discussion forum.

FAQ

Q: Should I email the professor or any persons involved with this course directly?
A: No. If you feel the need to contact the course staff involved in this course, please do so through the Discussion Forum.

Q: Do I need to buy any personal materials to take this course?
A: No. You do not need to purchase textbooks or any materials to aid you in completing the course.

Q: I've never taken an edX course before and this is confusing. What do I do?
A: There is a pre-course edX walkthrough that beginners can watch. It explains in detail how to use the edX platform. For further information, please visit the demo edX course.

Q: I found a mistake in the course. Where do I report it?
A: On the Wiki page, there is a specific section for "Errata." You can go there, edit the page, and post information concerning any errors or issues you have found. We will try to fix them as soon as possible.

Q: How do I learn more about the mathematics discussed in Module x?
A: Many of the modules discuss topics that can be studied in much more detail. If you find a topic especially interesting and would like to know more, then please post a question on the discussion forum. If we know of a good reference or resource, then we will post it on the wiki.

Time Zones

A note about time references: Time will be reported by course staff as Eastern Daylight Time, North America (EDT). Any times listed by edX, such as due dates listed on the course site, will be reported in Universal Time Coordinated (UTC). The course staff will make every effort to make times and time zones as clear as possible. There are various time zone converters on the web such as http://www.timeanddate.com/worldclock/converter.html.

Honor Code

The edX platform assumes a certain level of decorum and responsibility from those taking this course. Please review the edX Honor Code, which is reproduced below.

By enrolling in an edX course, I agree that I will:

- Complete all mid-terms and final exams with my own work and only my own work. I will not submit the work of any other person.
- Maintain only one user account and not let anyone else use my username and/or password.
- Not engage in any activity that would dishonestly improve my results, or improve or hurt the results of others.
- Not post answers to problems that are being used to assess student performance.

Unless otherwise indicated by the instructor of an edX course, learners on edX are encouraged to:

- Collaborate with others on the lecture videos, exercises, homework and labs.
- Discuss with others general concepts and materials in each course.
- Present ideas and written work to fellow edX learners or others for comment or criticism.

Credits and Acknowledgements

As with any major effort, this course would not be possible without large contributions from many sources. We would like to extend a special thanks to the various teams who have put in uncountable hours of work to help create this course. Specifically, we want to thank the following people and organizations that have contributed a large amount of effort to make this course become a reality: Romy Ruukel, Tim Brenner, Vanessa Ruano for managing this process and being responsible for every aspect of the making of this course; Joe Dwyer for editing the annotated slide videos that appear in this course and for filming and editing the about video; Kellan Reck for his graphics in the about video; Courtney Teixeira who drew the images on the title cards; Andrew Abrahamson and Adam Brilla of BU's Metropolitan College who helped us with our tablet capture in their media room; Kacie Cleary of BU's Information Services and Technology who helped us with tablet capture in Mugar Memorial Library; Daniel Shank for accuracy checking; Professor John Polking of Rice University for letting us use his program pplane in this course; MathWorks for providing licenses for MATLAB during the course; John Kotwicki, Brandon Armstrong, and especially Erin Byrne of MathWorks for their assistance with MATLAB; Hubert Hohn who worked with us designing and implementing DETools, software that we use when we teach differential equations; Cengage Learning for providing partial support during the development of DETools; and the Digital Learning Initiative and the Department of Mathematics and Statistics at Boston University for supporting Paul Blanchard and Patrick Cummings during the development of this course.

This course would not have been possible if the National Science Foundation had not partially funded the Boston University Differential Equations Project from 1993 to 1998.

Many undergraduate and graduate students have worked on the BU Differential Equations Project over the years: Gareth Roberts, Alex Kasman, Brian Persaud, Melissa Vellela, Sam Kaplan, Bill Basener, Sebastian Marotta, Stephanie R. Jones, Adrian Vajiac, Daniel Cuzzocreo, Duff Campbell, Lee Deville, J. Doug Wright, Dan Look, Nuria Fagella, Nick Benes, Adrian Iovita, Kinya Ono, and Beverly Steinhoff.

Paul Blanchard would especially like to thank his colleagues and coauthors, Robert L. Devaney and Glen R. Hall, for many years of enjoyable collaboration on the development of materials used to teach differential equations.

Terms of Service

For further information, please review the edX Terms of Service (https://www.edx.org/edx-terms-service).